Chin. Phys. B Vol. 20, No. 12 (2011) 127401

The Wiedemann–Franz law in a normal metal–superconductor junction

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(Received 5 June 2011; revised manuscript received 26 July 2011)

In this paper the influence of superconducting correlations on the thermal and charge conductances in a normal metal–superconductor (NS) junction in the clean limit is studied theoretically. First we solve the quasiclassical Eilenberger equations, and using the obtained density of states we can acquire the thermal and electrical conductances for the NS junction. Then we compare the conductance in a normal region of an NS junction with that in a single layer of normal metal (N). Moreover, we study the Wiedemann–Franz (WF) law for these two cases (N and NS). From our calculations we conclude that the behaviour of the NS junction does not conform to the WF law for all temperatures. The effect of the thickness of normal metal on the thermal conductivity is also theoretically investigated in the paper.

Keywords: quasiclassical approach, Eilenberger equations, Wiedemann–Franz law, thermal and charge conductances

PACS: 74.45.+c, 74.45.+r, 74.25.Fy, 74.25.Bt **DOI:** 10.1088/1674-1056/20/12/127401

1. Introduction

Charge and thermal transport in mesoscopic superconducting structures have attracted many researchers in the last decade.[1−19] The transport properties of normal and superconducting structures have been studied separately in many textbooks.[20−23] In addition, the charge and spin transport of normal metal–superconductor (NS) and superconductor normal-metal superconductor (SNS) structures are investigated in many theoretical and experimental studies.[24−34] It is well known that when a superconductor and a normal metal come together, superconductivity penetrates into the normal metal at a distance of the order of the coherence length ξ. This phenomenon, which is called proximity effect, has been studied in Refs. [35]–[37]. The effect of the proximity effect on the transport properties of the NS junction has been considered in Ref. [36]. Although the main interest in NS junctions for researchers is charge and spin transport, the thermal properties of such junctions can be interesting. In this paper we investigate the charge and thermal conductances of the NS junction at the same time, and also the Wiedemann–Franz (WF) law for the NS junction. We compare our results for charge and thermal transport and WF law for the NS junction with those for a single normal metal. The electrical and thermal conductances of a normal metal at low temperatures depend on the scattering of electrons. So as the carriers of both electrical and thermal currents are the same (electrons), these two conductances are related.[38] In Ref. [39] it was concluded that, although the charge conductance of a normal metal at low temperatures is constant ($G_q = \text{const.}$), the thermal conductance has a linear dependence on the temperature ($\kappa \propto T$). The relationship between thermal and charge conductances is given by the WF law, $\kappa/G = LT$, where the constant of proportionality, L, is called the Lorenz number.[20,21,39] The WF law for different normal materials has also been studied experimentally in Refs. [40]–[43].

In addition, charge and thermal conductances in a superconducting sample are interesting.[44−48] Although the investigation of charge transport in superconductors started immediately after the discovery of superconductivity, the thermal conductivity of superconductors has attracted attention in the last decades.[49−52] In particular, it was found that the WF law is violated by the formation of a coherent ground state in the superconductor.[53−55] In Ref. [38] the behaviour of the thermal transport in a superconductor was investigated theoretically. In addition, the charge conductivity of a junction was discussed by Blonder, Tinkham and Klapwijk (BTK) theoretically.[56] BTK calculated charge conductance at low temperatures by adding the Andreev reflection[57] in the charge transport of a junction into the Bogoliubov de-Gennes

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(BdG) formalism.[36] In Refs. [58] and [59] BTK theory was developed for the case of heat transport in a junction. The results of BTK have been confirmed by the experiments in Ref. [60].

