Chaos in complex motor networks induced by Newman–Watts small-world connections*

Wei Du-Qu(韦笃取)[a)†], Luo Xiao-Shu(罗晓曙)[a)], and Zhang Bo(张 波)[b)]

[a)] *College of Electronic Engineering, Guangxi Normal University, Guilin* 541004, *China*

[b)] *College of Electric Power, South China University of Technology, Guangzhou* 510640, *China*

(Received 1 June 2011; revised manuscript received 20 June 2011)

We investigate how dynamical behaviours of complex motor networks depend on the Newman–Watts small-world (NWSW) connections. Network elements are described by the permanent magnet synchronous motor (PMSM) with the values of parameters at which each individual PMSM is stable. It is found that with the increase of connection probability p, the motor in networks becomes periodic and falls into chaotic motion as p further increases. These phenomena imply that NWSW connections can induce and enhance chaos in motor networks. The possible mechanism behind the action of NWSW connections is addressed based on stability theory.

Keywords: complex networks, small-world connections, chaos, permanent magnet synchronous motor

PACS: 89.75.Hc, 05.45.Ac, 05.45.Pq **DOI:** 10.1088/1674-1056/20/12/128903

1. Introduction

Permanent magnet synchronous motors (PMSMs) have superior features such as compact size, high torque/inertia ratio, high torque/weight ratio, absence of rotor losses, and advantages of higher efficiency compared with induction motors.[1−3] Over the past few years, the secure and stable operation of PMSMs, which is an essential requirement of industrial automation manufacturing, has received considerable attention. For instance, Hemati *et al.*[4] investigated the conditions leading to bifurcation and chaos in the PMSM and found that the systems under investigation possess multiple equilibria which profoundly affect their global stability and dynamic characteristics. Li *et al.*[5] studied the dynamics of PMSM-based stability theory of a nonlinear system and found that when the values of parameters change, the PMSM experiences complex behaviours such as limit cycles, chaos oscillation, etc. Reference [6] analysed the stability of PMSMs in the uncontrolled and controlled cases inspired by the Lorenz system. Coria *et al.*[7] examined the localization problem of compact invariant sets of the system describing dynamics of the PMSM, and illustrated localization analysis of chaotic attractors existing for some parameters of the PMSM by numerical simulation. Jing *et al.*[8] investigated the dynamics in PMSMs with non-smooth air gap by theoretical analysis and numerical simulations and found some new results including the cyclic fold bifurcation, double-scroll chaotic attractors, ribbon-chaotic attractor, as well as intermittent chaos. Very recently, we have investigated the effects of current time-delayed feedback on the stability of PMSMs where the delay time is both fixed and varied. It was found that in comparison with the static delay time feedback, the variable delay time feedback can obtain stabilization of unstable steady states over a much larger domain of parameters.[9] On the other hand, with the development of larger-scale industrial processes, device-level communication networks brought new possibilities for advanced monitoring, control, and diagnostics. These networks offered the opportunity for greatly simplified wiring, eliminating the bundles of control interwiring, and corresponding complex interwiring diagrams, while the complexity of operation in industrial driven systems has increased. So, it is significant to study the effects of network topology on the dynamics of driven motors.

Over the past decade, investigation of complex networks has witnessed dramatic advances owing to their relevance to a variety of engineering, physical,

*Project supported by the Key Program of the National Natural Science Foundation of China (Grant No. 50937001), the National Natural Science Foundation of China (Grant Nos. 10862001 and 10947011), and the Construction of Key Laboratories in Universities of Guangxi, China (Grant No. 200912).

