Distributed quantum computation with superconducting qubit via *LC* circuit using dressed states*

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A scheme is proposed where two superconducting qubits driven by a classical field interacting separately with two distant *LC* circuits connected by another *LC* circuit through mutual inductance, are used for implementing quantum gates. By using dressed states, quantum state transfer and quantum entangling gate can be implemented. With the help of the time-dependent electromagnetic field, any two dressed qubits can be selectively coupled to the data bus (the last *LC* circuit), then quantum state can be transferred from one dressed qubit to another and multi-mode entangled state can also be formed. As a result, the promising perspectives for quantum information processing of mesoscopic superconducting qubits are obtained and the distributed and scalable quantum computation can be implemented in this scheme.

Keywords: superconducting qubit, quantum state transfer, quantum entangling gate

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1. Introduction

In recent years, much attention has been paid to quantum computers, which are based on the superposition principle of quantum mechanics. This type of new machines can solve some problems much more effectively than classical computers. So much interest and effort have been devoted to the construction of the two-quantum-bit gates for quantum computers. Such gates have been demonstrated in cavity quantum electrodynamics (cavity QED),[1] ion trap,[2] and nuclear magnetic resonance (NMR)[3] experiments, and its fidelity was essential with the time going on, which must exist for which the qubits are decoupled from the modes at a high degree of accuracy.

Recently, the schemes have been proposed for quantum communication between two atoms trapped in distant optical cavities connected by an optical fibre. In Ref. [4], Serafini *et al.* suggested a scheme for realizing the quantum gates between two two-level atoms in distant optical cavities coupled by an optical fibre. The scheme is based on the Rabi oscillation of the whole system composed of atoms, cavity modes, and fibre modes. Yin and Li[5] generalized the idea to the multi-atom case. Because what are studied in the above articles can function only in the limit case where the atom–cavity coupling is much smaller than the cavity–fibre coupling, so the scheme[6] beyond that limit needs to be investigated. But due to the de-coherence arising from the "bus" and the environment, it is substantially difficult to meet the fault–tolerant requirement. In cavity QED, the main obstacles are cavity decay, fibre length, and atomic spontaneous emission. Based on those cases and practically realizable experiments, superconducting qubits coupled by a data bus are promising candidates for distributed quantum computation.

In Ref. [7], the interaction between a single superconducting flux qubit and an *LC* circuit has been experimentally realized. The Hamiltonian is mathematically identical to that of the Jaynes–Cumming model, in which the *LC* circuit acts as a quantum harmonic

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oscillator. Meanwhile two theoretical approaches[8,9] by using the time-dependent electromagnetic field (TDEF) have been proposed to control the coupling between two qubits. In Ref. [10], according to the "dressed qubits" proposal,[8] a coupling (decoupling) method between a superconducting and an LC circuit, in which a controllable TDEF is used, has been studied. By choosing appropriate parameters for the TDEF, the dressed qubits formed by qubit plus the TDEF, can be coupled to the LC circuit and thus the LC circuit can exchange information with the assistance of the TDEF.

However, we have noted that the above schemes have some drawbacks. In Refs. [4]–[10], the authors restrained their investigations within the short-fibre regime in order to achieve a strong coupling between the cavities. This would be a serious obstacle for the remote transmission of the photon. On the other hand, it also requires ultrahigh-Q cavities ($\approx 10^{11}$) to suppress the leakage of photons, which is not easy to achieve in experiment. What is more, their schemes for realizing logical gates are based on the control of the interaction time, and so it is difficult to achieve precise control. In order to overcome these shortages mentioned above, in this paper, according to the ideas of atom–cavities-fibre[4−6] and Refs. [7]–[10], an alternative scheme is proposed for performing the distributed quantum computation between two three-junction flux qubits interacting separately with two LC circuits connected by another LC circuit through mutual inductance. As it is implemented by the mesoscopic materials, it is much easier to achieve the strong coupling and overcome the limitation of short-fibre. Meanwhile, using the TDEF to control the coupling, we can realize the quantum information transfer and control more accurately.

The remainder of the present paper is organized as follows. The model and the effective Hamiltonian are given in Section 2. The implementation of distributed quantum computation is investigated in Section 3. Finally, the conclusions are given in Section 4.

