Entanglement dynamics of two-qubit systems in different quantum noises*

Pan Chang-Ning(潘长宁)[a)†], Li-Fei(李 飞)[b)],
Fang Jian-Shu(方见树)[a)], and Fang Mao-Fa(方卯发)[c)]

[a)] *School of Science, Hunan University of Technology, Zhuzhou* 412008, *China*
[b)] *Department of Physics, Hunan University of Science and Technology, Xiangtan* 411201, *China*
[c)] *College of Physics and Information Science, Hunan Normal University, Changsha* 410081, *China*

(Received 5 August 2010; revised manuscript received 18 August 2010)

The entanglement dynamics of two-qubit systems in different quantum noises are investigated by means of the operator-sum representation method. We find that, except for the amplitude damping and phase damping quantum noise, the sudden death of entanglement is always observed in different two-qubit systems with generalized amplitude damping and depolarizing quantum noise.

Keywords: entanglement dynamics, quantum noise, operator-sum representation
PACS: 03.65.Ud, 42.50.Lc **DOI:** 10.1088/1674-1056/20/2/020304

Entanglement is a pure quantum mechanical feature, it can be viewed as a consequence of non-local coherence. If two or more subsystems are entangled, the whole state vector cannot be expressed by a product of the subsystems. And the quantum correlations of the observable physical properties always exist in the entangled systems, even though subsystems are far spatially separated. Any measurements on one subsystem can not only give information about the other subsystems, but also provide probabilities of manipulating it, and it is widely seen as the main reason for the computational advantage of quantum over classical algorithms. In recent years, there has always been growing interest in the complex quantum phenomenon of quantum entanglement[1,2] due to the extensive applications in quantum information processing,[3,4] such as quantum teleportation,[5,6] quantum secret key distribution,[7,8] quantum dense coding,[9,10] etc. Quantum entanglement properties, however, are very fragile: realistic quantum systems are not closed, due to unavoidable interactions with the environment, their entanglement and coherence can be irretrievably lost. Recent studies[11−15] have shown a novel property of disentanglement, when a pair of entangled qubits exposed to the noisy environments is being considered, quantum entanglement eventually disappears in the finite time. It is different from the usual decoherence in asymptotic time. This phenomenon is called "entanglement sudden death" (ESD). The entanglement completely disappears so that the useful quantum information is thoroughly lost in the finite time, hence the ESD is more harmful than the asymptotical disentanglement for utilizing the quantum entangled resource in practical quantum computation and quantum information. From the above discussion, some questions naturally arise in our mind: in which quantum noise does the ESD always occur for any initial state? And the converse, for which kind of initial entangled states does the disentanglement exhibit asymptotical behaviour in any noisy environments? If these questions can be solved, then different initial states can be chosen as the entanglement resources for different noisy environments to avoid the ESD; so that we have adequate time to finish quantum information processing.

In this paper, we address the effect of the specific quantum noise for the entanglement, including the amplitude damping, generalized amplitude damping, depolarizing and phase damping, and investigate in which kind of noisy environment and initial state, the ESD occurs.

To investigate the dynamic evolution of quan-

*Project supported by the Natural Science Foundation of Hunan Province of China (Grant No. 10JJ3088), the Key Research Foundation of the Education Bureau of Hunan Province of China (Grant No. 08A015), and the Funds of the Hunan Education Bureau of China (Grant Nos. 10C0616 and 08C344).

†Corresponding author. E-mail: panchangning2000@sina.com

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tum entanglement, we need a concrete measurement of the entanglement. For any bipartite system, the concurrence[16] is more convenient than other measuring methods. The concurrence is given by

$$C = \mathrm{Max}\left\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right\}, \quad (1)$$

where the quantities λ_i are the eigen-values of the matrix $M = \rho\left(\sigma_y^1 \otimes \sigma_y^2\right)\rho^*\left(\sigma_y^1 \otimes \sigma_y^2\right)$, and $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$; here ρ^* is the complex conjugate of the density matrix ρ and $\sigma_y^1(\sigma_y^2)$ is the second Pauli matrix. The concurrence varies from $C = 0$ for the separable state to $C = 1$ for the maximally entangled state. If the initial density matrix has an X structure,[13] the concurrence has a simple analytic expression

$$C = \mathrm{Max}\left\{0, 2\left(\sqrt{\rho_{23}\rho_{32}} - \sqrt{\rho_{11}\rho_{44}}\right), 2\left(\sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{22}\rho_{33}}\right)\right\}. \quad (2)$$