†Corresponding author. E-mail: weiduqu@mailbox.gxnu.edu.cn

© 2011 Chinese Physical Society and IOP Publishing Ltd

biological, and social phenomena.[10−12] Small-world (SW) topology is particularly useful in providing insights into the implications of complex connectivity, which is characterized by a given fraction of so-called long-range couplings or shortcut links that connect distant units of the system, while all other units are coupled in a diffusive-like manner.[13−15] Most works on SW networks have focused on understanding how the topological connections influence the system's dynamic behaviours. For instance, it is shown that the introduction of SW connections can induce and enhance the activity in Hindmarsh–Rose neural networks,[16] enhance phase synchronization in Chua networks,[17] and discrete neural networks,[18] etc. All these works show that the existence of the SW connectivity plays a vital role in the system's dynamics. The main aim of this work is to investigate how the Newman–Watts small-world (NWSW) connectivity affects the dynamical behaviours of motor networks, whose topology is constructed by randomly adding shortcuts to an originally nearest-neighbour coupled network.[19] The network element is described by the three-dimensional PMSM model with parameters admitting stable operation.[4−6] We mainly concentrate on how the topological randomness p would affect the dynamics of system. It is found that with the increase of the probability p of the network topology, motor systems transfer from steady to periodic motion and fall into chaos as p is further increased. That is, NWSW connectivity can induce and enhance chaos in motor networks. The possible mechanism behind the action of NWSW connectivity is also addressed based on Lyapunov stability theory.

2. The network structure

The dimensionless mathematical model of the PMSM is given by[4−6]

$$\begin{cases} \dfrac{\mathrm{d}I_q}{\mathrm{d}t} = -I_q - I_d\omega + \gamma\omega + V_q, \\ \dfrac{\mathrm{d}I_d}{\mathrm{d}t} = -I_d + I_q\omega + V_d, \\ \dfrac{\mathrm{d}\omega}{\mathrm{d}t} = \sigma(I_q - \omega) - T_L, \end{cases} \tag{1}$$

where, I_q, I_d, and ω are state variables, which denote quadrature (q)-, direct (d)-axis current and angle speed of the motor, respectively; $\sigma = bL/JR$ and γ are system operating parameters, which are positive; $V_q = K_T L/bR^2 u_q$ and $V_d = K_T L/bR^2 u_d + K_T^2/bR + \gamma$ are control signals, where $K_T = 0.31$ Nm/A, $L = L_q = L_d = 14.25$ mH, $R = 11.6\ \Omega$, $\psi = 0.96$, $J = 4.7 \times 10^{-6}$ K·g·m^2, and $b = 0.023$ N(rad/s). Considering the case that the system is uncontrolled and the motor is running freely under no loading conditions, namely, $V_q = 0$, $V_d = 0$, and $T_L = 0$,[4−6] the system (1) becomes:

$$\begin{cases} \dfrac{\mathrm{d}I_q}{\mathrm{d}t} = -I_q - I_d\omega + \gamma\omega, \\ \dfrac{\mathrm{d}I_d}{\mathrm{d}t} = -I_d + I_q\omega, \\ \dfrac{\mathrm{d}\omega}{\mathrm{d}t} = \sigma(I_q - \omega). \end{cases} \tag{2}$$

Introducing SW connectivity, we can write the motor network as

$$\begin{cases} \dfrac{\mathrm{d}I_q^i}{\mathrm{d}t} = -I_q^i - I_d^{ii}\omega^i + \gamma^i\omega^i + \dfrac{C}{N}\displaystyle\sum_{j=1}^{N} a_{ij}(I_q^i - I_q^j), \\ \dfrac{\mathrm{d}I_d^i}{\mathrm{d}t} = -I_d^i + I_q^i\omega^i + \dfrac{C}{N}\displaystyle\sum_{j=1}^{N} a_{ij}(I_d^i - I_d^j), \\ \dfrac{\mathrm{d}\omega^i}{\mathrm{d}t} = \sigma^i(I_q^i - \omega^i), \end{cases} \tag{3}$$

where indices $i = 1, \ldots, N$ specify the motors; $(C/N)\displaystyle\sum_{j=1}^{N} a_{ij}(I_q^i - I_q^j)$ and $(C/N)\displaystyle\sum_{j=1}^{N} a_{ij}(I_d^i - I_d^j)$ are the coupling term of the complex motors, where C is the coupling strength, and the matrix a_{ij} encodes the connection topology: if a connection exists between motors i and j, $a_{ij} = a_{ji} = 1$; otherwise, $a_{ij} = a_{ji} = 0$. Also, $a_{ii} = 0$ for all i. In this motor network, the NWSW connection patterns is achieved as follows.[19] It starts from a ring lattice with $N = 200$ motors and $k = 2$ nearest neighbours, and then links are randomly added between non-nearest vertices with probability p. For $p = 0$, this reduces to the original nearest-neighbour coupled network; for $p = 1$, it becomes a globally coupled network. Note that for a given p there are a lot of network realizations. Figure 1 illustrates an example of NWSW network formation with 10 nodes. The equivalent circuit for two arbitrarily coupled PMSMs in the networks are shown in Fig. 2.