2. Model

First, we consider two distant LC circuits interacting separately with two three-junction flux qubit through their mutual inductance (Fig. 1). The two LC circuits will be henceforth labeled by the letters A and B, then the LC circuits are connected by another LC circuit through mutual inductance. With the two qubits working at their optimal points, two time-dependent magnetic fields (TDMF) $\Phi^j_{e(t)}$ $(j=1,2)$ with frequencies w^j_L $(j=1,2)$ are applied to the two qubits independently, we note that the nonlinear coupling strength is zero between the qubit, LC circuit and TDMF right now. Then the total Hamiltonian of this system can be written as

$$\begin{aligned} H = \sum_{j=1}^{2} \Big[& \frac{\hbar}{2} w_q^j \sigma_z^j + \hbar w_f^j a_j^\dagger a_j + \hbar\chi_j (a_j \sigma_j^\dagger + a_j^\dagger \sigma_j) \\ & + \hbar\lambda_j (\mathrm{e}^{-\mathrm{i}\, w_L^j t} \sigma_j^\dagger + \mathrm{e}^{\mathrm{i}\, w_L^j t} \sigma_j) \Big] \\ & + \hbar w_f^3 a_3^\dagger a_3 + \hbar\varepsilon [a_3^\dagger (a_1 + a_2) \\ & + a_3 (a_1^\dagger + a_2^\dagger)] \end{aligned} \tag{1}$$

in the rotating-wave approximation. Here with the second quantization, the j-th LC circuit Hamiltonian has been changed from $H_j = \frac{1}{2} L_j \phi_j^2 + Q_j^2/2C_j$, $(j=1,2,3)$ to $H_j = \hbar w_f^j (a_j^\dagger a_j + 1/2)$, $(j=1,2,3)$, where the ladder operators a_j and $a_j^\dagger$ of the j-th LC circuit are defined as $a_j = \sqrt{Cw/\hbar}\phi_j + \mathrm{i}\sqrt{1/\hbar Cw} Q_j$, $(j=1,2,3)$ and $a_j^\dagger = \sqrt{Cw/\hbar}\phi_j - \mathrm{i}\sqrt{1/\hbar Cw} Q_j$, $(j=1,2,3)$, ϕ_j is the magnetic flux of the j-th LC circuit and Q_j is the charge stored on the capacitor C_j of the circuit with the self-inductance L_j, the frequency of the j-th circuit is $w_j = \sqrt{1/L_j C_j}$, $(j=1,2,3)$. The interaction Hamiltonian with two LC circuits has been changed from $H_{(j3)I} = M_{j3}\phi_3\phi_j$, $(j=1,2)$ to $H_{(j3)I} = \varepsilon(a_j a_3^\dagger + a_j^\dagger a_3)$, $(j=1,2)$, where the coupling constants ε_{j3} $(j=1,2)$ are related with inductances M_{j3} $(j=1,2)$, we let $\varepsilon_{13} = \varepsilon_{23} = \varepsilon$. The qubit operators are defined as $\sigma_z^j = |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma_j^\dagger = |e\rangle\langle g|$, $\sigma_j = |g\rangle\langle e|$, $(j=1,2)$ with using its ground state $|g\rangle_j$ and first exited state $|e\rangle_j$, and the qubit frequencies w_q^j $(j=1,2)$ can be expressed as $\hbar w_q^j = 2\sqrt{I_j^2(\Phi_e^j - \Phi_0^j/2)^2 + T_{RL(j)}^2}$, $(j=1,2)$ with the bias flux Φ_e^j and the qubit loop-current I_j, the parameter $T_{RL(j)}$ denotes the tunnel coupling between the two potential wells of the j-th qubit.

If the two TDMFs are not applied to the qubit, for the case $w_q^j - w_f^j \gg |\chi|_j$, $(j=1,2)$ and in the large detuning condition, the qubit and the LC circuit can be considered as two independent subsystems. Now the two TDMFs are applied to the qubit, two dressed qubit can be formed by the two applied TDMFs and the qubit.