In order to describe the dynamics of two qubits interacting independently with individual environment, we use the quantum operator-sum representation approach,[17] in which quantum operations ε can be represented in an elegant form. Given an initial state of two qubits $\rho(0)$, the reduced state of the systems, after interaction of systems-environments, can be rewritten as

$$\rho(t) = \varepsilon\left(\rho\left(0\right)\right) = \sum_{k,v} E_{k,v}\rho\left(0\right)E_{k,v}^{\dagger}, \quad (3)$$

where the operators $E_{k,v} = E_k \otimes E_v$,[17] and $E_{\{k\}} \equiv \langle e_k| U |e_0\rangle$ is an operator on the one-qubit of the principal system, with $\sum_{k,v} E_{k,v}^{\dagger}E_{k,v} = I$; $|e_k\rangle$ is an orthonormal basis for the state space of the environment. The operator-sum representation will be repeatedly used in the following discussion.

In the remainder of this scenario, we first consider the case of the systems interacting with the generalized amplitude damping (GAD) and amplitude damping; then the depolarizing noise is discussed; finally, we consider the dephasing noise.

Generalized amplitude damping The GAD is the description of energy dissipation effects due to loss of energy from the quantum system to environments at a finite temperature (such as thermal bath at an arbitrary temperature). It is described by the Kraus operators[17] $E_0 = \sqrt{q}\,\mathrm{diag}(1, \sqrt{1-\gamma})$, $E_1 = \sqrt{q\gamma}(\sigma_x + \mathrm{i}\sigma_y)/2$, $E_2 = \sqrt{(1-q)}\,\mathrm{diag}(\sqrt{1-\gamma}, 1)$, and $E_3 = \sqrt{(1-q)\,\gamma}(\sigma_x - \mathrm{i}\sigma_y)/2$, where $\gamma = 1 - \mathrm{e}^{-\Gamma t}$, with Γ denoting the decay rate,[17] and q defining the probability distribution of the stationary state when $t \to \infty$, σ_x (σ_y) being the first (the second) Paul matrix. To find which class of entangled states is more robust and to investigate the effect of the entanglement when the maximally entangled state is mixed with the additional state $|00\rangle$ or $|11\rangle$ in this noisy channel, we consider the different initial states

$$\rho_{\mathrm{A1}}\left(0\right) = \left(1 - \alpha_{\mathrm{A1}}\right)\left(|10\rangle + |01\rangle\right)\left(\langle 10| + \langle 01|\right)/2 + \alpha_{\mathrm{A1}}\,|00\rangle\,\langle 00|\,, \quad \left(0 \leq \alpha_{\mathrm{A1}} < 1\right), \quad (4)$$

$$\rho_{\mathrm{A2}}\left(0\right) = \left(1 - \alpha_{\mathrm{A2}}\right)\left(|10\rangle + |01\rangle\right)\left(\langle 10| + \langle 01|\right)/2 + \alpha_{\mathrm{A2}}\,|11\rangle\,\langle 11|\,, \quad \left(0 \leq \alpha_{\mathrm{A2}} < 1\right) \quad (5)$$

with $\alpha_{\mathrm{A}i} = 0$ $(i = 1, 2)$ meaning the maximally entangled state $(|10\rangle + |01\rangle)/\sqrt{2}$, for the sake of clarity of expression, we denote the above states by A1 and A2, and

$$\rho_{\mathrm{B1}}\left(0\right) = \alpha_{\mathrm{B1}}(|11\rangle + |00\rangle)\left(\langle 11| + \langle 00|\right)/4 + \left(2 - \alpha_{\mathrm{B1}}\right)|00\rangle\,\langle 00|/2, \quad \left(0 < \alpha_{\mathrm{B1}} \leq 2\right), \quad (6)$$

