Enhancing the synchronizability of networks by rewiring based on tabu search and a local greedy algorithm*

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By considering the eigenratio of the Laplacian matrix as the synchronizability measure, this paper presents an efficient method to enhance the synchronizability of undirected and unweighted networks via rewiring. The rewiring method combines the use of tabu search and a local greedy algorithm so that an effective search of solutions can be achieved. As demonstrated in the simulation results, the performance of the proposed approach outperforms the existing methods for a large variety of initial networks, both in terms of speed and quality of solutions.

Keywords: synchronizability, network rewiring, tabu search, local greedy, complex networks

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1. Introduction

The study of complex networks pervades all fields of sciences and has been applied to manage many real-world problems, such as virus spreading and control in computer networks,[1] human interaction in social networks,[2,3] study of financial issues,[4] and so on.

One of the key research topics in complex networks is the synchronization or consensus of dynamical networks,[5,6] not only because of the theoretical interest but also its practical usage. To quantify the ability and degree of synchronization of a network, a term called synchronizability is commonly used,[7–13] and a close relationship between network structures and synchronizabilty has been explored.[12,14–16] There are different implications of high synchronizability, such as the ease for synchronization or achieving faster convergence time.[17] Based on the introduction of the master stability function (MSF) in Ref. [18], the synchronizability of a network is now known to be closely related with the largest and smallest nonzero eigenvalues (λ_N and λ_2) of the Laplacian matrix which describes the connection of nodes in the network.

In order to enhance the synchronizability, many different methods have been proposed recently. This is usually achieved by assigning proper weights to the links and/or the coupling strengths,[19,20] by rewiring some links[13,21–28] or by removing some nodes.[29]

In this paper, we are interested in improving the synchronizability while keeping the total numbers of nodes and edges in a network unchanged. Therefore, the edge-rewiring method is focused on. The easiest way to rewire an edge is based on a random approach[21] but its efficiency is obviously questionable. In Refs. [13] and [25], the Laplacian matrix is studied and it is reported that the change of eigenvalue λ_i after adding an edge e_{pq} can be approximated by $(u_{i,p} - u_{i,q})^2$, where $u_{i,p}$ and $u_{i,q}$ are the values associated with nodes p and q in the eigenvector corresponding to λ_i. Based on this finding, it is then proposed in Refs. [13], [25], and [26] that the edge e_{pq} with maximum $(u_{N,p} - u_{N,q})^2$ (or $|u_{N,p} - u_{N,q}|$) is to be deleted, while the edge e_{st} with largest $(u_{2,s} - u_{2,t})^2$ (or $|u_{2,s} - u_{2,t}|$) is to be added. A different approach is used for deleting edges in Ref. [12]. The edge connecting nodes with a maximum degree sum is suggested to be deleted, as it is noticed that λ_N is related to the maximum degree of the graph. A similar approach for edge deletion is suggested in Ref. [28]. However, since the design concept in Ref. [28] is based on the observation that homogeneous properties usually make a network more synchronizable,[21] edges are added to connect nodes with the lowest degrees.

To develop a relatively complete algorithm with iterative improvement, the rewiring scheme is commonly incorporated with some optimization approaches, such as simulated annealing,[12,13,21–23]

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greedy heuristic[25,26,28] or tabu search.[20]

Unfortunately, since a global search is needed to select the "best" edges for adding or deleting, the computational complexities of the aforementioned methods are usually very high. Let M and N be the numbers of rewiring iterations and the network size, respectively. Then, it can be easily derived that the computational complexities of the methods developed in Refs. [13], [24]–[26] are $O(MN^2)$. A few proposals have lower complexity. For example, the complexity is $O(MN)$ for the method suggested in Ref. [28]. However, as shown in our simulation later, its performance is poor because it may be trapped in local minima on regular networks.

the aim of this paper is thus to propose a fast and effective algorithm for improving the synchronizability of a given network via rewiring. The rest of this paper is organized as follows. In Section 2, some preliminaries are given and the problem is formulated. The proposed algorithm is then described in Section 3. Experimental results are presented in Section 4 to demonstrate the effectiveness of this algorithm and, finally, conclusions are drawn in Section 5.

