Dark states and Aharonov–Bohm oscillations in multi-quantum-dot systems*

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We study the formation of dark states and the Aharonov–Bohm effect in symmetrically/asymmetrically coupled three- and four-quantum-dot systems. It is found that without a transverse magnetic field, destructive interference can trap an electron in a dark state. However, the introduction of a transverse magnetic field can disrupt the dark state, giving rise to oscillation in current. For symmetrically structured quantum-dot systems, the oscillation has a period of one flux quanta. But for asymmetrically structured dot systems, the period of oscillation is halved. In addition, the dephasing due to charge noise also blocks the formation of dark states, while it does not change the period of oscillation.

Keywords: quantum dot, dark state, Aharonov–Bohm effect

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1. Introduction

Dark state[1] is a specific coherent superposition of quantum states which is immune to further excitation due to destructive quantum interference. It has played an important role in studying a number of coherent phenomena in physics such as electromagnetically induced transparency (EIT),[2−5] coherent population trapping (CPT),[6−11] lasing without inversion,[12−14] slow propagation of light in a medium,[15,16] optical information storage,[17,18] and so on. It also has intrinsical advantages for quantum computation and quantum information processing.

Aharonov–Bohm (AB)[19] effect is also a basic type of quantum-coherent effect, which has received considerable attention in recent years from both theoretical[20−23] and experimental aspects.[24,25] The analogy between real and artificial atoms (i.e., quantum dots, abbreviated to QDs below) suggests the transfer of concepts from atomic physics to ultrasmall semiconductor structures. So the basic quantum mechanical effects such as CPT and AB effect may also be realized and controlled in artificial microscopic devices (QD systems).

In this paper, we study the creation of dark states and AB effect in symmetrically/asymmetrically coupled three- and four-QD systems. We find it possible to create dark states and observe current oscillations in these multi-QD systems. Moreover, there are different characteristics of current oscillations for symmetrically and asymmetrically structured QD systems. The rest of the present paper is organized as follows. In the next section, we first give the general interaction model for a coupled N-QD system in the presence of an external magnetic field. Then in Sections 3, 4 and 5, we study the formations of dark states and AB oscillations in three- and four-QD systems respectively, where the stationary currents for zero-field and nonzero-field are given explicitly, and the influence of dephasing due to charge noise is considered. Finally, a conclusion is presented in Section 6.

2. Interaction model of a multi-QD system

For unification, we first give the interaction model of a coupled multi-QD system. Consider N QDs which couple into a ring-like geometry as shown in Fig. 1. A transverse magnetic field $\boldsymbol{B}$ orthogonally threads the ring so as to induce a magnetic flux Φ. Assume that the system works in the strong Coulomb block-

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ade regime so that there is at most one excess electron in the multi-QD system at any time. The spin degree of freedom is not relevant to our discussion and electron will be assumed to be spinless. The interaction Hamiltonian of the corresponding model can be written as[26]

$$H[\phi] = t\sum_{j=1}^{N}(\mathrm{e}^{\mathrm{i}\,\phi/N}a_j^{\dagger}a_{j+1} + \mathrm{h.c.}), \tag{1}$$

where a_j is the electron annihilation operator at the j-th dot which meets the circular condition $a_{N+1} = a_1$. The flux Φ does not exert a force on the electron, but can change the local phase of its wave function due to AB effect. The phase $\phi = 2\pi\Phi/\Phi_0$ with magnetic flux quanta $\Phi_0 = h/e$ is responsible for this effect. We assume that the tunneling strengths between all neighbouring dots are equal to t, which may be taken to be real and positive due to the time-reversal symmetry in the absence of magnetic field. By choosing a gauge transformation $c_{j+1}^{\dagger} = a_j^{\dagger}\mathrm{e}^{-\mathrm{i}\,\phi j/N}$, the interaction Hamiltonian can be transformed as

$$H[\phi] = t(c_1^{\dagger}c_2\mathrm{e}^{\mathrm{i}\,\phi} + c_2^{\dagger}c_3 + \cdots + c_N^{\dagger}c_1 + \mathrm{h.c.}). \tag{2}$$