The individual dynamics of the PMSM, which depends on the parameters σ and γ, has been investigated fully in Refs. [4]–[6]. The results of those investigations showed that with the change of parameters, the individual PMSM experienced steady states, limit cycles, period-doubling bifurcation, chaos, etc. In this paper, in order to study the effect of the NWSW connections on the dynamics of motor networks, we fix $\sigma^i = 4$ and generate $\gamma^i \in [4, 8]$ at random, such that the original state of each individual PMSM is stable with nonidentical properties.

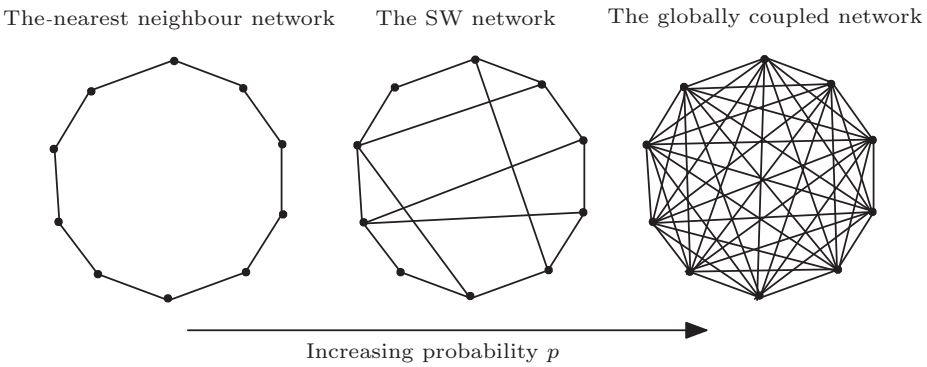


Fig. 1. An example of the Newman–Watts small-world formation with 10 nodes.

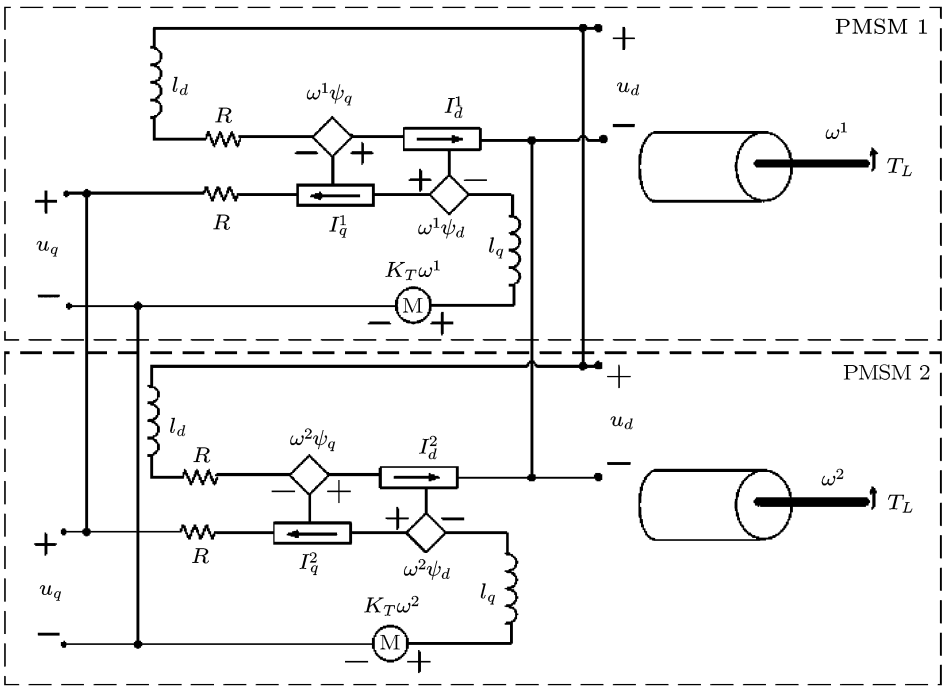


Fig. 2. The equivalent circuit for arbitrary two coupled PMSMs in the motor networks.