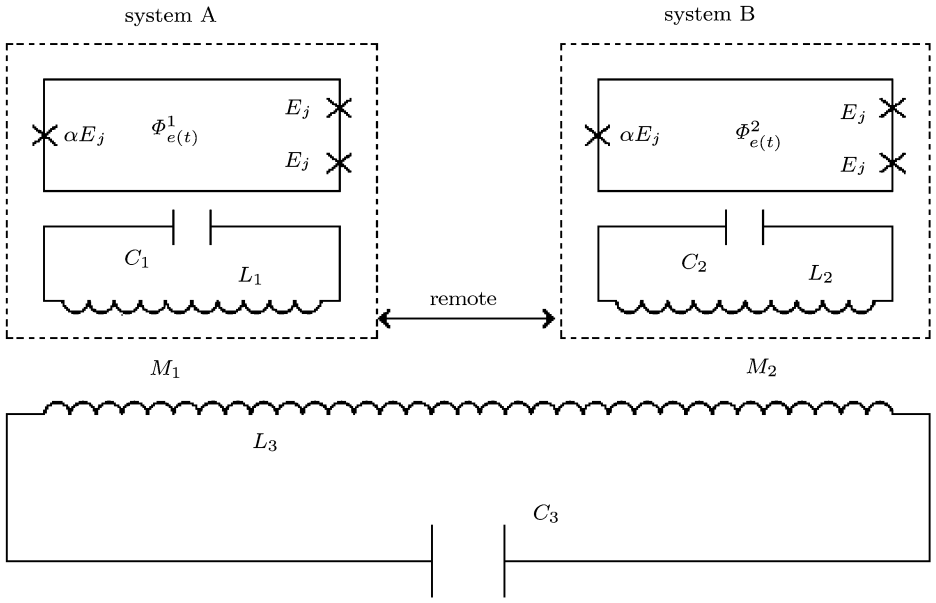


Fig. 1. Two three-junction flux qubits independently coupled to two LC circuits, which are connected by another LC circuit through their mutual inductance M. The DC bias magnetic flux through the i-th qubit is Φ^i_e. A time-dependent magnetic field $\Phi^i_{e(t)}$ can be independently applied to the i-th qubit, so the two qubit can be separately coupled to the two LC circuits.

In the rotating reference frame the Hamiltonian in Eq. (1) is transformed into the Hamiltonian H_e under a unitary transformation

$$U_1 = \exp\left[\left(-\mathrm{i}\sum_{j=1}^{2} w_L^j \sigma_z^j t\right)\Big/ 2\right]$$

as

$$H_e = U_1^\dagger H U_1 - \mathrm{i}\,\hbar U_1^\dagger(\partial U_1/\partial t) = H_e^1 + H_e^2 \quad (2)$$

with

$$\begin{aligned} H_e^1 &= \sum_{j=1}^{2}[\hbar w_f^j a_j^\dagger a_j + \hbar\chi_j(a_j\sigma_j^\dagger \mathrm{e}^{\mathrm{i}\,w_L^j t} + a_j^\dagger\sigma_j \mathrm{e}^{-\mathrm{i}\,w_L^j t})] \\ &\quad + \hbar w_f^3 a_3^\dagger a_3 + \hbar\varepsilon[a_3^\dagger(a_1+a_2) + a_3(a_1^\dagger + a_2^\dagger)], \\ H_e^2 &= \sum_{j=1}^{2}[\hbar\lambda_j(\sigma_j^\dagger + \sigma_j) + \frac{\hbar}{2}\Delta_j\sigma_z^j], \end{aligned} \quad (3)$$

and $\Delta_j = w_q^j - w_L^j$, $(j = 1, 2)$. Using the method similar to that used in Ref. [10], the Hamiltonian H_e^2 can be diagonalized and rewritten as

$$H_e^{(2)} = \sum_{j=1}^{2}\frac{\hbar}{2}\Omega_j\eta_z^j, \quad (4)$$

where the transition frequency

$$\Omega_j = \sqrt{(w_q^j - w_L^j)^2 + 4|\lambda_j|^2}, \quad \eta_z^j = |E\rangle_j\langle E| - |G\rangle_j\langle G|,$$

$\eta_j^+ = |E\rangle_j\langle G|$, and $\eta_j^- = |G\rangle_j\langle E|$ in the dressed states, with

$$|E\rangle_j = \cos\frac{\theta_j}{2}|e\rangle_j + \sin\frac{\theta_j}{2}|g\rangle_j, \quad (5a)$$

$$|G\rangle_j = -\sin\frac{\theta_j}{2}|e\rangle_j + \cos\frac{\theta_j}{2}|g\rangle_j \quad (5b)$$

with $\theta_j = \arctan(2|\lambda_j|/\Delta_j)$.

If we assume $w_f^j > w_L^j$ $(j = 1, 2)$, so the Hamiltonian H_e^1 can be rewritten as

$$\begin{aligned} H_e^{(1)} &= \sum_{j=1}^{2}\left\{\hbar w_f^j a_j^\dagger a_j + \hbar\chi_j\left[a_j\left(\eta_j^+\cos^2\frac{\theta_j}{2} - \eta_j^-\sin^2\frac{\theta_j}{2} - \eta_z^-\frac{\sin\theta_j}{2}\right)\mathrm{e}^{\mathrm{i}\,w_L^j t} + \mathrm{H.c.}\right]\right\} \\ &\quad + \hbar w_f^3 a_3^\dagger a_3 + \hbar\varepsilon[a_3^\dagger(a_1+a_2) + a_3(a_1^\dagger + a_2^\dagger)], \end{aligned} \quad (6)$$