Now the phase ϕ is only accumulated on the bond between dots 1 and 2. In the following sections, we will work in the natural basis $\{|j\rangle, j = 1, \ldots, N\}$, where $|j\rangle$ means the state with an extra electron in dot j and can be written as

$$|j\rangle = c_j^{\dagger}|\mathrm{vac}\rangle \tag{3}$$

with $c_j^{\dagger}$ being the creation operator and $|\mathrm{vac}\rangle$ a reference state in which all dots have no extra electrons.

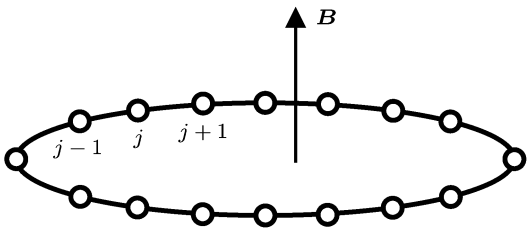


Fig. 1. The N QDs coupled into a ring with equal tunneling strength t between all neighbouring dots. The system consists of only one extra unpaired electron and is threaded by a magnetic field $\boldsymbol{B}$.

3. Three-QD system

As the symmetrically structured three-dot system has been studied already,[27] we thus begin with an asymmetrically structured three-dot system (see Fig. 2), which includes a source S and a drain D with rates Γ_1 and Γ_3 respectively. The tunneling rates between all neighbouring dots are equal to t. The total Hamiltonian for the system in the basis $\{|1\rangle, |2\rangle, |3\rangle\}$ (here $|j\rangle$ is defined by Eq. (3)) is

$$H = \begin{pmatrix} \Delta & t\mathrm{e}^{\mathrm{i}\,\phi} & t \\ t\mathrm{e}^{-\mathrm{i}\,\phi} & -\Delta & t \\ t & t & \epsilon \end{pmatrix}, \tag{4}$$

where we take the energy of the centre between levels $|1\rangle$ and $|2\rangle$ to be zero so that the energies of levels $|1\rangle, |2\rangle, |3\rangle$ may be denoted as Δ, $-\Delta$ and ϵ respectively. Within Born–Markov and infinite-bias approximations, the density matrix $\rho(t)$ of the system that contains entries not only for the three single electron states $|j\rangle$ with $j = 1, 2, 3$, but also for the empty state $|0\rangle$ follows master equation,

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}[H, \rho] + \sum_{k=1,3}\Big\{D_k\rho D_k^{\dagger} - \frac{1}{2}D_k^{\dagger}D_k\rho - \frac{1}{2}\rho D_k^{\dagger}D_k\Big\}, \tag{5}$$

where the quantum jump operators $D_1 = \sqrt{\Gamma_1}|1\rangle\langle 0|$ and $D_3 = \sqrt{\Gamma_3}|0\rangle\langle 3|$ describe irreversible tunneling of electrons into and out of the system with rates Γ_i. In this paper, we assume $\Gamma_1 = \Gamma_3 = \Gamma$. Subject to $\mathrm{d}\rho/\mathrm{d}t = 0$, we can obtain the stationary state $\rho(\infty)$. Thus the stationary average current through the system reads $\langle I\rangle = \Gamma\rho_{33}(\infty)$. If $\langle I\rangle = 0$, we say that the system is in a dark state. In the following, we will develop our study for two cases: without magnetic field ($\Phi = 0$) and with magnetic field ($\Phi \neq 0$) separately.