2. Problem statement and preliminaries

Consider a network of N nodes, which is represented by an undirected and unweighted graph $G = (V, E)$ with a set of nodes V and a set of edges E. The node dynamics are described by

$$\dot{x_i}(t) = F(x_i) - \sigma \sum_{j=1}^{N} L_{ij} H(x_j), \tag{1}$$

where $i = 1, 2, \ldots, N$, x_i is the state vector of node i with the isolated dynamics specified by $F(\cdot)$, σ is the coupling strength, $H(\cdot)$ is the coupling function, and $L = (L_{ij})$ is the coupling matrix known also as the Laplacian matrix.[1)]

The nodes in network (1) are said to be (asymptotically) synchronized if

$$x_1(t) \to x_2(t) \to \cdots \to x_N(t) \to s(t), \quad t \to \infty, \tag{2}$$

where $s(t)$ is a solution of the node: $\dot{s}(t) = F(s)$. It is remarked that $s(t)$ can be an equilibrium point, a periodic orbit or a chaotic attractor.

With the synchronous state $x_i(t) \to s(t)$ $\forall i$, the variational equation of the dynamical network (1) can be diagonalized into N blocks of the following form:[11,25]

$$\dot{\xi}_i = [JF(s) - \sigma\lambda_i JH(s)]\xi_i, \tag{3}$$

where J denotes the Jacobian operator.

Let $LE_{\max}$ be the largest Lyapunov exponent of Eq. (3). Then a synchronized region Ω can be specified with $LE_{\max} < 0$. The synchronized state $s(t)$ is locally stable if and only if the spectrum $\sigma\lambda_k \subseteq \Omega$, $k = 2, 3, \ldots, N$, where $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$ are the eigenvalues of L.

According to the boundness of Ω, three types of networks can be specified.[26]

(i) Type 1 network: Ω is unbounded with $(\alpha, +\infty)$ (with $\alpha > 0$) and the synchronization can be achieved if the eigenvalue $\lambda_2 > \alpha/\sigma$. Larger λ_2 leads to better synchronizability.

(ii) Type 2 network: Ω is bounded with $[\alpha_1, \alpha_2]$ for some finite α_1 and α_2. The synchronizability is dependent on the eigenratio $R = \lambda_N/\lambda_2$ of L. The smaller the R is, the better the synchronizability is.

(iii) Type 3 network: $\Omega = \emptyset$ implies that synchronization cannot be achieved by the nature itself without external input.

In this paper, we focus on Type 2 networks, aiming to improve the synchronizability of a given network that keeps the total numbers of nodes (N) and the average degree ($\langle k \rangle$) unchanged. This can also be considered as a problem of designing a network with best synchronizability.

Since a smaller eigenratio stands for better synchronizability of a network described by Eq. (1), maximizing the synchronizability of G is thus equilivalent to minimizing the corresponding eigenratio $R(G)$. Below, a new algorithm is proposed to attain the minimized eigenratio by modifying a given undirected and unweighted network via rewiring and keeping the total numbers of nodes and edges unchanged.

3. A combination of greedy rewiring and memory tabu search

To enhance the synchronizability of a network, as reported in Ref. [24], the edge connecting nodes with the largest sum of degrees is to be deleted, while the edge e_{pq} with maximal quantity $(u_{2,p} - u_{2,q})^2$ is to be added. As designed in Section 2, $u_{2,p}$ and $u_{2,q}$ are the

[1)] $L_{ij} = -1$ if $(i, j) \in E$ and $L_{ij} = 0$ otherwise. The diagonal entries $L_{ii} = \deg(i)$ where $\deg(i)$ represents the degree of node i.

entries associated with nodes p and q in the eigenvector corresponding to λ_2.

Although this approach is effective for obtaining network with small eigenratio, the computational complexity is heavy, as the edges to be deleted or added are to be exhaustively searched over the entire network.

Inspired by the introduction of tabu list in Ref. [27], a low computational effort but effective optimization approach is proposed below for searching for the best possible eigenratio of a network with given size and average degree. This new algorithm combines the tabu search and the greedy algorithm, and its operational procedures are described in detail as follows:

Step 1 Initially (i.e., $t = 0$), a tabu list of L networks, $T_{t=0} = \{G_t^{(1)}, G_t^{(2)}, \ldots, G_t^{(L)}\}$, is randomly generated. Each network $G_t^{(i)}$ consists of N nodes and the average degree is $\langle k \rangle$.

Step 2 The eigenratio, $R_t^{(i)}$, for each $G_t^{(i)}$ $(i = 1, 2, \ldots, L)$ in the list T_t is computed. Define $G_{\min}$ as the global optimal network with the minimum $R_{\min}$ obtained so far at time t.

Step 3 For every $G_t^{(i)}$ $(i = 1, 2, \ldots, L)$ in T_t, the following local greedy search is performed:

(I) Let $\tilde{G}^{(j)} = \emptyset$ and $\tilde{R}^{(j)} = \infty$.