in the dressed-state basis of Eqs. (5a) and (5b). We let $k_j = \chi_j\cos^2\theta$. Now $\eta_j^-\sin^2(\theta_j/2)a_j\mathrm{e}^{\mathrm{i}\,w_L^j t}$, $\eta_z^-(\sin\theta/2)$ $a_j\mathrm{e}^{\mathrm{i}\,w_L^j t}$ and their conjugates have been neglected, because there is no way to conserve energies in these terms and they should be neglected in the use of the rotating-wave approximation. The effective Hamiltonian in Eq. (2) can be rewritten as

$$H_{\mathrm{eff}} = \sum_{j=1}^{2}\left[\hbar w_f^j a_j^\dagger a_j + \frac{\hbar}{2}\Omega_j\eta_z^j + \hbar k_j(a_j\eta_j^+\mathrm{e}^{\mathrm{i}\,w_L^j t} + \mathrm{H.c.})\right] + \hbar w_f^3 a_3^\dagger a_3 + \hbar\varepsilon[a_3^\dagger(a_1+a_2) + a_3(a_1^\dagger + a_2^\dagger)]. \quad (7)$$

We can let $\Omega_1 = w_f^1 - w_L^1$, and $\Omega_2 - \delta = w_f^2 - w_L^2$ and then rewrite the Hamiltonian of Eq. (7) in the interaction picture as

$$H_{\text{eff(I)}} = \exp\left(\frac{\mathrm{i}\,H_0 t}{2}\right)(H_{\text{eff}} - H_0)\exp\left(\frac{-\mathrm{i}\,H_0 t}{2}\right)$$

with

$$H_0 = \frac{\hbar}{2}(w_f^2 - w_L^2)\eta_z^2 + \frac{\hbar}{2}\Omega_1\eta_z^1 + \hbar w_f^3 a_3^\dagger a_3 + \sum_{j=1}^{2}\hbar w_f^j a_j^\dagger a_j,$$

so

$$\begin{aligned} H_{\text{eff(I)}} = {} & \delta|E\rangle_2\langle E| + \sum_{j=1}^{2}(k_j\eta_j^+ a_j + k_j\eta_j^- a_j^\dagger) \\ & + \varepsilon[a_3^\dagger(a_1 + a_2) + a_3(a_1^\dagger + a_2^\dagger)]. \end{aligned} \tag{8}$$

Now with the coherent evolution described by Eq. (8), the distributed quantum computational possibilities are discussed in the following.

3. Distributed quantum computation

First, quantum state transfer (QST)[11] can be achieved. In other words, with the initial state being $(\alpha|G\rangle_1 + \beta|E\rangle_1) \otimes |G\rangle_2 \otimes |000\rangle_{LC}$, the goal of the achieved state transfer is to turn into the state $(\alpha|G\rangle_2 + \beta|E\rangle_2) \otimes |G\rangle_1 \otimes |000\rangle_{LC}$. Here, $|n_1 n_3 n_2\rangle_{LC}$ denotes the j-th LC circuit with $a_j^\dagger a_j = n_j$, ($j = 1, 2, 3$), Let $\delta = 0$, and for simplicity, the whole system is considered to have only one excitation. Now the time evolution of the system remains in the subspace spanned by $\forall_1 \in \{|\varphi_1\rangle, |\varphi_2\rangle, |\varphi_3\rangle, |\varphi_4\rangle, |\varphi_5\rangle\} \equiv \{|G\rangle_1 \otimes |E\rangle_2 \otimes |000\rangle, |E\rangle_1 \otimes |G\rangle_2 \otimes |000\rangle, |G\rangle_1 \otimes |G\rangle_2 \otimes |001\rangle, |G\rangle_1 \otimes |G\rangle_2 \otimes |010\rangle, |G\rangle_1 \otimes |G\rangle_2 \otimes |100\rangle\}$, the Hamiltonian in this subspace has five eigenvalues:

$$\begin{aligned} E_1 &= 0, \\ E_{2,3} &= \mp\sqrt{2k_1^2 + 2k_2^2 + 4\varepsilon^2 - 2\sqrt{k_1^4 + k_2^4 + 4\varepsilon^4 - 2k_1^2 k_2^2}}, \\ E_{4,5} &= \mp\sqrt{2k_1^2 + 2k_2^2 + 4\varepsilon^2 + 2\sqrt{k_1^4 + k_2^4 + 4\varepsilon^4 - 2k_1^2 k_2^2}}, \end{aligned}$$