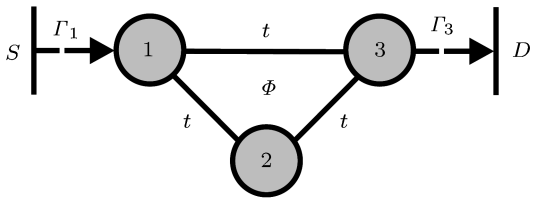


Fig. 2. Three QDs coupled coherently to one another via coupling t and incoherently to a source and a drain with rates Γ_1 and Γ_3 respectively, where dot 1 is connected to source and dot 3 to drain. A perpendicular magnetic flux Φ threads the system.

For zero magnetic field ($\Phi = 0$), by tuning the backgates under the dots so as to $\epsilon = 0$ (i.e., the energy of state $|3\rangle$ is zero), we have the stationary current from the stationary solution of Eq. (5) as

$$\langle I\rangle = \frac{4\Gamma t^2\Delta^2}{\Gamma^2 t^2 - \Gamma^2 t\Delta + 16t^2\Delta^2 + \Gamma^2\Delta^2 - 4t\Delta^3 + 4\Delta^4}. \tag{6}$$

It is clearly that $\Delta = 0$ will induce $\langle I\rangle = 0$, which means that the system is in a dark state. By taking the stationary solution of Eq. (5) under conditions $\Delta = 0$ and $\epsilon = 0$ (i.e., the levels $|1\rangle, |2\rangle$ and $|3\rangle$ degenerate), we obtain the dark state as

$$|\Psi_{\rm dark}\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle). \tag{7}$$

The physical meaning of this dark state is very obvious: the electronic density at the drain dot is zero, thus the stationary current through the device is exactly zero. Note that this dark state has the same form as that of Ref. [27], but the system devices are different: the geometry of the three QDs in Ref. [27] is symmetrical, but here ours is asymmetrical.

The application of magnetic field will lift the current blockade of dark state and give rise to oscillation in the current. With $\Delta = \epsilon = 0$, we have the stationary current in this case as

$$\langle I(\phi)\rangle = \frac{4\Gamma t^2\sin^2\phi}{\Gamma^2 + 2t\Gamma\sin\phi + 16t^2\sin^2\phi}. \tag{8}$$

The behaviour that $\langle I(\phi)\rangle$ oscillates with ϕ is shown in Fig. 3 (solid line). Now only at some discrete flux that satisfies $\Phi/\Phi_0 = 0.5n$ with n positive integers, can the dark states be maintained. Otherwise, they are broken. The current oscillation arises due to the accumulation of a phase difference ϕ between different paths through the device. It is clear that the period of current oscillation is half of a flux quanta, i.e., $\Phi_0/2$. Compared with the symmetrically structured three-QD system,[27] where the period of current oscillation is one flux quanta, the asymmetrically structured three-QD system halves the period of current oscillation. Interestingly, this rule also holds for the four-QD systems discussed below. Compared with the traditional AB effect,[19] this type of current oscillation in QD systems may be easier to observe through the antiresonance current.

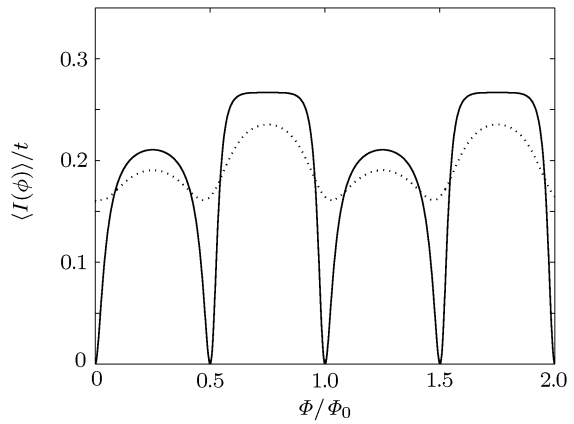


Fig. 3. The stationary current through the three-dot system, showing pronounced oscillations as a function of the applied flux. Solid line is for no dephasing (where we set $t = \Gamma$ and $\Delta = \epsilon = 0$) and dot line for dephasing (where we set $t = \Gamma = \gamma$ and $\Delta = \epsilon = 0$).