(II) Randomly choose a node a in $G_t^{(i)}$.

(III) For each edge e_{ab} of node a in $G_t^{(i)}$, calculate the quantity $P_{\text{cut}} = k_a + k_b$, where k_a and k_b stand for the degrees of nodes a and b, respectively. The edge with the largest P_{cut} is to be deleted and the obtained network is denoted as G_{cut}.

(IV) Based on λ_2, determine whether or not the network G_{cut} is connected.[30] If it is disconnected, the network is discarded. Otherwise, connect the node a to the nonadjacent node c in G_{cut} for which the quantity $P_{\text{add}} = (u_{2,a} - u_{2,c})^2$ is largest. The resultant network is denoted as G_{new}.

(V) Compute the eigenratio $R_{\text{new}} = \lambda_N/\lambda_2$ of G_{new}. If $R_{\text{new}} < \tilde{R}^{(j)}$, accept the rewiring and set $\tilde{G}^{(j)} = G_{\text{new}}$, $\tilde{R}^{(j)} = R_{\text{new}}$.

(VI) Repeat steps (II)–(V) for Q iterations.

Step 4

If $\tilde{R}^{(j)} \leq R_{\min}$, then $G_{\min} = \tilde{G}^{(j)}$, $R_{\min} = \tilde{R}^{(j)}$ and $G_{t+1}^{(j)} = \tilde{G}^{(j)}$.

If $\tilde{R}^{(j)} \leq R_t^{(j)}$ or a tabu condition is satisfied, then the network in the tabu list is updated by $G_{t+1}^{(j)} = \tilde{G}^{(j)}$ and $R_{t+1}^{(j)} = \tilde{R}^{(j)}$. Otherwise, there is no update in the tabu list, i.e. $G_{t+1}^{(j)} = G_t^{(j)}$ and $R_{t+1}^{(j)} = R_t^{(j)}$. The tabu condition is specified as in Ref. [27]: $|\tilde{R}^{(j)} - R_t^{(j)}|/\tilde{R}^{(j)} > \delta$, where δ is a random number in the range $[0.50,\ 0.75]$.

Step 5 Set $t = t + 1$ and repeat Steps 2–4 until $t = M$.

4. Simulation results

4.1. Network models

Different kinds of networks have been used to verify the effectiveness of the proposed algorithm for the improvement of synchronizability. They include:

i) regular network: a regular network with average degree of 8 ($\langle k \rangle = 8$) is generated for our study.

ii) BA scale-free network: the BA scale-free networks are constructed based on the method presented in Ref. [31]. Starting with a fully connected network of size m_0, at each step, a new node is added and m edges are connected to some existing nodes. The connection of the new node to node i is governed by the probability $P_i = (k_i + B)/\sum_j (k_j + B)$, where k_i is the degree of node i and B is a tunable real value. In our simulation, it is assumed that $m_0 = 9,\ m = 8$ and thus $\langle k \rangle = 8$.

iii) SW small-world network: the SW small world networks are constructed by following the procedures given in Ref. [32]. Considering a regular ring of nodes, where each node is connected to its nearest n neighbours, and the edges of each node are then randomly rewired to connect to other nodes with a probability P. Duplication of edges and self-loops are not allowed. In our simulation, $n = 4$ and hence $\langle k \rangle = 8$.

iv) ER random network: given N nodes, the ER random networks are built by randomly connecting every pair of nodes with a probability of p. In our simulation, $p \times N = 8$, but only the networks with $\langle k \rangle = 8$ are used.

v) HK scale-free network: the HK networks[33] are also growing scale-free networks, similar to BA networks. The major difference is that a triad formation process is included. When an edge is to connect a new node v to an existing node w based on the BA preferential attachment scheme, another edge is added between v and a random chosen neighbour of w. This can enhance the clustering and thus networks with power-law degree distribution and high clustering can be generated.

Remark 1 In our simulation, the average degree of each type of initial networks is 8 so that a fair comparison can be made for different types of initial networks.

Figure 1 shows the eigenratio ($R_{\min}$) against iterations (t) for different types of initial networks when the proposed algorithm (with $L = Q = 1$) is applied. As it can be observed, $R_{\min}$ is greatly improved, indicating the effectiveness of the proposed algorithm for the improvement of network synchronizability. Based on the results for the minimum (Min), the maximum (Max), the mean (Mean) values, and the corresponding standard deviations (SD) of the eigenratio tabulated in Table 1, it is concluded that the final eigenratios obtained from different types of initial networks are close to each other. Moreover, the small standard derivation also indicates that the proposed algorithm is quite robust to the change of the initial network topologies.