In the present paper we consider a system which consists of a normal metal layer with a finite thickness connected to a superconductor (Fig. 1). We investigate the behaviour of the charge and thermal conductances for the NS junction. In particular, we compare the charge and thermal transport of the NS junction with those of a single normal metal, and the investigation of the validity of the WF law for the NS junction is also an important part of the paper. The rest of this paper is organized as follows. In Section 2 we introduce our mathematical formalism, which is based on the quasiclassical Eilenberger approach. Section 3 is devoted to our numerical results, and we end with some conclusions in Section 4.

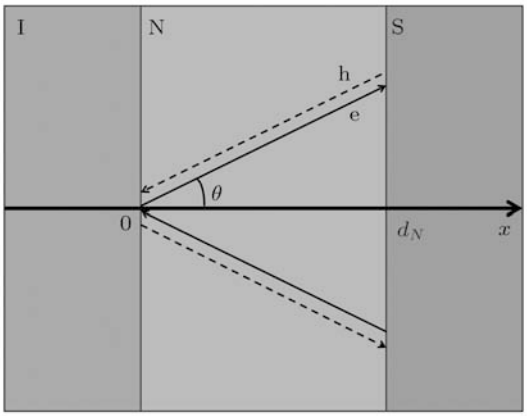


Fig. 1. An NS junction consisting of a normal metal (N) layer with a finite thickness connected to a bulk of superconductors (S). The semiclassical trajectories for the quasiparticles, with usual reflections at the IN interface and Andreev reflections at the NS interface, are shown in this figure. The experimental setup was planned for the investigation of the proximity effect in the NS junction, and also for the study of the variation of the order parameter inside the film of the normal metal.

2. Theory

In our calculations the physical variation of the system is only in one dimension (1D), x. Also, the thickness, d_N, is larger than the Fermi wavelength $\lambda_f, (d_N \gg \lambda_f)$, but smaller than the elastic mean-free path, $\ell, (d_N \ll \ell)$, which allows us to use 1D quasiclassical theory in the clean limit.[61] Then we use the quasiclassical Eilenberger equations and obtain the density of states (DOS) in the normal part of the NS junction. Using the obtained DOS, we also study the electrical and thermal transport and the WF law for this system. Quasiclassical Eilenberger equations describing the coherent states of the ballistic structures, including s-wave superconductivity, are as follows:[61−63]

$$\boldsymbol{v}_{\rm F}.\nabla\hat{G}_\omega(\boldsymbol{v}_{\rm F},\boldsymbol{r})+[\omega\hat{\tau}_3+\hat{\Delta}(\boldsymbol{v}_{\rm F},\boldsymbol{r}),\hat{G}_\omega(\boldsymbol{v}_{\rm F},\boldsymbol{r})]=0, \quad (1)$$

where $\hat{\Delta}=\begin{pmatrix}0 & \Delta\\ \Delta^\dagger & 0\end{pmatrix}$ and $\hat{G}_\omega(\boldsymbol{v}_{\rm F},\boldsymbol{r})=\begin{pmatrix}g_\omega & f_\omega\\ f_\omega^\dagger & -g_\omega\end{pmatrix}$ are the matrix of the order parameter and Green's matrix, respectively. Both the matrix of the order parameter and Green's matrix depend on the electron velocity on the Fermi surface $\boldsymbol{v}_{\rm F}$ and the coordinate $\boldsymbol{r}$. The Green's function also depends on the Matsubara frequency $\omega=(2n+1)\pi T$, with n being an integer number and T the temperature. $\hat{\tau}_3$ is the Pauli matrix. In addition, the Green's function components satisfy the normalization conduction

$$g_\omega^2+f_\omega f_\omega^\dagger=1. \quad (2)$$

In general, Δ depends on the electron velocity on the Fermi surface $\boldsymbol{v}_{\rm F}$, and is determined by the self-consistent equation:[61,62]

$$\Delta(\boldsymbol{v}_{\rm F},\boldsymbol{r})=2\pi N(0)T\sum_{\omega>0}\langle V(\boldsymbol{v}_{\rm F},\boldsymbol{v}'_{\rm F})f_\omega(\boldsymbol{v}'_{\rm F},\boldsymbol{r})\rangle_{\boldsymbol{v}'_{\rm F}}, \quad (3)$$