In the above discussion of current oscillation, we have not yet included the influence of dephasing due to charge noise, which may be considered through the introduction of the jump operators

$$D_i^\gamma = \sqrt{\gamma}|i\rangle\langle i|, \quad i = 1, 2, 3. \tag{9}$$

For simplicity, we assume the decoherence rate γ to be the same for each dot. Entering these jump operators into Eq. (5) in the same way as doing the jump operators D_i, we plot the stationary current as a function of ϕ in Fig. 3 (dot line). It is shown that under the influence of dephasing, though the period of current oscillation does not vary, the oscillation amplitude changes visibly and the dark states vanish completely.

4. Symmetrical four-QD system

For a symmetrically structured four-QD system, we arrange the structure as shown in Fig. 4. The total Hamiltonian in the basis $\{|1\rangle, |2\rangle, |3\rangle, |4\rangle\}$ can be written as

$$H = \begin{pmatrix} \Delta & t{\rm e}^{{\rm i}\,\phi} & 0 & t \\ t{\rm e}^{-{\rm i}\,\phi} & -\omega & t & 0 \\ 0 & t & 0 & t \\ t & 0 & t & \omega \end{pmatrix}, \tag{10}$$

where we assume that the energies of states $|1\rangle$, $|2\rangle$, $|3\rangle$ and $|4\rangle$ are Δ, $-\omega$, 0 and ω respectively, which can be reached by tuning the backgates under the dots. The meanings of other parameters are the same as before. The quantum equation of motion in this case is still described by Eq. (5), with the Hamiltonian H replaced

by Eq. (10). We will still assume $\Gamma_1 = \Gamma_3 = \Gamma$.

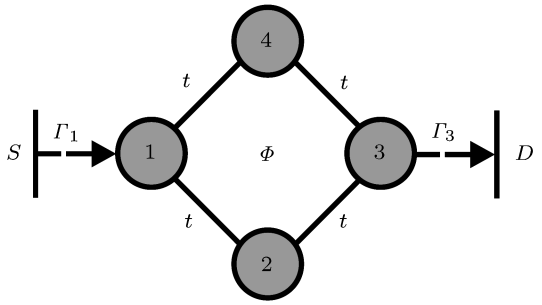


Fig. 4. Four QDs coupled coherently to one another via coupling t and incoherently to source and drain with rate Γ_i. A perpendicular magnetic flux Φ threads the system.

Without magnetic field ($\Phi = 0$), the stationary average current now becomes

$$\langle I\rangle = \frac{16\Gamma t^4\Delta^2}{A} \tag{11}$$

with

$$\begin{aligned} A =\ & 16t^2\Delta^2\omega^2 + 4\Gamma^2t^2\Delta^2 + 16t^2\Delta^4 \\ & + 64t^4\Delta^2 + 4\Gamma^2t^4 + 4\Gamma^2t^2\omega^2 \\ & + 4\Delta^6 + \Gamma^2\Delta^4 - 8\Delta^4\omega^2 \\ & - 2\Gamma^2\Delta^2\omega^2 + 4\Delta^2\omega^4 + \Gamma^2\omega^4. \end{aligned} \tag{12}$$

Thus $\Delta = 0$ induces $\langle I\rangle = 0$, and a dark state

$$|\Psi^{\rm s}_{\rm dark}\rangle = \frac{1}{\sqrt{\omega^2+2t^2}}(\omega|1\rangle + t|2\rangle - t|4\rangle) \tag{13}$$

is created. This dark state is a superposition of all dot states except for the drain one.