and the corresponding eigenvectors (with $k_1 = k_2 = k$) are

$$|\phi_1\rangle = -\frac{\varepsilon}{\sqrt{k^2 + 2\varepsilon^2}}|\varphi_1\rangle - \frac{-\varepsilon}{\sqrt{k^2 + 2\varepsilon^2}}|\varphi_2\rangle + \frac{k}{\sqrt{k^2 + 2\varepsilon^2}}|\varphi_4\rangle, \tag{9}$$

$$|\phi_2\rangle = \frac{1}{2}|\varphi_1\rangle - \frac{1}{2}|\varphi_2\rangle - \frac{1}{2}|\varphi_3\rangle + \frac{1}{2}|\varphi_5\rangle, \tag{10}$$

$$|\phi_3\rangle = -\frac{1}{2}|\varphi_1\rangle + \frac{1}{2}|\varphi_2\rangle - \frac{1}{2}|\varphi_3\rangle + \frac{1}{2}|\varphi_5\rangle, \tag{11}$$

$$|\phi_4\rangle = -\frac{k}{2\sqrt{k^2 + 2\varepsilon^2}}|\varphi_1\rangle - \frac{k}{2\sqrt{k^2 + 2\varepsilon^2}}|\varphi_2\rangle + \frac{1}{2}|\varphi_3\rangle - \frac{\varepsilon}{\sqrt{k^2 + 2\varepsilon^2}}|\varphi_4\rangle + \frac{1}{2}|\varphi_5\rangle, \tag{12}$$

$$|\phi_5\rangle = \frac{k}{2\sqrt{k^2 + 2\varepsilon^2}}|\varphi_1\rangle + \frac{k}{2\sqrt{k^2 + 2\varepsilon^2}}|\varphi_2\rangle + \frac{1}{2}|\varphi_3\rangle + \frac{\varepsilon}{\sqrt{k^2 + 2\varepsilon^2}}|\varphi_4\rangle + \frac{1}{2}|\varphi_5\rangle. \tag{13}$$

Thus, the initial state $|\Psi_{(0)}\rangle \equiv |\varphi_2\rangle$ can be rewritten as

$$|\varphi_2\rangle = \frac{k}{2\sqrt{k^2 + 2\varepsilon^2}}|\phi_5\rangle - \frac{k}{2\sqrt{k^2 + 2\varepsilon^2}}|\phi_4\rangle + \frac{1}{2}|\phi_3\rangle - \frac{1}{2}|\phi_2\rangle - \frac{k}{\sqrt{k^2 + 2\varepsilon^2}}|\phi_1\rangle.$$

The time evolution of the system is governed by the equation

$$\mathrm{i}\,\hbar\frac{\partial}{\partial t}|\Psi_{(t)}\rangle = H|\Psi_{(t)}\rangle. \tag{14}$$

Then we have

$$\begin{aligned} |\Psi_{(t)}\rangle = {} & \frac{2\varepsilon^2 - (k^2 + 2\varepsilon^2)\cos(kt) + k^2\cos(t\sqrt{k^2 + 2\varepsilon^2})}{2(k^2 + 2\varepsilon^2)}|\varphi_1\rangle + \frac{2\varepsilon^2 + (k^2 + 2\varepsilon^2)\cos(kt) + k^2\cos(t\sqrt{k^2 + 2\varepsilon^2})}{2(k^2 + 2\varepsilon^2)}|\varphi_2\rangle \\ & + \frac{-\mathrm{i}(k^2 + 2\varepsilon^2)\sin(kt) + \mathrm{i}\,k^2\sin(t\sqrt{k^2 + 2\varepsilon^2})}{2(k^2 + 2\varepsilon^2)}|\varphi_3\rangle + \frac{\varepsilon k[\cos(t\sqrt{k^2 + 2\varepsilon^2}) - 2]}{2(k^2 + 2\varepsilon^2)}|\varphi_4\rangle \\ & + \frac{\mathrm{i}(k^2 + 2\varepsilon^2)\sin(kt) + \mathrm{i}\,k^2\sin(t\sqrt{k^2 + 2\varepsilon^2})}{2(k^2 + 2\varepsilon^2)}|\varphi_5\rangle. \end{aligned} \tag{15}$$

If the parameters k and ε fulfill the following condition:

$$kt = n\pi, \quad (n = 1, 2, 3\ldots), \tag{16}$$

$$\sqrt{1 + 2\left(\frac{\varepsilon}{k}\right)^2} = 2m, \quad (m = 1, 2, 3\ldots). \tag{17}$$

The initial state $|\Psi_{(0)}\rangle \equiv |\varphi_2\rangle$ can evolves into the state $|\Psi_{(t)}\rangle \equiv |\varphi_1\rangle$. As a result, with the initial state $(\alpha|G\rangle_1 + \beta|E\rangle_1) \otimes |G\rangle_2 \otimes |000\rangle_{LC}$ and because equations (16) and (17) are fulfilled, the state $(\alpha|G\rangle_2 + \beta|E\rangle_2) \otimes |G\rangle_1 \otimes |000\rangle_{LC}$ can be easily evolved into. Therefore, the QST can be implemented. The results and discussion are similar to those in Ref. [5].