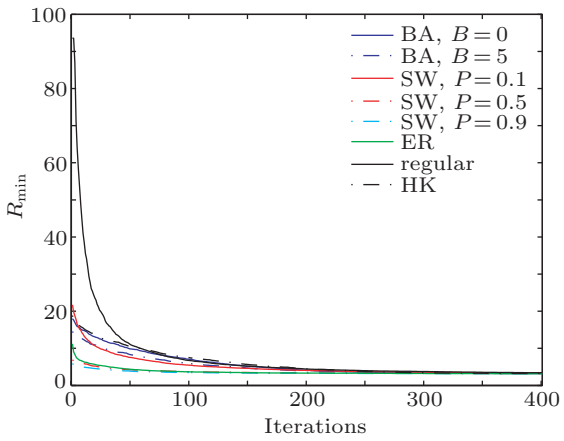


Fig. 1. (colour online) The eigenratio $R_{\min}$ against iterations for different initial networks with $N = 100$ for $\langle k \rangle = 8$. The plots are based on the average of 10 realizations.

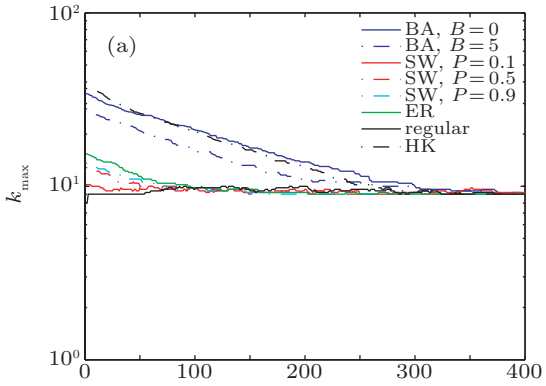
As commented in Ref. [26], some structural characteristics of a network are also closely related to its synchronizability. Here, the network properties including the maximum node degree $k_{\max}$, the difference between $k_{\max}$ and $k_{\min}$ (Diff_k), the maximum node betweenness centrality ($\mathrm{NBC}_{\max}$), the difference between $\mathrm{NBC}_{\max}$ and $\mathrm{NBC}_{\min}$ ($\mathrm{Diff}_{\mathrm{NBC}}$), the maximum edge betweenness centrality ($\mathrm{EBC}_{\max}$), the difference between $\mathrm{EBC}_{\max}$ and $\mathrm{EBC}_{\min}$ ($\mathrm{Diff}_{\mathrm{EBC}}$), the clustering coefficient (C), and the number of 3-loops (Loops-3), are plotted against iterations as shown in Fig. 2. The results are based on the average of 10 realizations of each type of initial networks.

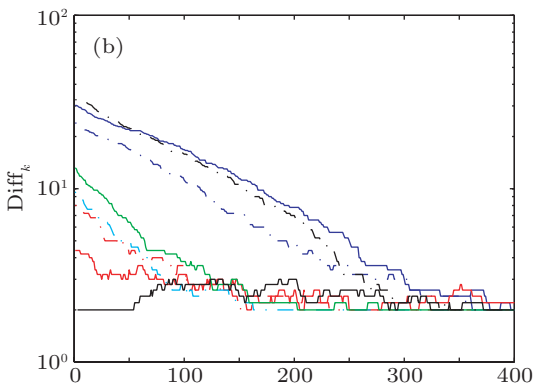
As shown in Fig. 2, the Diff_k, $\mathrm{Diff}_{\mathrm{NBC}}$ and $\mathrm{Diff}_{\mathrm{EBC}}$ are very small for the final networks, regardless of the types of initial networks. It implies that networks with high synchronizability belong to a class of homogenous networks with small variations on degree distributions, node and edge betweenness centralities. However, the clustering coefficient and the number of 3-loops (as shown in Fig. 2) are different for different initial networks, even though the resultant $R_{\min}$ (as illustrated in Table 1) are close to each other.

Table 1. The minimum, maximum, mean and standard deviation of $R_{\min}$ for different initial networks with $N = 100$ over 10 realizations.

Networks	Min	Max	Mean	SD
BA, $B = 0$	3.2651	3.3336	3.2906	0.0232
BA, $B = 5$	3.2208	3.2976	3.2604	0.0249
SW, $P = 0.1$	3.2981	3.4793	3.3748	0.0609
SW, $P = 0.5$	3.1608	3.2968	3.2086	0.0355
SW, $P = 0.9$	3.1771	3.2613	3.1870	0.0306
ER	3.1336	3.2167	3.1786	0.0267
Regular	3.3116	3.4516	3.3927	0.0610
HK	3.1526	3.2676	3.2275	0.0338

Note: the Diff_k and $\mathrm{Diff}_{\mathrm{NBC}}$ of regular networks are zeros. For clarity, these values are not included in Fig. 2(b) and 2(d) at $t = 0$.