where $N(0)$, $V(\boldsymbol{v}_{\rm F},\boldsymbol{v}'_{\rm F})$, and $\langle\cdots\rangle$ are the density of states at the Fermi surface in the normal metal, the interaction potential and the average over the direction of vector $\boldsymbol{v}_{\rm F}$, respectively. In this study, for simplicity we consider the spatial variation of the order parameter as a step function. The order parameter within the normal region is zero, $\Delta(0<x<d_N)=0$, and is constant inside the superconducting region, $\Delta(x>d_N)=$ const. Furthermore, the temperature dependence of $\Delta(T)$ can be calculated by[64]

$$\Delta(T)=V\sum_k\frac{\Delta(T)}{2E_k}\tanh\frac{E_k}{2k_BT}, \quad (4)$$

where $E_k=\sqrt{E^2+\Delta^2(T)}$ is the energy of the quasiparticle at the superconducting phase and $k_{\rm B}$ is the Boltzmann constant. An immediate result of this self-consistent equation is a relation between the critical temperature and the order parameter at zero temperature, expressed as $T_{\rm c}=\Delta_0/(1.765k_{\rm B})$.

Using Eilenberger equations (1) and boundary conditions at $x=0$ and d_N, and Green's function for superconducting bulks,

$$\hat{G}_\omega(\infty)=\frac{\omega\hat{\tau}_3+\hat{\Delta}}{\sqrt{\omega^2+\Delta^2}},$$

we obtain the diagonal term of Green's matrix.[65] So, for electron trajectories inside the normal metal, we obtain

$$g_\omega = \tanh\left[\omega\frac{l}{v_\mathrm{F}} + \sinh^{-1}\left(\frac{\omega}{\Delta}\right)\right], \tag{5}$$

where $l = d_N/\cos(\theta)$. Also, the density of states is calculated by the following formula:[65]

$$N(E) = N(0)\sum_{\omega>0}\mathrm{Re}\, g_\omega(\omega \longrightarrow E + \mathrm{i}\,\delta). \tag{6}$$

For the calculation of charge transport we apply a very small potential difference to both sides of the normal part of the junction. So, we use the following method:[66]

$$G_\mathrm{q} = \alpha\int N(E)\left(-\frac{\partial f_0}{\partial E}\right)\mathrm{d}E. \tag{7}$$

Also, by applying a small ΔT to the normal part of the junction, the thermal conductance can be obtained by the following equation:[66]

$$\kappa = \beta\int \frac{E^2}{T}N(E)\left(-\frac{\partial f_0}{\partial E}\right)\mathrm{d}E. \tag{8}$$

Where $N(E)$ shows the density of states of normal metal subject to the proximity of the superconductor. Also, f_0 is the equilibrium Fermi distribution function $f_0 = 1/(\exp(E/k_\mathrm{B}T)+1)$. Using the calculated electrical and thermal conductances, the WF law and Lorenz number for the NS junction can be compared with a normal metal. In our numerical calculations we set $k_\mathrm{B} = \alpha = \beta = 1$, and all length scales are normalized by superconducting coherence length ξ_s.

3. Discussion

It is well known that DOS plays a central role in thermal and charge transport in the junction. So using the obtained Green's function, we plotted the DOS in Fig. 2. We also plotted the Fermi distribution versus the temperature in Fig. 3. Then we used the self-consistent equation of gap (Eqs. (3) and (4)) for the calculation of the temperature dependence of the order parameter in Fig. 3. This self-consistent equation has been introduced for the first time in the framework of the Bardeen–Cooper–Schrieffer microscopic theory of superconductivity. Finally, using the obtained order parameter dependence on the temperature (Eq. (4)), Fermi distribution (Fig. 3) and DOS (Eq. (6) and Fig. 2), we investigated the temperature dependence of the transport properties of the mentioned SNS system.