When a magnetic field is applied to the system and with $\Delta = 0$, the stationary current becomes

$$\langle I(\phi)\rangle = \Gamma\frac{B}{C} \tag{14}$$

with

$$B = 8t^4(\omega^2\cos\phi - \omega^2 - t^2\sin^2\phi) \tag{15}$$

and

$$\begin{aligned} C =\ & -40t^6\sin^2\phi + (40t^4\omega^2 + 4\Gamma^2t^4)(\cos\phi - 1) \\ & + 8\Gamma t^4\omega\sin\phi - \Gamma^2t^2\omega^2(1+\cos\phi) - \Gamma^2\omega^4. \end{aligned} \tag{16}$$

The stationary current $\langle I(\phi)\rangle$ as a function of ϕ is plotted in Fig. 5 (solid line). It is clear that for this symmetrically structured four-QD system, the current oscillates in a regular way with flux period Φ_0 which is the same as that of symmetrically structured three-QD system.[27] Only at $\Phi = n\Phi_0$, will the current be zero where the CPT or a dark state is maintained.

By introducing phase noise operators

$$D_i^\gamma = \sqrt{\gamma}|i\rangle\langle i|, \quad i = 1,2,3,4, \tag{17}$$

for each dot, with $\Delta = \omega = 0$ (i.e., all levels $|1\rangle, |2\rangle, |3\rangle$ and $|4\rangle$ degenerate) and $\gamma = \Gamma = t$, the relation between stationary current and flux is pictured in Fig. 5 (dot line). Due to dephasing, the current blocking of dark state at integral multiple of Φ_0 cancels, but the period of oscillation remains invariable.

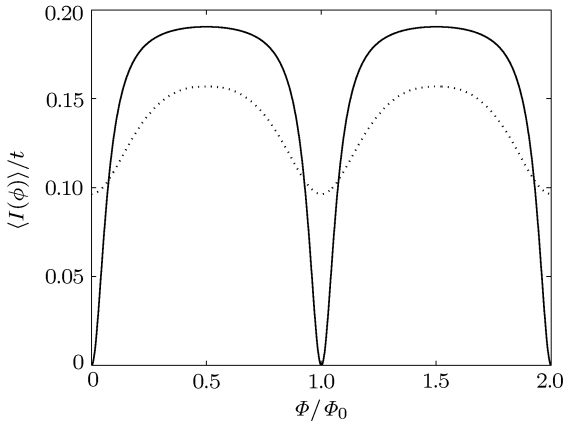


Fig. 5. Stationary current through the four-dot system, showing pronounced oscillations as a function of the applied flux. Solid line is for no dephasing (where we set parameters $t = \Gamma$ and $\Delta = \omega = 0$) and dot line for dephasing (where we set parameters $t = \Gamma = \gamma$ and $\Delta = \omega = 0$).

5. Asymmetric four-QD system

Finally, we consider an asymmetrically structured four-dot system (see Fig. 6). The quantum equation of motion in this case may still be described by Eq. (5), with the Hamiltonian H still given by Eq. (10) and the summation index replaced by $k = 1, 4$. We still assume that the tunneling rates for the source and drain are equal, i.e., $\Gamma_1 = \Gamma_4 = \Gamma$.

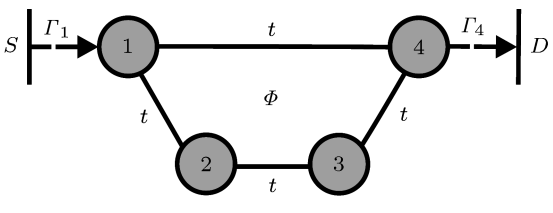


Fig. 6. Four QDs coupled coherently to one another via coupling t and incoherently to source and drain with rate Γ_i in an asymmetrical structure. A perpendicular magnetic flux Φ threads the system.