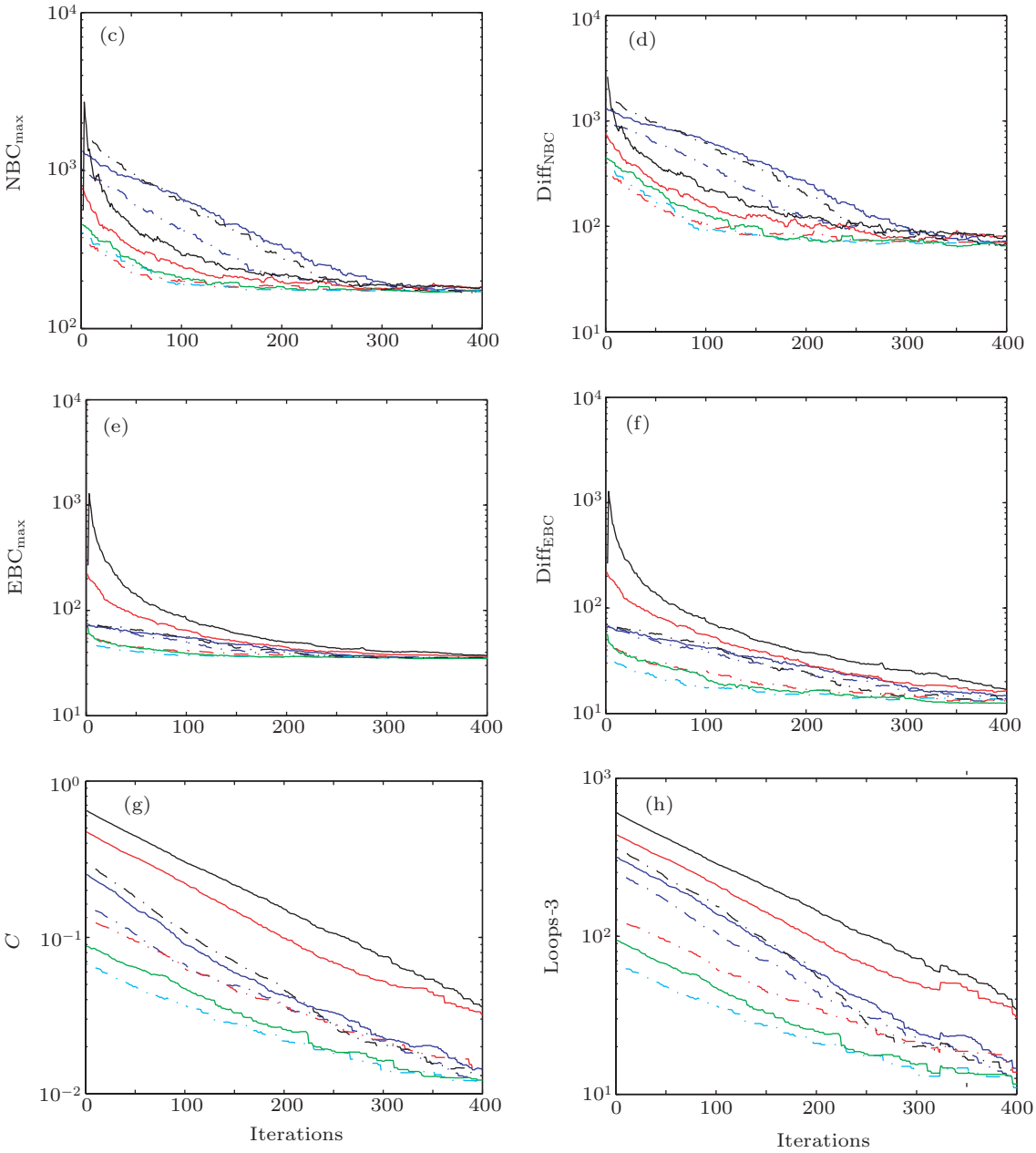


Fig. 2. (colour online) Various network properties against iterations for different types of initial networks with $N = 100$. The legend is the same for all the graphs and it is given in panel (a) for k_{max}, (b) for Diff$_k$, (c) for NBC$_{max}$, (d) for Diff$_{NBC}$, (e) for EBC$_{max}$, (f) for Diff$_{EBC}$, (g) for C, and (h) for loops-3.