$$\rho_{\mathrm{B2}}\left(0\right) = \alpha_{\mathrm{B2}}(|11\rangle + |00\rangle)\left(\langle 11| + \langle 00|\right)/4 + \left(2 - \alpha_{\mathrm{B2}}\right)|11\rangle\,\langle 11|/2, \quad \left(0 < \alpha_{\mathrm{B2}} \leq 2\right) \quad (7)$$

with $\alpha_{\mathrm{B}i} = 0$ $(i = 1, 2)$ meaning the maximally entangled state $(|11\rangle + |00\rangle)/\sqrt{2}$, denoting the states by B1 and B2.

According to Eq. (3), we can straightforwardly calculate the density dynamics of the states Ai and Bi $(i = 1, 2)$ as Eqs. (A1)–(A4). Using formula (3), we obtain the concurrence of two classes of states, respectively:

$$C_{\mathrm{A}i} = \mathrm{Max}\left\{0, 2\left|\rho_{23}^{\mathrm{A}i}\right| - 2\sqrt{\rho_{11}^{\mathrm{A}i}\rho_{44}^{\mathrm{A}i}}\right\}, \quad C_{\mathrm{B}i} = \mathrm{Max}\left\{0, 2\left|\rho_{14}^{\mathrm{B}i}\right| - 2\sqrt{\rho_{22}^{\mathrm{B}i}\rho_{33}^{\mathrm{B}i}}\right\}.$$

In Fig. 1, we plotted the time evolution of the concurrence of two different kinds of states for various values of the parameter α, taking $q = 0.8$. We find that, when $\alpha_{\mathrm{A}i} = \alpha_{\mathrm{B}i} = 0$, the two systems are in the ideal maximally entangled states; the entanglement of the state $(|10\rangle + |01\rangle)/\sqrt{2}$ is more robust than the state $(|00\rangle + |11\rangle)/\sqrt{2}$ in the noisy channel. However, when $\alpha_{\mathrm{A}i} \neq 0$, $\alpha_{\mathrm{B}i} \neq 0$, i.e., the maximally entangled states are mixed with the additional state $|00\rangle$ or $|11\rangle$, two other interesting features can be observed: (i) the entanglement is more robust when the states are mixed with the state $|00\rangle$ other than the state $|11\rangle$, irrespective of which kind of maximally entangled states is $((|10\rangle + |01\rangle)/\sqrt{2}$ or $(|11\rangle + |00\rangle)/\sqrt{2})$; (ii) although the initial entanglement degrees are equal, taking $\alpha_{\mathrm{A}1} = \alpha_{\mathrm{A}2} = 0.25$, $\alpha_{\mathrm{B}1} = \alpha_{\mathrm{B}2} = 1.5$, the entanglement of states Bi can last more time, thus the states Ai are more fragile than the states Bi when the additional state, $|00\rangle$ or $|11\rangle$, is mixed with them. But the entanglement always appears the sudden-death for the two different entangled states in the GAD no matter whether there is the additional state $|00\rangle$ ($|11\rangle$) or not.

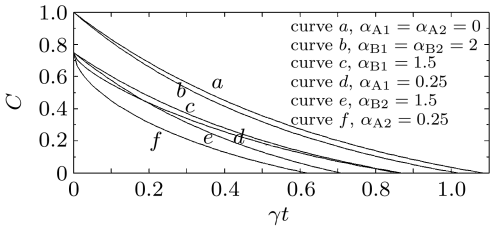


Fig. 1. The time evolution of the concurrence of two classes of states Ai and Bi ($i = 1, 2$) in the GAD, $q = 0.8$: curve a for $\alpha_{\mathrm{A}1} = \alpha_{\mathrm{A}2} = 0$, curve b for $\alpha_{\mathrm{B}1} = \alpha_{\mathrm{B}2} = 2$, curve for c $\alpha_{\mathrm{B}1} = 1.5$, curve d for $\alpha_{\mathrm{A}1} = 0.25$, curve e for $\alpha_{\mathrm{B}2} = 1.5$, curve f for $\alpha_{\mathrm{A}2} = 0.25$.