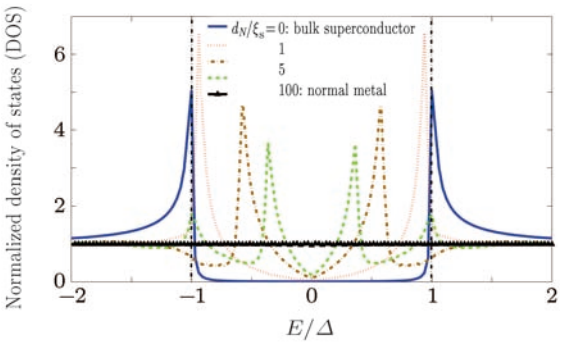


Fig. 2. (colour online) Normalized densities of state versus E/Δ in the normal part of the NS junction for several values of normal metal thickness d_N. $d_N = 0$ also describes the bulk superconductor density of states, and $d_N \gg 1$ describes the normal metal DOS. In this figure and the next figures all length scales are normalized by superconducting coherence length ξ_s.

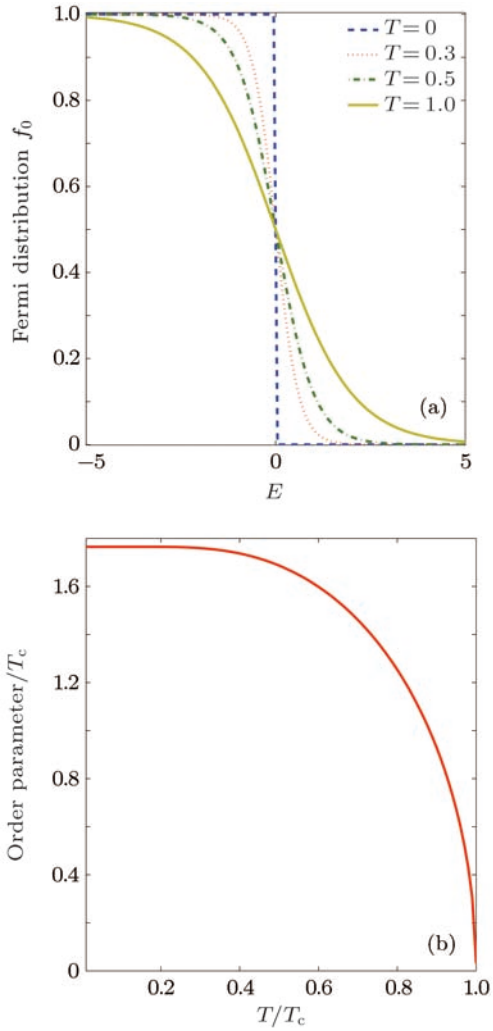


Fig. 3. (colour online) Curves for (a) Fermi distribution f_0 versus energy (E) at different temperatures. Energies are measured from the Fermi energy. (b) Order parameter dependence on temperature $\Delta(T)$.