Secondly, quantum entangling gate (QEG)[4] can be achieved. In other words, the entangled states $(|E\rangle_1 \otimes |G\rangle_2 \pm |G\rangle_1 \otimes |E\rangle_2)/\sqrt{2}$ can be created. By using the canonical transformation[4]

$$\begin{aligned} a_1 &= \frac{1}{2}(c_+ + c_- + \sqrt{2}c), \\ a_2 &= \frac{1}{2}(c_+ + c_- - \sqrt{2}c)\mathrm{e}^{\mathrm{i}\,\gamma}, \\ a_3 &= \frac{1}{\sqrt{2}}(c_+ - c_-), \end{aligned} \tag{18}$$

where $c_\mp$ are non-resonant modes of frequencies $\Omega \pm \sqrt{2}\varepsilon$, c is resonant mode of frequency Ω, and the phase γ depends on the third LC circuit, and letting $\varepsilon \gg k_1$, $\varepsilon \gg k_2$, in the rotating approximation, Hamiltonian (7) in the interaction picture after canonical transformation can be expressed as

$$\begin{aligned} H' = {} & \delta\eta_2^z + \frac{1}{\sqrt{2}}[k_1(\eta_1^- c^+ + \eta_1^+ c) \\ & - k_2(\eta_2^- c^+ \mathrm{e}^{-\mathrm{i}\,\gamma} + \eta_2^+ c\,\mathrm{e}^{\mathrm{i}\,\gamma})]. \end{aligned} \tag{19}$$

Through Eq. (19), the interaction of the qubits with the non-resonant modes is highly suppressed and the system reduces to two superconducting qubits resonantly coupled through a single harmonic oscillator. Now let $\delta = 0$, and $\gamma = 0$, the time evolution of the system will remain in the subspace spanned by $\forall_1 \in \{|\varphi_1\rangle, |\varphi_2\rangle, |\varphi_3\rangle\} \equiv \{|G\rangle_1 \otimes |E\rangle_2 \otimes |0\rangle, |E\rangle_1 \otimes |G\rangle_2 \otimes |0\rangle, |G\rangle_1 \otimes |G\rangle_2 \otimes |1\rangle\}$, where $|0\rangle_c$ and $|1\rangle_c$ are number states of the normal mode c. Using the above method similar to QST, with the initial state $|E\rangle_1 \otimes |G\rangle_2 \otimes |0\rangle$, the state of the system at time t is written

$$\begin{aligned} |\Psi_{(t)}\rangle = {} & \frac{k_1^2 + k_2^2\cos(t\sqrt{(k_1^2 + k_2^2)/2})}{k_1^2 + k_2^2}|\varphi_1\rangle \\ & + \frac{k_1 k_2 - k_1 k_2\cos(t\sqrt{(k_1^2 + k_2^2)/2})}{k_1^2 + k_2^2}|\varphi_2\rangle \\ & + \frac{k_1\sin(t\sqrt{(k_1^2 + k_2^2)/2})}{\sqrt{k_1^2 + k_2^2}}|\varphi_3\rangle. \end{aligned} \tag{20}$$

If the parameters k_1, k_2 and t fulfill the following condition: $k_1 t = \pi/\sqrt{2 + \sqrt{2}}$ and $k_2/k_1 = 1 + \sqrt{2}$, the initial state $|\Psi_{(0)}\rangle \equiv |\varphi_2\rangle$ can evolve into the state $|\Psi_{(t)}\rangle \equiv (|\varphi_1\rangle + |\varphi_2\rangle)/\sqrt{2}$; if the parameters k_1, k_2 and t fulfill the following condition: $k_1 t = \pi/\sqrt{2 - \sqrt{2}}$ and $k_2/k_1 = \sqrt{2} - 1$, the initial state $|\Psi_{(0)}\rangle \equiv |\varphi_2\rangle$ can evolve into the state $|\Psi_{(t)}\rangle \equiv (|\varphi_2\rangle - |\varphi_1\rangle)/\sqrt{2}$. Therefore, the QEG can be implemented. The results and discussion are similar to those in Ref. [5].