When $q = 1$, this case corresponds to the usual amplitude damping with $T = 0$ K.[17] For example, the photon state in the interferometer or cavity is subject to scattering, and attenuation can be well characterized by this quantum noise. The time evolution of entanglement is shown in Fig. 2, the parameter of α is taken as given above. We find that the entanglement can survive for an infinite time period for two maximally entangled states or the maximally entangled states mixed with the additional state $|00\rangle$. However, instead of mixing with the $|00\rangle$, we mix the state$|11\rangle$ with them, thus the entanglement disappears in the finite time, and the sudden-death entanglement results from the additional state $|11\rangle$ in this noisy channel.

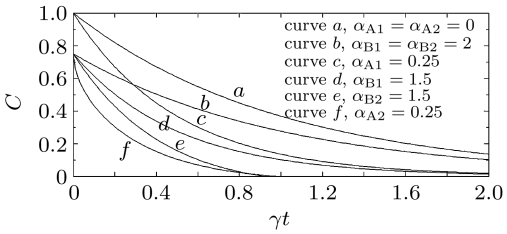


Fig. 2. The time evolution of the concurrence of two classes of states Ai and Bi ($i = 1, 2$) in the usual amplitude damping channel: curve a for $\alpha_{\mathrm{A}1} = \alpha_{\mathrm{A}2} = 0$, curve b for $\alpha_{\mathrm{B}1} = \alpha_{\mathrm{B}2} = 2$, curve c for $\alpha_{\mathrm{A}1} = 0.25$, curve d for $\alpha_{\mathrm{B}1} = 1.5$, curve e for $\alpha_{\mathrm{B}2} = 1.5$, curve f for $\alpha_{\mathrm{A}2} = 0.25$.

Depolarizing channel The depolarizing channel represents a type of quantum noise in which the single qubit is replaced by the completely mixed state $I/2$ (I is the identity matrix of a single qubit) with probability p, the effect of the quantum noise is given by the Kraus operators:[17] $E_0 = \sqrt{1 - 3\gamma/4}I$, $E_1 = \sqrt{\gamma/4}\sigma_1$, $E_2 = \sqrt{\gamma/4}\sigma_2$, $E_3 = \sqrt{\gamma/4}\sigma_3$, with γ being defined above.[17] Assuming the initial states given by Eqs. (4)–(7), we obtain the density matrix of time t as the Eqs. (A5)–(A8). From Eq. (2), we calculate the concurrences as follows:

$$C_{\mathrm{A}1} = C_{\mathrm{A}2} = \mathrm{Max}\left\{0, 2\left|\rho_{23}^{\mathrm{A}i}\right| - 2\sqrt{\rho_{11}^{\mathrm{A}i}\rho_{44}^{\mathrm{A}i}}\right\},$$

$$C_{\mathrm{B}1} = C_{\mathrm{B}2} = \mathrm{Max}\left\{0, 2\left|\rho_{14}^{\mathrm{B}i}\right| - 2\sqrt{\rho_{22}^{\mathrm{B}i}\rho_{33}^{\mathrm{B}i}}\right\},$$

where i takes 1 or 2. In Fig. 3 we plot the concurrences for the case where both ideal maximally entangled states are mixed with the states $|00\rangle$ or $|11\rangle$ (taking the parameter α as given above). We see that not only both ideal maximally entangled states appear identical disentanglement behaviour, but also each of them has the same disentanglement no matter whether the additional state is $|00\rangle$ or $|11\rangle$, that is to say, both

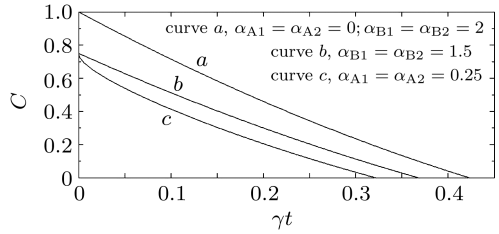


Fig. 3. Time evolution of the concurrence of two classes of states Ai and Bi ($i = 1, 2$) in the deplolarizing channel: curve a for $\alpha_{\mathrm{A}1} = \alpha_{\mathrm{A}2} = 0, \alpha_{\mathrm{B}1} = \alpha_{\mathrm{B}2} = 2$, curve b for $\alpha_{\mathrm{B}1} = \alpha_{\mathrm{B}2} = 1.5$, curve c for $\alpha_{\mathrm{A}1} = \alpha_{\mathrm{A}2} = 0.25$.