In Fig. 4 we plotted the thermal conductance κ (using Eq. (8)) of system versus temperature T. We compared the NS junction results with those of a single normal metal which was studied in Ref. [39]. In Fig. 4 it is clear that the thermal conductance of a normal metal N is proportional to temperature, i.e. $\kappa \propto T$. This linear dependence coefficient is $\pi^2/3 = 3.28$ (Fig. 4 and Ref. [20]). The thermal behaviour of the NS junction is interesting. As in Fig. 4, the thermal conductance of the NS junction for low temperatures $T \leq 0.4T_c$ is less than that of pure normal metal, but for high values of temperature $T > 0.4T_c$ it is greater than the normal metal one. In addition, for the proximity system the temperature dependence is not linear. Also, at temperatures close to T_c, the thermal conductance of the NS junction in terms of temperature is oscillatory. As expected, the thermal conductances for N and NS tend to the same value at temperature $T = T_c$. Also, using Eq. (7), we investigate the charge conductance dependence on the temperature. As is clear in Fig. 5, charge conductance is constant for normal metal, and the variation of temperature does not affect the charge conductance considerably. In addition, in Fig. 5 we see that the charge conductance of the normal layer in the NS junction is less than that of the same layer of normal metal. At temperatures close to T_c, the charge conductance diagram has some oscillations. Also, the conductances for N and NS will be the same at temperature $T = T_c$. Now, using the obtained thermal and charge conductances, we investigate the validity of the WF law and calculate the Lorenz ratio. As seen in Fig. 6, the normal metal WF ratio has a linear dependence on the temperature, and the L number is $\pi^2/3 = 3.28$. As shown in Fig. 6, at low temperatures ($\approx 0-0.05T_c$), the WF law is satisfied by the normal part of the NS junction. At temperatures close to the T_c, oscillations emerge in the conductance curve.

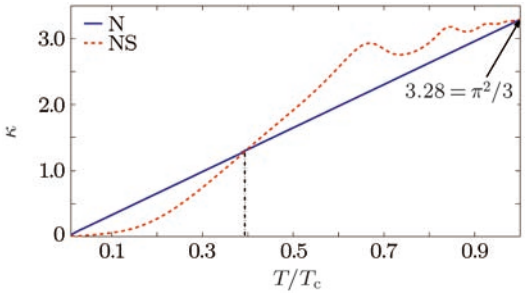


Fig. 4. (colour online) Thermal conductances versus T/T_c for a normal metal (N) and NS junction for $d_N/\xi_s = 1$. The thermal conductance of a normal metal, N, is proportional to temperature ($\kappa \propto T$). The coefficient of the linear relation is $\pi^2/3 = 3.28$. The thermal conductance of the NS junction for low temperatures, $T/T_c \leq 0.4$, is less than that of pure normal metal, but for high values of temperature, $T/T_c > 0.4$, it is greater than that of normal metal. This is a non-monotonic effect of proximity on thermal conductance in the normal region.

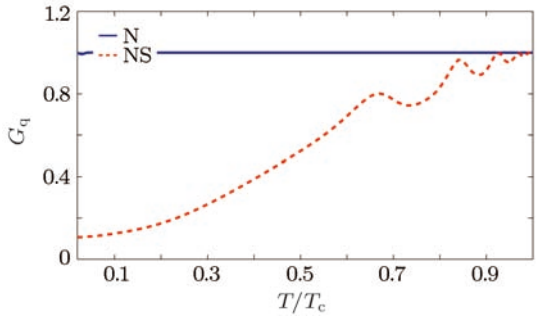


Fig. 5. (colour online) Charge conductances versus T/T_c for a single normal metal (N) and NS junction for $d_N/\xi_s = 1$. Charge conductance is constant in the single normal metal, G_q = const. In addition, the charge conductance of the normal part in the NS junction is less than that of the single normal metal layer.

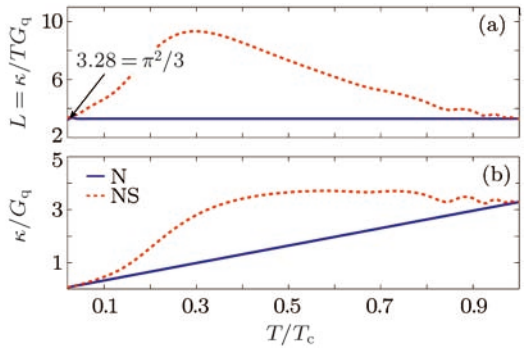


Fig. 6. (colour online) (a) WF law and (b) L number versus T/T_c for single normal metal (N) and NS junction for $d_N/\xi_s = 1$. The WF ratio has a linear dependence on temperature for normal metal and the L number is $\pi^2/3$. At low temperatures ($\approx 0 - 0.05T_c$), the WF law is satisfied by the NS junction. Also, at temperatures close to T_c, oscillations emerge in the conductance curve.