3. Numerical results and mechanism analysis

The results discussed hereafter are obtained by varying p. For each p, 20 network realizations are generated. In each network realization, the initial values of 200 PMSM dynamical variables are newly chosen randomly. We numerically integrate Eq. (3) using the fourth-order Runge–Kutta method with a time step of $h = 0.01$. In the numerical analysis, three conventional indices, namely, the time series, phase diagram, and power spectrum (PS) are adopted to describe the NWSW connection-induced responses in the motor networks, where the phase diagram is the evolution of a set of trajectories emanating from various initial conditions in the state space. When the solution reaches a stable state, the asymptotic behaviour of the phase trajectories is particularly interesting and the transient behaviour in the system is neglected. The PS is utilized to capture the frequency content of a stochastic process and helps us to distinguish between stable periodic, quasi-periodic, and chaotic behaviour for a dynamical system.[20] We obtain the following results.

Figures 3–5 show the dynamical behaviours of any motor in the network at $C = 0.5$ with $p = 0$, $p = 0.05$, and $p = 0.5$, respectively. From Figs. 3(a)–3(c) one can see clearly that in the absence of NWSW connections the motor is operating in the stable regime. With the increase of p, the periodic motion of mo-

tor systems appears (as shown in Figs. 4(a)–4(c)). From Figs. 5(a)–5(c) we can see that the PS transfers to a noise-like spectrum which shows the NWSW-induced chaos in motor networks. These phenomena imply that NWSW connections can induce and enhance chaos in motor networks.

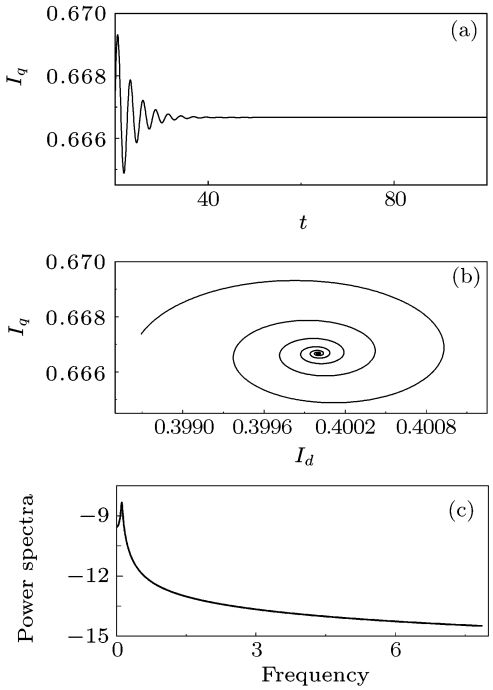


Fig. 3. Stable motion in motor networks when $p = 0$: (a) time series, (b) phase diagram, and (c) the log of the power spectrum.

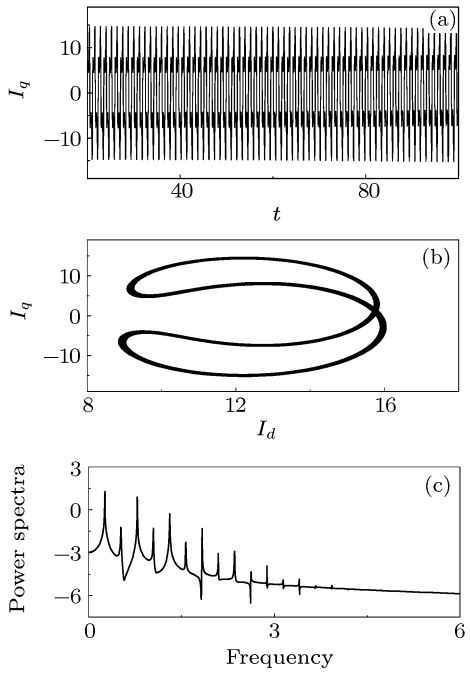


Fig. 4. NWSW-induced period motion in motor networks with $p = 0.05$: (a) time series, (b) phase diagram, and (c) the log of the power spectrum.