additional states mixed with the ideal maximally entangled states have the same influence on the evolution of entanglement. On the other hand, the state $(|00\rangle + |11\rangle)/\sqrt{2}$ mixed with the state $|00\rangle$ or $|11\rangle$ can

last for a longer time than the state $(|10\rangle + |01\rangle)/\sqrt{2}$ when they have the equal initial entanglement degrees for the proper parameter α. In addition, the depolarizing channel always induces the sudden death of entanglement for all of values α.

Phase damping The dephasing channel is a uniquely quantum mechanical noise process, which describes the loss of quantum coherence without exchange of energy,[17] for example, what happens when a photon scatters randomly as it travels through a waveguide can be characterized by this quantum noise. The Kraus operators are given by $E_0 = \mathrm{diag}\left(1, \sqrt{1-\gamma}\right)$ and $E_1 = \mathrm{diag}\left(1, \sqrt{\gamma}\right)$, where $\gamma = 1 - \mathrm{e}^{-\Gamma t}$, Γ denoting the decay rate.[17] We can calculate analytically the concurrence of the initial entangled states Ai and Bi $(i = 1, 2)$ in terms of Eqs. (2) and (3):

$$C_{\mathrm{A1}} = C_{\mathrm{A2}} = (1 - \alpha_{\mathrm{A1}})(1 - \gamma),$$
$$C_{\mathrm{B1}} = C_{\mathrm{B2}} = \alpha_{\mathrm{B1}}(1 - \gamma)/2.$$

It is clear from the above expression that all states have the same disentanglement behaviour if they have an equal initial entanglement degree. So the effect of the additional states is identical to both maximally entangled states. Furthermore, as shown in Fig. 4, the sudden death of entanglement never happens for any initial state in this environment, (the values $\alpha_{\mathrm{A}} = \alpha_{\mathrm{B}} = 0$; $\alpha_{\mathrm{A}} = 0.25$, $\alpha_{\mathrm{B}} = 0.5$; $\alpha_{\mathrm{A}} = 0.5$, $\alpha_{\mathrm{B}} = 1.0$ are taken from Fig. 4).

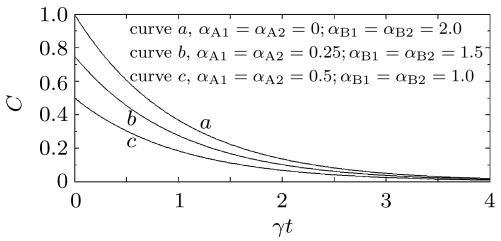


Fig. 4. Time evolution of the concurrence of two classes of states Ai and Bi $(i = 1, 2)$ in the phase damping channel: curve a for $\alpha_{\mathrm{A1}} = \alpha_{\mathrm{A2}} = 0; \alpha_{\mathrm{B1}} = \alpha_{\mathrm{B2}} = 2$, curve b for $\alpha_{\mathrm{A1}} = \alpha_{\mathrm{A2}} = 0.25; \alpha_{\mathrm{B1}} = \alpha_{\mathrm{B2}} = 1.5$, curve c for $\alpha_{\mathrm{A1}} = \alpha_{\mathrm{A2}} = 0.5; \alpha_{\mathrm{B1}} = \alpha_{\mathrm{B2}} = 1.0$.

In conclusion, we have investigated the dynamics of entanglement for the ideal two-qubit maximally entangled state and the mixed states with the additional states in different quantum noise environments. The concurrence is used as a measure of entanglement. We observed that the sudden death always appears in the GAD and depolarizing channel, moreover, the influence of the additional state $|00\rangle$ and $|11\rangle$ to the ideal entangled state is the same in the depolarizing channel. However, in the usual amplitude damping,[17] only when the additional state $|11\rangle$ is mixed with, can the sudden death be observed; otherwise, we always have asymptotic decay of entanglement. For the case of two-qubits system in the dephasing channel, the disentanglement presents the same behaviour if the initial states have equal entanglement degree no matter which kind of ideal maximally entangled state is mixed with the state $|00\rangle$ or $|11\rangle$, and the entanglement sudden death never happens for any values of α.