4.2. Comparison with the previous works

To further investigate the effectiveness of the proposed algorithm, its performance is compared with several existing approaches, including those based on the global greedy rewiring methods given in Refs. [13], [24], [26], and [27] and the network rebuilding method described in Ref. [28]. Figure 3 depicts the results for the proposed algorithm with $L = Q = 1$ and $L = Q = 2$, against all the other mentioned methods, based on the average of 10 realizations. The R_{min} obtained by different methods after 1000 iterations are also given in Table 2 for comparison. From Table 2, it is clearly shown that our approach (even with $L = Q = 1$) outperforms all the others. The performance is further improved if L and Q are increased, which also implies an increase of the number of search in the algorithm.

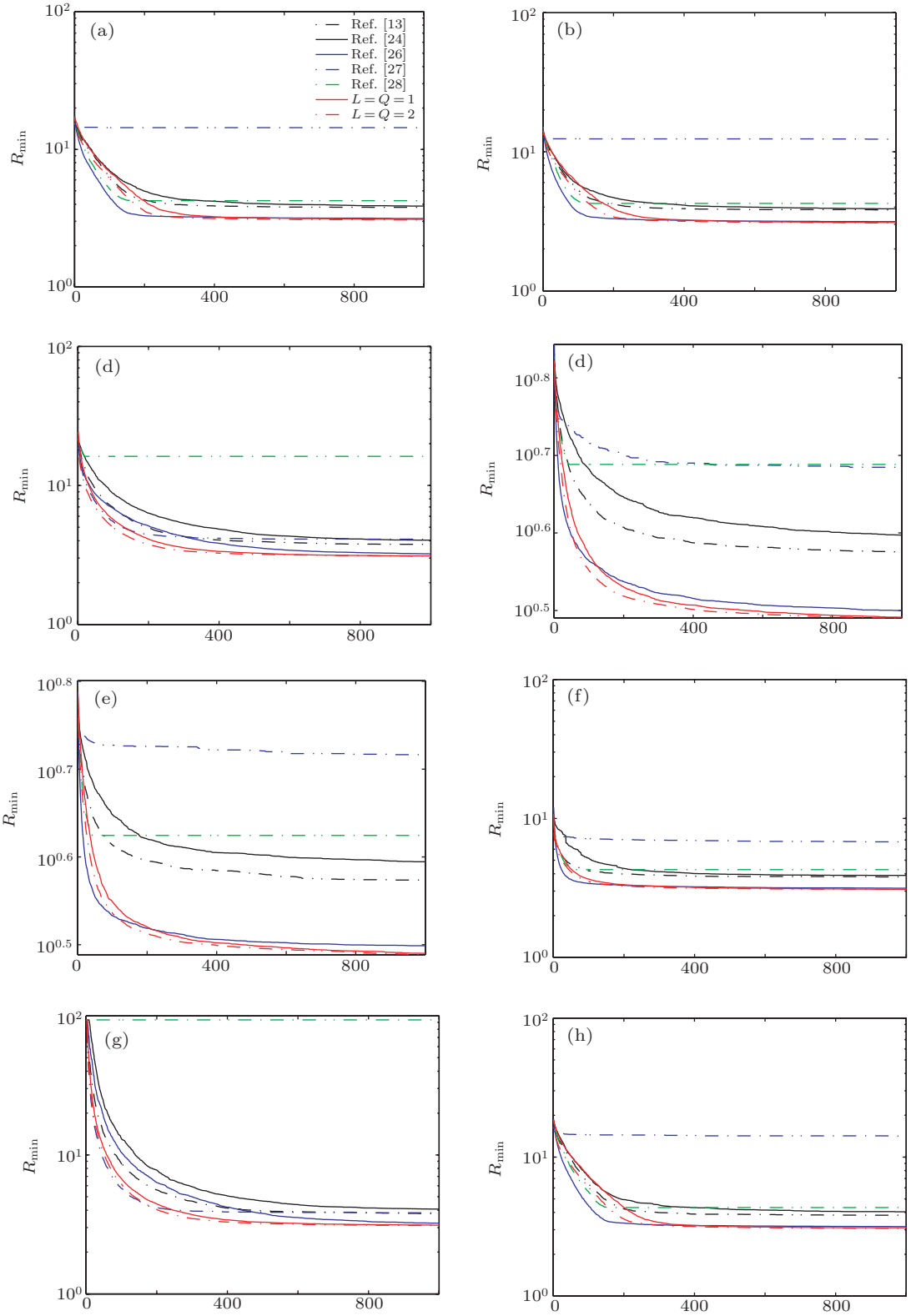


Fig. 3. (colour online) The eigenratio $R_{\min}$ against iterations for different initial networks using different methods. The legend is the same for all the graphs and it is given in (a). Panel (a) for BA network with $B = 0$, (b) for BA network with $B = 5$, (c) for SW network with $P = 0.1$, (d) for SW network with $P = 0.5$, (e) for SW network with $P = 0.9$, (f) for ER network with $P = 0.08$, (g) for regular network and (h) for HK network.