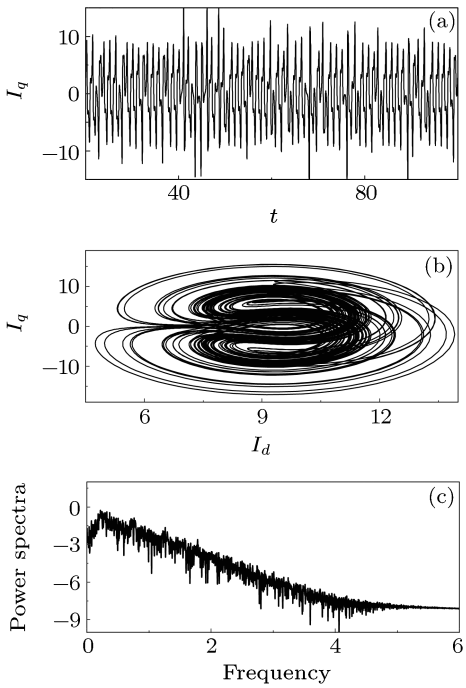


Fig. 5. NWSW-induced chaotic motion in motor networks with $p = 0.5$: (a) time series, (b) phase diagram, and (c) the log of the power spectrum.

The possible mechanism behind the action of NWSW connections may be analysed as follows based on the Lyapunov stability theory. We consider a general complex dynamical network consisting of such N linearly coupled identical nodes. The network is specified by[21]

$$\dot{X}^i(t) = f(X^i(t)) - \frac{C}{N}\sum_{j=1}^{N} a_{ij}X^j(t),$$
$$i = 1, 2, \ldots, N, \tag{4}$$

where $X^i(t) = [x_1^i(t), x_2^i(t), \ldots, x_n^i(t)]^{\mathrm{T}} \in R_n$ are the state variables of node i, and C is the coupling strength; coupling matrix $A = (a_{ij})_{N\times N} \in R^{N\times N}$ represents the coupling configuration of the network, where $a_{ij} = a_{ij} = 1$ if there is a connection between node i and node j $(i \neq j)$; otherwise, $a_{ij} = a_{ij} = 0$ $(i \neq j)$ and

$$a_{ii} = -\sum_{j=1, i\neq j}^{N} a_{ij} = -\sum_{j=1, i\neq j}^{N} a_{ij}.$$

We assume that each isolated node in the network (4) is in stable steady state before they are coupled together. Hence, 0 is an eigenvalue of matrix A and all the other eigenvalues of A (denoted by λ_i, $i = 1, 2, \ldots, N$) are real and are strictly negative:

$$0 = \lambda_1 > \lambda_2 \geq \cdots \geq \lambda_N. \tag{5}$$

The Lyapunov exponents h_i $(i = 1, 2, \ldots, N)$ of each node are negative; we let

$$0 \geq h_{\max} = h_1 \geq h_2 \geq \cdots \geq h_N. \tag{6}$$

Rewriting the dynamical network (4) in a vector form:

$$\dot{X}^i(t) = f(X(t), C), \tag{7}$$

whose Lyapunov exponents are

$$\eta_i = \lim_{t\to\infty} \frac{1}{t} \ln |Df(t, X_0) \cdot \eta_i|, \quad i = 1, 2, \ldots, N. \tag{8}$$

Where $Df(t, X_0)$ is the Jacobian matrix and X_0 is a random initial value, and η_i $(i = 1, 2, \ldots, N)$ are a set of orthonormal vectors in the tangent space of the ordinary differential equations. The Lyapunov exponents η_i of the coupled dynamical networks (4) should satisfy[22,23]

$$\eta_i(\lambda_K) = h_i - C\lambda_K, \quad K = 1, 2, \ldots, N. \tag{9}$$

Due to relations (5) and (6) and $C > 0$, we have

$$\eta_1(\lambda_K) \geq \eta_2(\lambda_K) \geq \cdots \eta_N(\lambda_K), \ K = 1, 2, \ldots, N, \tag{10}$$

$$\eta_i(\lambda_1) \leq \eta_i(\lambda_2) \geq \cdots \eta_i(\lambda_N), \ i = 1, 2, \ldots, N. \tag{11}$$

If the coupled network (4) is chaotic, then there is at least one positive Lyapunov exponent in inequalities (10) and (11), so the maximum of η_i is $\eta_i(\lambda_N) = h_i - C\lambda_N = h_{\max} - C\lambda_N > 0$, that is, there is a positive Lyapunov exponent in the dynamical network (4) when $C > |h_{\max}|/\lambda_N$. So we obtain the following sufficient conditions for the emergence of chaos in the dynamical network (4):

$$C > \frac{|h_{\max}|}{\lambda_N}, \tag{12}$$

where $h_{\max}$ is an invariable for each node, λ_N is the smallest eigenvalue of the coupling matrix A which denotes the topology of complex networks. Eventually, the values of λ_N are determined by topology probability p and system size N. The dependences of