Appendix: Solution of density matrix elements in different quantum noisy environments

$$\rho_{11}^{\mathrm{A1}}(t) = (1 - \gamma + q\gamma)^2 \rho_{11}^{\mathrm{A1}}(0) + 2q\gamma(1 - \gamma + q\gamma)\rho_{22}^{\mathrm{A1}}(0),$$
$$\rho_{22}^{\mathrm{A1}}(t) = \rho_{33}^{\mathrm{A1}}(t) = \gamma(1 - q)(1 - \gamma + q\gamma)\rho_{11}^{\mathrm{A1}}(0) + \left[(1 - \gamma)\left(1 - 2q + 2q^2\right) + 2q(1 - q)\left(1 - \gamma + \gamma^2\right)\right]\rho_{22}^{\mathrm{A1}}(0),$$
$$\rho_{44}^{\mathrm{A1}}(t) = 1 - \rho_{11}^{\mathrm{A1}}(t) - 2\rho_{22}^{\mathrm{A1}}(t), \quad \rho_{23}^{\mathrm{A1}}(t) = \rho_{32}^{\mathrm{A1}}(t) = (1 - \gamma)\rho_{23}^{\mathrm{A1}}(0); \tag{A1}$$
$$\rho_{11}^{\mathrm{A2}}(t) = 2q\gamma(1 - \gamma + q\gamma)\rho_{22}^{\mathrm{A2}}(0) + q^2\gamma^2\rho_{44}^{\mathrm{A2}}(0),$$
$$\rho_{22}^{\mathrm{A2}}(t) = \rho_{33}^{\mathrm{A2}}(t) = \gamma q(1 - q)\rho_{44}^{\mathrm{A2}}(0) + \left[(1 - \gamma)\left(1 - 2q + 2q^2\right) + 2q(1 - q)\left(1 - \gamma + \gamma^2\right)\right]\rho_{22}^{\mathrm{A1}}(0),$$
$$\rho_{44}^{\mathrm{A2}}(t) = 1 - \rho_{11}^{\mathrm{A2}}(t) - 2\rho_{22}^{\mathrm{A2}}(t), \quad \rho_{23}^{\mathrm{A2}}(t) = \rho_{32}^{\mathrm{A2}}(t) = (1 - \gamma)\rho_{23}^{\mathrm{A2}}(0); \tag{A2}$$
$$\rho_{11}^{\mathrm{B1}}(t) = (1 - \gamma + q\gamma)^2 \rho_{11}^{\mathrm{B1}}(0) + q^2\gamma^2\rho_{14}^{\mathrm{B1}}(0),$$
$$\rho_{22}^{\mathrm{B1}}(t) = \rho_{33}^{\mathrm{B1}}(t) = \gamma(1 - q)(1 - \gamma + q\gamma)\rho_{11}^{\mathrm{B1}}(0) + q\gamma(1 - r)\rho_{14}^{\mathrm{B1}}(0),$$
$$\rho_{44}^{\mathrm{B1}}(t) = 1 - \rho_{11}^{\mathrm{B1}}(t) - 2\rho_{22}^{\mathrm{B1}}(t), \quad \rho_{14}^{\mathrm{B1}}(t) = \rho_{41}^{\mathrm{B1}}(t) = (1 - \gamma)\rho_{14}^{\mathrm{B1}}(0); \tag{A3}$$