Differing from Ref. [24], our local greedy search relies on a random process for selecting a node for rewiring. Though it may affect the speed of the convergence of $R_{\min}$ initially, this random process helps to escape from local minima and greatly reduce the computational time. This is especially prominent when regular network

or SW network with a small rewiring probability P is simulated. As illustrated by Table 2, the $R_{\min}$ obtained by Ref. [24] for SW ($P = 0.1$ or 0.5) and for regular network is relatively larger than those for BA and ER networks, but they are all much better with our approach. The trap to local minima appears to be a serious problem for some other methods too. For example, the method described in Ref. [28] may present a very poor result (as shown in Table 2, $R_{\min} = 93.5695$ for regular network) because the same edge is deleted and added back again.

Table 2. $R_{\min}$ obtained by different algorithms with different initial networks after 1000 iterations.

Networks	Ref. [13]	Ref. [24]	Ref. [26]	Ref. [27]	Ref. [28]	$L = Q = 1$	$L = Q = 2$
BA, $B = 0$	3.9073	3.9937	3.2220	14.3962	4.0547	3.1774	3.1533
BA, $B = 5$	3.8803	3.9564	3.2110	12.2996	4.0613	3.1813	3.1697
SW, $P = 0.1$	3.7698	4.0388	3.2197	4.0983	16.2028	3.1133	3.0984
SW, $P = 0.5$	3.7623	3.9551	3.1628	4.8391	4.8801	3.1002	3.0921
SW, $P = 0.9$	3.7453	3.9308	3.1542	5.2058	4.2117	3.0923	3.0818
ER	3.7916	3.8894	3.1491	6.7811	4.2834	3.0927	3.0894
Regular	3.7748	4.0748	3.2283	3.8235	93.5695	3.1210	3.1027
HK	3.8176	4.0415	3.1619	14.1727	4.3369	3.1014	3.0772

The use of a tabu list also helps to lead to a better search of solutions and hence some suggestions given in Ref. [27] have been followed. However, as shown in Table 2, our implemented greedy algorithm is much more effective than the edge exchange algorithm used in Ref. [27].

The time complexity of our approach is also compared with others. As shown in Table 3, when $Q = L = 1$, the run-time is very close to the method suggested in Ref. [28] and faster than all the others. If we increase both Q and L to two, the speed is still faster than those algorithms given in Refs. [13], [24], and [27]. In either case, the obtained networks have better synchronizability as already shown in Table 2.

Table 4 presents the computational complexity of different algorithms. Unlike all the others, the complexity of our proposed algorithm depends also on the parameters L and Q. However, this reduces to $O(MN)$ when $L = Q = 1$, which is similar to the complexity of the algorithm given in Ref. [28] and is much lower than those algorithms proposed in Refs. [13], [24], and [26].

Table 3. Runtime (sec) for different algorithms.

Networks	Ref. [13]	Ref. [24]	Ref. [26]	Ref. [27]	Ref. [28]	$L = Q = 1$	$L = Q = 2$
BA, $B = 0$	59.6454	59.9605	22.8822	57.1166	10.1525	9.5317	39.0970
BA, $B = 5$	56.3164	58.3007	21.7263	58.4895	9.6346	9.3101	36.8864
SW, $P = 0.1$	58.0667	58.9746	22.8198	54.9498	9.5254	9.4786	38.0299
SW, $P = 0.5$	58.4879	59.5830	22.8916	54.7548	9.6409	9.2509	38.4917
SW, $P = 0.9$	56.3819	56.9528	22.5453	53.5598	9.6502	9.0886	38.0954
ER	59.4738	60.6563	22.9415	55.5956	10.1089	9.8062	39.4277
Regular	59.0495	60.3505	23.6217	56.2883	8.0403	9.5161	40.3325
HK	59.0557	60.6937	23.4688	58.0851	9.9248	9.4693	38.8817

Table 4. Computational complexity for different algorithms.