Without magnetic field ($\Phi = 0$), the stationary average current now becomes

$$\langle I\rangle = \frac{4\Gamma t^2\Delta^2(\Delta+2\omega)^2}{E} \tag{18}$$

with

$$E = 64\Delta^2 + 16t^4 + 4\Delta^4 + 8\omega - 16\Delta^3\omega^3 + 32\Delta^2\omega^4 + \Delta^4\Gamma^2 - 12\omega^2 + 4\Delta^6 - 8t^2\Delta\Gamma^2\omega + 112t^2\Delta^2\omega^2 + 32\Delta^3\omega t^2 + 4\Delta^3\omega\Gamma^2 + 5\Delta^2\omega^2\Gamma^2. \quad (19)$$

When $\Delta = 0$, a dark state with the form

$$|\Psi^{\rm a}_{\rm dark}\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |3\rangle) \quad (20)$$

is created. Interestingly, this state is a superposition of only states of dots 1 and 3, including neither the state of drain dot 4 nor the one of dot 2. It has clearly a difference in structural form from the dark state of Eq. (13) for the symmetrically structured four-QD system where only the state of drain dot is not included.

When a magnetic field is applied to the system and with $\Delta = 0$, the stationary current becomes

$$\langle I(\phi)\rangle = \Gamma\frac{8t^2\sin^2\phi}{G} \quad (21)$$

with

$$G = 4\Gamma\omega\sin\phi + 32\omega^2(1-\cos\phi) + \Gamma^2(3-\cos\phi) + 40t^2\sin^2\phi. \quad (22)$$

The stationary current $\langle I(\phi)\rangle$ as a function of ϕ is plotted in Fig. 7 (solid line). It is clear that the current oscillates in a regular way with a period of half of a flux quanta, $\Phi_0/2$, which is the same as that of the asymmetrically structured three-QD system discussed above. At $\Phi = n\Phi_0/2$, the current is still zero where the CPT or dark state is maintained.

As in the symmetrically structured four-QD system, by introducing the jump operators of Eq. (17) into quantum equation of motion, under the parameter conditions $\Delta = \omega = 0$ and $\gamma = \Gamma = t$, we plot the stationary current versus flux in Fig. 7 (dot line). It is again shown that due to dephasing, the current blocking of dark state at integral multiple of $\Phi_0/2$ disappears, while the period of oscillation remains unchanged.

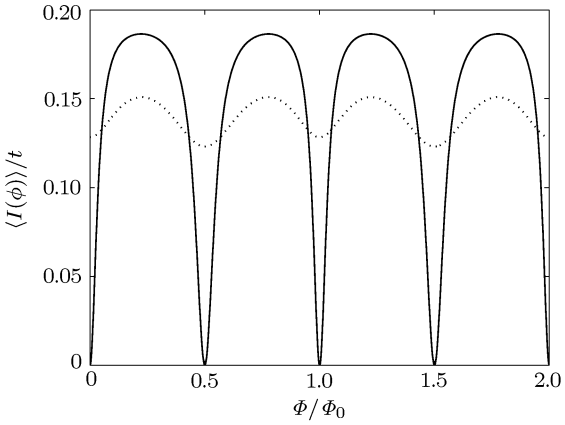


Fig. 7. The stationary current through the asymmetric four-dot system, showing pronounced oscillations as a function of the applied flux. Solid line is for no dephasing (where we set parameters $t = \Gamma$ and $\Delta = \omega = 0$) and dot line for dephasing (where we set parameters $t = \Gamma = \gamma$ and $\Delta = \omega = 0$).

6. Conclusion

We have studied the formation of dark states and the AB effect in coupled three- and four-QD systems, where two basic types of quantum coherence, coherent population trapping and AB effect, interplay each other, giving rise to dark states and current oscillations. The concrete form of dark states and the period of current oscillations depend on QD geometric structure. We have shown that the period of current oscillation for asymmetrically structured three- or four-QD system is twice that for the corresponding symmetrically structured case. The dephasing due to charge noise breaks the formation of dark states, but does not change the period of current oscillation. For systems with more QDs, we find that similar phenomena of dark states and AB current oscillations also exist. We expect these mesoscopic quantum coherence to be applied to quantum information science.

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