Now we study the influence of the thickness of the normal region d_N on conductivity. As seen in Fig. 7, for any particular thickness d_N/ξ_s, at a temperature such as T_L, the thermal conductivity of the NS junction is equal to that of the normal metal. The increase in thickness, d_N/ξ_s, reduces the temperature, T_L, and for large values of d_N that are close to that of normal metal, T_L tends to zero. Here, we want to make a theoretical plan for the experimental setup. As in Ref. [51], we select copper for the normal part of our setup, with a 50 nm thickness and $L = 500$ nm. Also, our superconductor is Al, with a thickness larger than the superconducting coherence length and an energy gap of $\Delta_0 \simeq 2$ K.

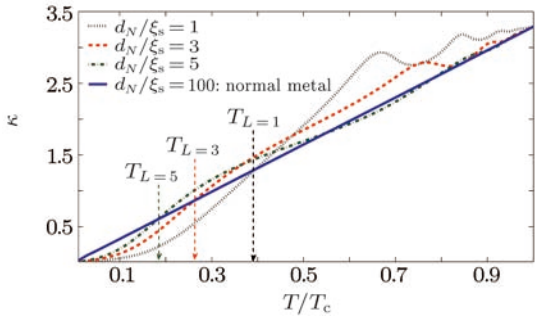


Fig. 7. (colour online) The effects of the thickness of the normal region d_N on the thermal conductance. For some particular thickness of the normal layer, d_N/ξ_s, at a specified temperature such as T_L, the thermal conductance of an NS junction is equal to that of the single normal metal layer. Also, by increasing the thickness of the normal film, d_N/ξ_s, the specified temperature T_L decreases. Thus for thick layers of normal metal, the thermal conductance tends to the thermal conductance of the single normal layer and T_L tends to zero. This case is understandable because thick normal film superconductors do not play any role.

4. Conclusion

In this paper we investigated the thermal and charge conductances for the NS junction. We found that for the NS junction thermal conductance increases by increasing the temperature, and at temperatures close to T_c some oscillations appear. These oscillations can be the result of the superconducting proximity effect inside the normal metal. In addition, the thermal conductance of the NS junction is equal to that of the normal metal at zero temperature, and at a finite temperature T_L. We also investigated the effect of the thickness of normal metal d_N on the thermal conductance, and found that temperature T_L reduces to zero by increasing d_N. Thus $T_L \longrightarrow 0$ for $d_N \gg \xi_s$. In addition charge conductance, which is almost constant for normal metal, for the NS junction increases by increasing the temperature. It should be mentioned that the charge conductance of the NS junction is less than that of the normal metal, and the charge conductance for high temperatures has oscillations like thermal conductances. Next we found that the WF law is satisfied for the NS junction at low temperatures ($\approx 0 - 0.05T_c$), and for high temperature the WF law is violated. Finally, we discovered that the proximity effect has an important influence on the thermal and charge conductances in the normal film. This effect shows a clear difference between the conductances of the normal film in the proximity of a superconductor and a single normal layer.

Acknowledgement

We would like to thank Yu A Kolesnichenko for the thorough, constructive and helpful comments, discussions and suggestions on the manuscript.