Ref. [13]	Ref. [24]	Ref. [26]	Ref. [27]	Ref. [28]	Ours
$O(MN^2)$	$O(MN^2)$	$O(MN^2)$	$O(ML)$	$O(MN)$	$O(MLQN)$

Note: M is the number of rewiring iterations and N is network size. L is the length of the tabu list and Q is the number of trials for the local greedy search for each element in the tabu list.

5. Conclusion

In this paper, a fast and effective hybrid algorithm which combines the tabu list and the local greedy algorithm, has been proposed for enhancing the synchronizability of networks. The networks in the tabu list are modified by rewiring edges, based on the information of node-degree sums and eigenvectors corresponding to the second smallest eigenvalue of the Laplacian matrix. As demonstrated by the simulation results, a connected network with smallest eigenratio can be obtained regardless of the initial network topology and it is better than those obtained by other existing methods. It is also shown that the computational complexity of our algorithm is the lowest and its speed is the fastest based on the run-time tests.

References

[1] Pastor S R and Vespignani A 2001 *Phys. Rev. Lett.* **86** 3200

[2] Klemm K, Eguiluz V M, Toral R and Miguel M S 2003 *Phys. Rev.* E **67** 026120

[3] Trpevski D, Tang W K S and Kocarev L 2010 *Phys. Rev.* E **81** 056102

[4] Tse C K, Liu J, Lau F C M and He K 2010 *J. Empir. Finance* **17** 2099

[5] Saber R O and Murray R M 2004 *IEEE Trans. Autom. Control* **49** 1520

[6] Arenas A, Guilera A D, Kurths J, Moreno Y and Zhou C S 2008 *Phys. Rep.* **469** 93

[7] Li Z Q, Duan Z S and Chen G R 2009 *Chin. Phys.* B **18** 1674

[8] Zou Y L and Chen G R 2009 *Chin. Phys.* B **18** 3337

[9] Tu L L 2011 *Chin. Phys.* B **20** 030504

[10] Milanese A, Sun J and Nishikawa T 2010 *Phys. Rev.* E **81** 046112

[11] Zhao M, Chen G R, Zhou T and Wang B H 2007 *Front. Phys.* **2** 460

[12] Jalili M, Rad A A and Hasler M 2008 *Phys. Rev.* E **78** 016105

[13] Jalili M and Rad A A 2009 *Chaos* **19** 028101

[14] Feng C F, Xu X J, Wu Z X and Wang Y H 2008 *Chin. Phys.* B **17** 1951

[15] Ma X J, Wang Y and Zheng Z G 2009 *Acta Phys. Sin.* **58** 4426 (in Chinese)

[16] Duan Z S, Wang W X, Liu C and Chen G R 2009 *Chin. Phys.* B **18** 3122

[17] Korniss G, Novotny M A, Guclu H, Toroczkai Z and Rikvold P A 2003 *Science* **299** 677

[18] Pecora L M and Carroll T L 1998 *Phys. Rev. Lett.* **80** 2109

[19] Chavez M, Hwang D U, Amann A, Hentschel H G E and Boccaletti B 2005 *Phys. Rev. Lett.* **94** 218701

[20] Wang X, Lai Y C and Lai C H 2007 *Phys. Rev.* E **75** 056205

[21] Donetti L, Hurtado P I and Munoz M A 2005 *Phys. Rev.* E **95** 188701

[22] Donetti L, Neri F and Munoz M A 2006 *J. Stat. Mech.: Theor. Exp.* **8** P08007

[23] Donetti L, Hurtado P I and Munoz M A 2008 *J. Phys.* A: *Math. Theor.* **41** 224008

[24] Rad A A, Jalili M and Haslerc M 2008 *Chaos* **18** 037104

[25] Hagberg A and Schult D A 2008 *Chaos* **18** 037105

[26] Wang L F, Wang Q L, Kong Z and Jing Y W 2010 *Chin. Phys.* B **19** 080207

[27] Wang B, Zhou T, Xiu Z L and Kim B J 2007 *Phys. J.* B **60** 89

[28] Mishkovski I, Righero M, Biey M and Kocarev L 2010 *Proceedings of* 2010 *IEEE International Symposium on Circuits and Systems* Paris, May, p. 681

[29] Watanabe T and Masuda N 2010 *Phys. Rev.* E **82** 046102

[30] Lu W 2007 *Chaos* **17** 023122

[31] Barabsi A L and Albert R 1999 *Science* **286** 509

[32] Watts D and Strogatz S 1998 *Nature* **393** 440

[33] Holme P and Kim B J 2002 *Phys. Rev.* E **65** 026107