$$\rho_{11}^{\mathrm{B2}}(t)=(1-\gamma+q\gamma)^2\rho_{11}^{\mathrm{B2}}(0)+q^2\gamma^2\rho_{44}^{\mathrm{B2}}(0),$$
$$\rho_{22}^{\mathrm{B2}}(t)=\rho_{33}^{\mathrm{B2}}(t)=\gamma(1-q)(1-\gamma+q\gamma)\rho_{11}^{\mathrm{B2}}(0)+q\gamma(1-r)\rho_{44}^{\mathrm{B2}}(0),$$
$$\rho_{44}^{\mathrm{B2}}(t)=1-\rho_{11}^{\mathrm{B2}}(t)-2\rho_{22}^{\mathrm{B2}}(t),\quad \rho_{14}^{\mathrm{B2}}(t)=\rho_{41}^{\mathrm{B2}}(t)=(1-\gamma)\rho_{14}^{\mathrm{B2}}(0); \tag{A4}$$
$$\rho_{11}^{\mathrm{A1}}(t)=(1-\gamma/2)^2\rho_{11}^{\mathrm{A1}}(0)+\left(\gamma-\gamma^2/2\right)\rho_{22}^{\mathrm{A1}}(0),$$
$$\rho_{22}^{\mathrm{A1}}(t)=\rho_{33}^{\mathrm{A1}}(t)=\left(\gamma/2-\gamma^2/2\right)\rho_{11}^{\mathrm{A1}}(0)+\left(1-\gamma+\gamma^2/2\right)\rho_{22}^{\mathrm{A1}}(0),$$
$$\rho_{44}^{\mathrm{A1}}(t)=1-\rho_{11}^{\mathrm{A1}}(t)-2\rho_{22}^{\mathrm{A1}}(t),\quad \rho_{23}^{\mathrm{A1}}(t)=\rho_{32}^{\mathrm{A1}}(t)=(1-\gamma)^2\rho_{23}^{\mathrm{A1}}(0); \tag{A5}$$
$$\rho_{11}^{\mathrm{A2}}(t)=\rho_{44}^{\mathrm{A1}}(t),\quad \rho_{22}^{\mathrm{A2}}(t)=\rho_{33}^{\mathrm{A2}}(t)=\rho_{22}^{\mathrm{A1}}(t)=\rho_{33}^{\mathrm{A1}}(t),$$
$$\rho_{44}^{\mathrm{A2}}(t)=\rho_{11}^{\mathrm{A1}}(t),\quad \rho_{23}^{\mathrm{A2}}(t)=\rho_{32}^{\mathrm{A2}}(t)=\rho_{23}^{\mathrm{A1}}(t)=\rho_{32}^{\mathrm{A1}}(t); \tag{A6}$$
$$\rho_{11}^{\mathrm{B1}}(t)=(1-\gamma/2)^2\rho_{11}^{\mathrm{B1}}(0)+\gamma^2/4\rho_{44}^{\mathrm{B1}}(0),$$
$$\rho_{22}^{\mathrm{B1}}(t)=\rho_{33}^{\mathrm{B1}}(t)=\left(\gamma/2-\gamma^2/2\right)\rho_{11}^{\mathrm{B1}}(0)+\left(\gamma/2-\gamma^2/2\right)\rho_{44}^{\mathrm{B1}}(0),$$
$$\rho_{44}^{\mathrm{B1}}(t)=1-\rho_{11}^{\mathrm{B1}}(t)-2\rho_{22}^{\mathrm{B1}}(t),\quad \rho_{14}^{\mathrm{B1}}(t)=\rho_{41}^{\mathrm{B1}}(t)=(1-\gamma)^2\rho_{14}^{\mathrm{B1}}(0); \tag{A7}$$
$$\rho_{11}^{\mathrm{B2}}(t)=\rho_{44}^{\mathrm{B1}}(t),\quad \rho_{22}^{\mathrm{B2}}(t)=\rho_{33}^{\mathrm{B2}}(t)=\rho_{22}^{\mathrm{B1}}(t)=\rho_{33}^{\mathrm{B1}}(t),$$
$$\rho_{44}^{\mathrm{B2}}(t)=\rho_{11}^{\mathrm{B1}}(t),\quad \rho_{14}^{\mathrm{B2}}(t)=\rho_{41}^{\mathrm{B2}}(t)=\rho_{14}^{\mathrm{B1}}(t)=\rho_{41}^{\mathrm{B1}}(t). \tag{A8}$$

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