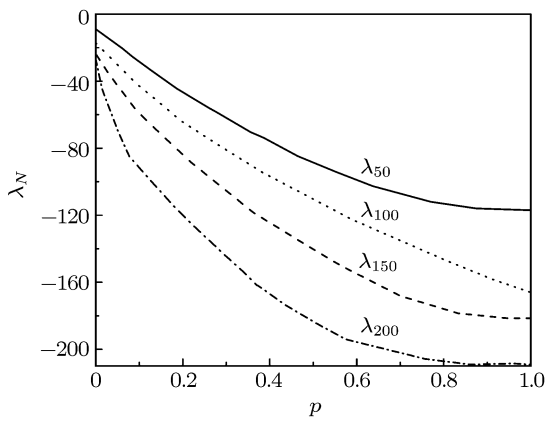


Fig. 6. The dependences of the smallest eigenvalue λ_N on p and N in PMSM network.

λ_N on p and N in a PMSM network are shown in Fig. 6, from which one can see clearly that with the optimal combination for p and N, the sufficient conditions for the emergence of chaos is satisfied.

4. Conclusion

To gain new insights into the effects of NWSW connections on dynamical networks, in this paper we studied the small-world response in motor networks where each individual PMSM is stable. The time series, phase diagram, and power spectrum are employed to measure the evolution of dynamical response. It is found that with the increase of connection probability p, the motor in networks becomes periodic and falls into chaotic motion as p is further increased. These phenomena imply that NWSW connections can induce and enhance chaos in motor networks. The possible mechanism behind the action of NWSW connections was addressed based on the Lyapunov stability theory.

References

[1] Wei D Q, Luo X S, Wang B H and Fang J Q 2007 *Phys. Lett.* A **363** 71

[2] Wei D Q and Zhang B 2009 *Chin. Phys.* B **18** 1399

[3] Wei D Q, Zhang B, Qiu D Y and Luo X S 2009 *Acta Phys. Sin.* **58** 6026 (in Chinese)

[4] Hemati N and Kwatny H 1993 *Proc.* 32*nd Conf. Decision and Control*, San Antonio, Texas, December, p. 475

[5] Li Z, Park J B, Joo Y H, Zhang B and Chen G 2002 *IEEE Trans. Circ. Syst.*-I **49** 383

[6] Zahera A A 2008 *Chaos* **18** 13111

[7] Coria L N and Starkov K E 2009 *Commun. Nonlinear Sci. Numer. Simul.* **14** 3879

[8] Jing Z J, Yu C Y and Chen G R 2004 *Chaos, Solitons and Fractals* **22** 831

[9] Wei D Q, Zhang B, Qiu D Y and Luo X S 2010 *IEEE Trans. Circ. Syst.-II* **57** 456

[10] Newman M E J 2000 *J. Stat. Phys.* **101** 819

[11] Albert R and Barabasi A L 2002 *Rev. Mod. Phys.* **74** 47

[12] Qin S and Dai G Z 2009 *Chin. Phys.* B **18** 383

[13] Watts D J and Strogatz S H 1998 *Nature* **393** 440

[14] Dorogovtsev S N and Mendes J F F 2003 *Evolution of Networks: From Biological Nets to the Internet and WWW* (Oxford, UK: Oxford University Press)

[15] Wang L F, Wang Q L, Kong Z and Jing Y W 2010 *Chin. Phys.* B **19** 080207

[16] Wei D Q, Luo X S and Qin Y H 2008 *Physica* A **387** 2155

[17] Wei D Q, Luo X S and Qin Y H 2009 *Chin. Phys.* B **18** 2184

[18] Wei D Q and Luo X S 2007 *Europhys. Lett.* **78** 68004

[19] Newman M E J and Watts D J 1999 *Phys. Rev.* E **60** 7332

[20] Qin Y H, Luo X S and Wei D Q 2010 *Chin. Phys.* B **19** 050511

[21] Yuan W J, Luo X S, Jiang P Q, Wang B H and Fang J Q 2008 *Chaos, Solitons and Fractals* **37** 799

[22] Li X, Chen G and Ko K T 2004 *Physica* A **338** 367

[23] Zhang H F, Wu R X and Fu X C 2006 *Chaos, Solitons and Fractals* **28** 472