Thirdly, in this paper compared with Ref. [4], this model also allows for the reliable emulation of the swap gate[12] and the "controlled-phase" gate[13] by adjusting the appropriate parameters k_1, k_2, ε and δ. And now we come to remark on the difference between the LC circuit and an optical fibre as a data bus. In order to let only one resonant mode of the fibre interact with the cavity modes and gain higher fidelity, the short fibre limit $2l\bar{v}/(2\pi c) \leq 1$ should be satisfied, where l is the length of the fibre and $\bar{v}$ is the decay rate of the cavity mode spectra into the continuum spectra of the fibre modes. But the LC circuit as a data bus is beyond the length limit, so more interests in the dissipation of the mutual inductance are worth investigating.

Finally, in the above discussion we show the basic mechanism of the quantum information processing between the superconducting qubits via the LC circuit with using dressed states. Therefore a scalable quantum circuit which is required for quantum computation (quantum information processing), can be constructed by N dressed qubits and the $(N+1)$-th LC circuit acting as a data bus shown in Fig. 2. The $(N+1)$-th LC circuit interacts with N dressed qubits through their mutual inductances. The distance between any two nearest dressed qubits is assumed to be so large that their interaction through the mutual inductance is negligibly small. Then, the total effective Hamiltonian can be written as

$$\begin{aligned} H_{\mathrm{eff(I)}} = {} & \sum_{j=1}^{N}(\delta_j|E\rangle_j\langle E|) \\ & + \sum_{j=1}^{N}(k_j\eta_j^+ a_j + k_j\eta_j^- a_j^\dagger) \\ & + \varepsilon\left[b^\dagger\left(\sum_{j=1}^{N} a_j\right) + b\left(\sum_{j=1}^{N} a_j^\dagger\right)\right]. \end{aligned} \tag{21}$$

With the same discussion to that in Ref. [10], if N TDMFs are not applied to the qubits, for the case $w_q^j - w_f^j \gg |\chi|_j$, $(j = 1, 2, \ldots, N)$, all N qubits are decoupled from their LC circuits, then dressed qubits cannot be formed, in other words, each qubit can be independently manipulated by the TDEF, which seems to act as a switch of the computation-process. To couple a qubit to the LC circuit for forming a dressed qubit, an appropriate TDEF needs to be applied, then the state of the dressed qubit can be transfer from one dressed qubit to another. Meanwhile the entangled state of N modes can be generated.

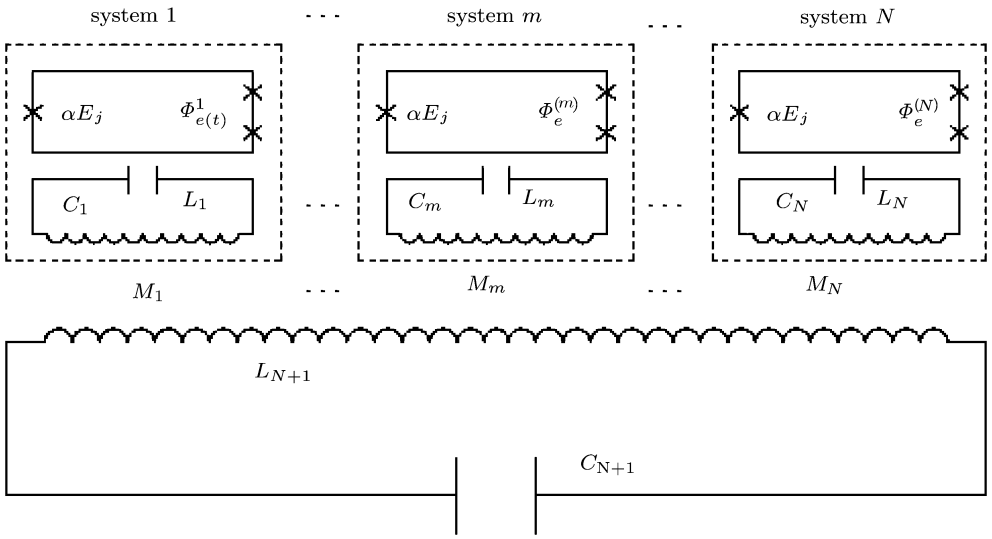


Fig. 2. N dressed qubits formed by N flux qubit plus their TDEF, which are coupled to the $(N+1)$-th LC circuit through their mutual inductances M_m $(m = 1, 2, \ldots, N)$. The bias magnetic flux through the m-th qubit is $\Phi_e^{(m)}$. A TDEF can be applied to any one of the qubits such that the dressed qubit can be coupled to the $(N+1)$-th LC circuit by the switch TDEF.