References

[1] Guron S, Pothier H, Birge N O, Esteve D and Devoret M H 1996 *Phys. Rev. Lett.* **77** 3025
[2] Courtois H, Charlat P, Gandit Ph, Mailly D and Pannetier B 1999 *J. Low Temp. Phys.* **116** 187
[3] Pannetier B and Courtois H 2000 *J. Low Temp. Phys.* **188** 599
[4] Moussy N, Courtois H and Pannetier B 2001 *Europhys. Lett.* **55** 861
[5] Dubos P, Courtois H, Pannetier B, Wilkhelm F K, Zaikin A D and Schn G 2001 *Phys. Rev.* B **63** 064502
[6] Rashedi G and Kolesnichenko Yu A 2004 *Phys. Rev.* B **69** 024516
[7] Rashedi G 2010 *Chin. Phys.* B **19** 107303
[8] Taddei F, Giazotto F and Fazio R 2005 *J. Comput. Theor. Nanosci.* **2** 329
[9] Del Maestro A, Rosenow B, Shah N and Sachdev S 2008 *Phys. Rev.* B **77** 180501
[10] Linder J, Yokoyama Y, Tanaka Y, Asano Y and Sudbϕ A 2008 *Phys. Rev.* B **77** 174505
[11] Durst A C and Sachdev S 2009 *Phys. Rev.* B **80** 054518
[12] Levchenko A 2010 *Phys. Rev.* B **81** 012507
[13] Andersson A and Lidmar J 2010 *Phys. Rev.* B **81** 060508
[14] Golubev D S and Zaikin A D 2010 *Phys. Rev.* B **82** 134508
[15] Ruiz H S, Lopez C and Badia-Majos A 2011 *Phys. Rev.* B **83** 134508
[16] Mahmoodi R, Shevchenko S N and Kolesnichenko Yu A 2002 *Low Temp. Phys.* **28** 184
[17] Mahmoodi R, Shevchenko S N and Kolesnichenko Yu A 2002 *Fiz. Nizk. Temp.* **28** 262
[18] Rashedi G and Kolesnichenko Yu A 2005 *Low Temp. Phys.* **31** 481
[19] Rashedi G and Kolesnichenko Yu A 2005 *Fiz. Nizk. Temp.* **31** 634
[20] Kittel C 2004 *Introduction to Solid State Physics* (Berkeley: University of California Press)
[21] Jones W and March N H 1685 *Theoretical Solid State Physics* (New York: Courier Dover Publications)
[22] Poole C P, Farach H A, Creswick R J and Prozorov R 2007 *Superconductivity* 2nd edn. (London: Academic Press)
[23] Poole C P 2000 *Handbook of Superconductivity* (San Diego: Academic Press)
[24] Alidoust M, Rashedi G, Linder J and Sudbo A 2010 *Phys. Rev.* B **82** 014532
[25] Rahnavard Y, Rashedi G and Yokoyama T 2010 *J. Phys: Condens. Matter* **22** 415701
[26] Rashedi G, Rahnavard Y and Kolesnichenko Yu A 2010 *Low Temp. Phys.* **36** 205
[27] Rashedi G, Rahnavard Y and Kolesnichenko Yu A 2010 *Fiz. Nizk. Temp.* **36** 262
[28] Golubov A A, Kupriyanov M Y and Il'ichev E 2004 *Rev. Mod. Phys.* **76** 411