Next, we change our perspective from theoretical discussion to physical implementation. As shown in Fig. 3, the Hamiltonian of multiple superconducting qubits coupled by a data bus is completely equivalent to that of atomic ensemble coupled to a quantized field, in mathematics. At present, there are three methods to couple multiple qubits, i.e., cavity QED, circuit QED and mechanical QED, which are shown in Fig. 4. In Refs. [4]–[6], the authors have discussed the distributed quantum computation in cavity QED, while in the present paper, we restrain our scheme in circuit QED and mechanical QED. We note that a single mode data bus coupled to two solid-state qubits could be demonstrated by the Jaynes–Cummings model. The data bus, which is the $(N+1)$-th LC circuit in our paper, can also be realized by quantum transmission line or nano-mechanical resonator. However, as the quantum correlation in solid-state systems is fragile and finite, for example, we can use a data bus with energy gap and strong correlation characteristics to couple two solid-state qubits. If the energy gap is much larger than the transition frequency of the qubit, then the information transfer could be realized without destroying the energy of the qubits system. Yet, for a solid-state system, the existence of energy gap means that the quantum correlation is finite, in other words, the existence of energy gap for the data bus and long distance transmission of information are inconsequent. In this respect, we need to consider this theoretical question: whether the quantum correlation and quantum entanglement are consistent? As mentioned above, there are some obstacles in realizing the long-distance distributed quantum computation by using superconducting qubits in experiment, and the intrinsical relationship between quantum correlation and quantum entanglement is a key issue that needs further investigations.

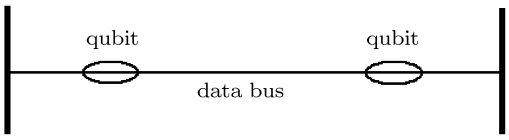


Fig. 3. Two qubits coupled by a data bus.

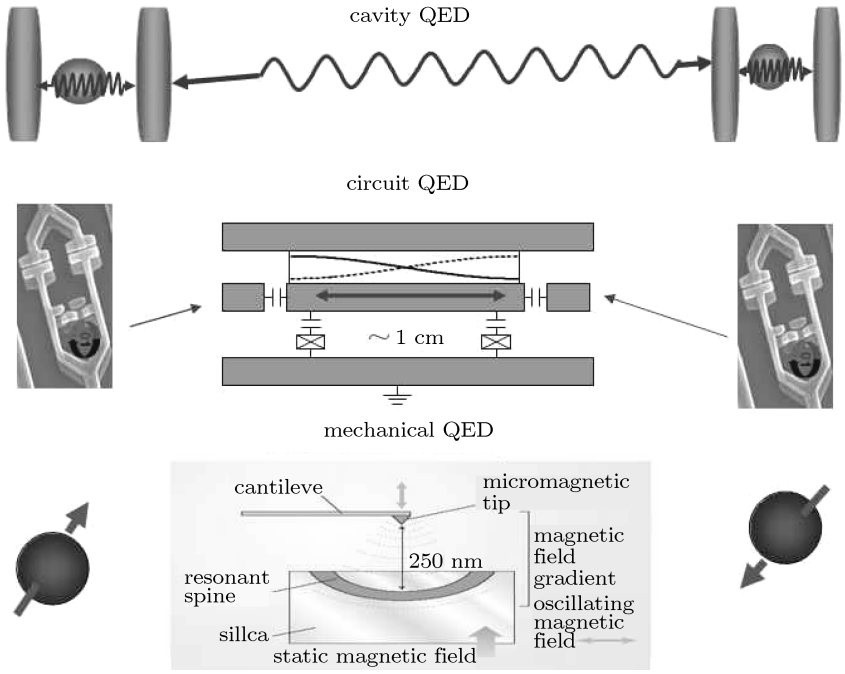


Fig. 4. Three methods to couple multiple qubits: cavity QED, circuit QED and mechanical QED.

4. Conclusions

A scheme is proposed where two superconducting qubits driven by the classical field interacting separately with two distant LC circuits connected by another LC circuit through mutual inductance are used for implementing quantum gates. By using dressed states, quantum state transfer, quantum entangling gate, swap gate and "controlled-phase" gate can all be implemented. With the help of TDEF, any two dressed qubits can be selectively coupled to the data bus (the last LC circuit), therefore, quantum state can be transferred from one dressed qubit to another, multi-mode entangled state can also be formed and thus quantum information can be transferred from one qubit to another. The above conclusion is based on the following assumptions: (i) all qubits are decoupled from the data bus when their detunings to the data bus are far large than their coupling constants to the LC circuits; (ii) the qubits can be independently manipulated by the TDEFs resonantly addressing them; (iii) flux dissipation is not taken into account between the LC circuits. The influence of the flux dissipation will be discussed somewhere.

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