[29] Belzig W, Bruder C and Schon G 1996 *Phys. Rev.* B **54** 9443
[30] Scheer E, Belzig W, Naveh Y, Devoret M H, Esteve D and Urbina C 2001 *Phys. Rev. Lett.* **86** 284
[31] Rubio-Bollinger G, de las Heras C, Bascones E, Agrat N, Guinea F and Vieira S 2003 *Phys. Rev.* B **67** 121407(R)
[32] Clarke J 1969 *Proc. R. Soc. London.* A **308** 447
[33] Ishida S, Nakajima M, Tomioka Y, Ito T, Miyazawa K, Kito H, Lee C H, Ishikado M, Shamoto S, Iyo A, Eisaki H, Kojima K M and Uchida S 2010 *Phys. Rev.* B **81** 094515
[34] Koerting V, Andersen B M, Flensberg K and Paaske J 2010 *Phys. Rev.* B **82** 245108
[35] de Gennes P G 1964 *Rev. Mod. Phys.* **36** 255
[36] de Gennes P G 1996 *Superconductivity of Metals and Alloys* (New York: Benjamin)
[37] Buzdin A I 2005 *Rev. Mod. Phys.* **77** 935
[38] Graf M J, Yip S K, Sauls J A and Rainer D 1996 *Phys. Rev.* B **53** 15147
[39] Wiedemann G and Franz R 1853 *Ann. Phys.* **89** 497
[40] Ronning F, Hill R W, Sutherland M, Hawthorn D G, Tanatar M A, Paglione J, Taillefer L, Graf M J, Perry R S, Maeno Y and Mackenzie A P 2006 *Phys. Rev. Lett.* **97** 067005
[41] Hill R W, Proust C, Taillefer L, Fournier P and Greene R L 2001 *Nature* (London) **414** 711
[42] van Harlingen D J, Heidel D F and Garland J C 1980 *Phys. Rev.* B **20** 1842
[43] Tanatar M A, Reid J P, Shakeripour H, Luo X G, Doiron-Leyraud N, Ni N, Bud'ko S L, Canfield P C, Prozorov R and Taillefer L 2010 *Phys. Rev. Lett.* **104** 067002
[44] Mattis D C and Bardeen J 1958 *Phys. Rev.* **111** 412
[45] Geilikman B T 1958 *Zh. Eksp. Teor. Fiz.* **34** 1042 [1958 *Sov. Phys. JETP* **34** L721]
[46] Kadanoff L P and Martin P 1961 *Phys. Rev.* **124** 670
[47] Tewordt L 1962 *Phys. Rev.* **128** 12
[48] Eckern U 1983 *J. Low Temp. Phys.* **50** 489
[49] Machida Y, Sakai S, Izawa K, Okuyama H and Watanabe T 2011 *Phys. Rev. Lett.* **106** 107002
[50] Kopnin N B, Mel'nikov A S, Pozdnyakova V I, Ryzhov D A, Shereshevskii I A and Vinokur V M 2007 *Phys. Rev.* B **57** 024514
[51] Rabani H, Taddei F, Bourgeois O, Fazio R and Giazotto F 2008 *Phys. Rev.* B **78** 012503
[52] Tewordt L and Fay D 2003 *Phys. Rev.* B **67** 134524
[53] Spivak B, Oreto P and Kivelson S A 2008 *Phys. Rev.* B **77** 214523
[54] Podolsky D, Vishwanath A, Moore J and Sachdev S 2007 *Phys. Rev.* B **75** 014520
[55] Houghton A, Lee S and Marston J B 2002 *Phys. Rev.* B **65** 220503
[56] Blonder G E, Tinkham M and Klapwijk T M 1982 *Phys. Rev.* B **25** 4515
[57] Andreev A F 1964 *Sov. Phys. JETP* **19** 1228
[58] Bardas A and Averin D 1995 *Phys. Rev.* B **52** 12873
[59] Devyatov I A, Kuprianov M Y, Kuzmin L S, Golubov A A and Willander M 2000 *J. Exp. Theor. Phys.* **90** 1050
[60] Kohen A and Deutscher G 2001 *Phys. Rev.* B **64** 060506
[61] Eilenberger G 1968 *Z. Phys.* **214** 195
[62] Larkin A I and Ovchinnikov Y N 1968 *Sov. Phys. JETP* **26** 1200
[63] Amin M H S, Omelyanchouk A N, Rashkeev S N, Coury M and Zagoskin A M 2002 *Physica* B: *Condens. Matter* **318** 162
[64] Schmidt V V, Muller P and Ustinov A V (eds.) 1997 *The Physics of Superconductors* (New York: Springer)
[65] Zareyan M, Belzig W and Nazarov Y V 2002 *Phys. Rev.* B **65** 184505
[66] Bennemann K H and Ketterson J B (eds.) 2008 *Superconductivity: Conventional and Unconventional Superconductors* Vol. 1 (New York: Springer) pp. 280–283