Persistent current in a magnetized Rashba ring*

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We theoretically study the persistent currents flowing in a Rashba quantum ring subjected to the Rashba spin–orbit interaction. By introducing uniform or nonuniform magnetization into the ring, we find that a nonzero persistent charge current circulates in the ring, which stems from the original equilibrium spin current due to the Rashba spin–orbit interaction. Because of broken time reversal symmetry, the two oppositely flowing spin-up and spin-down charge currents of the equilibrium spin current are no longer equal, and so a net persistent charge current can flow in the system. It is also found that the persistent current can be modulated by the Fermi energy, the Rashba spin–orbit interaction strength and the magnetization in the ring. Moreover, the magnetization perpendicular to the ring plane can optimize the current. The persistent current flowing in the ring is a manifestation of the nonzero equilibrium spin current existing in the ring.

Keywords: persistent current, equilibrium spin current, Rashba ring, magnetization

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1. Introduction

In the past two decades spintronics[1−5] has emerged as one of the most vigorously pursued areas of condensed matter physics, materials science and nanotechnology. The aim here is to utilize the spin degree of freedom of an electron to make spin-based devices, which are believed to have many advantages over conventional charge-based electronic devices, such as faster operational speed, lower energy dissipation and higher integration density. During the development of spintronics, the good generation, manipulation and detection of a spin-polarized current in nanostructures have become important issues and attracted enormous attention in the research community.

In order to control and manipulate spin coherently, researchers prefer the spin–orbit interaction (SOI) effect in paramagnetic semiconductor[6−10] to the cumbersome magnetic field, since the SOI effect makes spin control possible in very short lengths and timescales via electric fields, and therefore enables smooth integration with conventional electronics. The SOI coupling the spin degree of freedom of an electron to its orbital motion originates from the relativistic effect, and can be quite significant in some semiconductors, such as InGaAs/InAlAs heterostructures and InAs quantum wells.[7,8] Koga *et al.*[7] measured the Rashba SOI (RSOI) in a two-dimensional electron gas (2DEG) system at room temperature and confirmed that the RSOI strength can be effectively modulated by a perpendicular electric field to a large extent.

Based on such an RSOI in the 2DEG system, many important effects[11−13] have been predicted in the literature, for example Datta and Das[11] proposed a spin transistor by use of the spin precession due to the RSOI two decades ago. More recently the intrinsic spin Hall effect was theoretically predicted by Sinova *et al.*[12] in the 2DEG with the RSOI, as well as by Murakami *et al.*[13] in a three-dimensional p-doped semiconductor with the Luttinger SOI. The spin Hall effect is referred to as a transverse pure spin current generated by a longitudinal electric field, similar to the usual Hall effect. Since then, tremendous research interest[14−18] has been aroused and plenty of work has been dedicated to this subject.

In addition to the spin Hall current driven by an external electric force in the RSOI medium, the equilibrium spin current (ESC) was also proposed by

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many authors in RSOI systems.[19−23] For instance Rashba[19] first pointed out a nonzero background ESC flowing in an infinite RSOI system, and concluded that the ESC was not a real spin transport and that the conventional definition of the spin current was questionable. Recently Sonin[20] showed that this ESC was indeed a real spin transport and could be detected by measuring the mechanical torque on the system edges. Sun *et al.*[21] reached the same conclusion that the ESC was a real spin transport in a ring device with RSOI, and argued that the generated electric field could be used to detect the ESC. One of the authors found a net pure spin current flowing through two-terminal SOI junctions[22] with different types of SOIs, which was accounted for by the exchange coupling between the noncollinear pseudomagnetic fields from different SOIs.

ESC respects the time reversal symmetry and there is no accompanied charge current or spin accumulation, thus it is very difficult to detect. To prove the existence of the spin current, the desirable scheme[23−29] is to transform the spin current into directly measurable quantities such as charge current, voltage drop and spin accumulation. For instance, by using the inverse spin Hall effect,[27−29] several groups have demonstrated that the nonequilibrium pure spin current could be converted to a measurable voltage drop in a spin-Hall device. In contrast, however, at present the detection of ESC has not had enough attention and so whether it really exists is still an open issue.[30]

As is currently known, a persistent charge current (PCC) can flow in a mesoscopic quantum ring threaded by a magnetic flux.[31,32] This has already been confirmed experimentally by measuring the magnetization response with SQUID.[33,34] Thus, in the present work we study the possible PCC stemming from the ESC in a magnetized Rashba ring. The key observation is that the time reversal symmetry of the system is broken when the uniform or nonuniform magnetization is introduced into the ring, so that the equality of the up- and down-spin current components of the ESC is broken and the charge current can form. Certainly there should be no charge current in the ring with only RSOI or magnetization. The magnetization in the ring could be obtained through the magnetic proximity effect by mounting a ferromagnetic insulator. Alternatively, nonuniform magnetization could also be obtained by introducing magnetic impurities into the ring. According to the tight-binding lattice model and Green's function method, we show that a nonzero PCC can indeed circulate in the ring. The ESC with spin polarization out of the ring plane accounts for the generation of the PCC. It is also found that the PCC can be modulated by the Fermi energy of the system, the RSOI strength and the introduced magnetization.

The rest of this paper is organized as follows. In Section 2 we present the model Hamiltonian for the Rashba ring system and the general formulae to calculate the persistent current. In Section 3 the numerical investigations are presented to show the properties of the persistent current, and a brief conclusion is drawn in the final section.

2. Model and formulation

We consider a one-dimensional (1D) Rashba ring as shown schematically in Fig. 1, where the ring lies in the xy plane and the quantum spin axis of the system is set along the z axis perpendicular to the ring plane. The following noninteracting Hamiltonian is adopted to describe the system:

$$H = \boldsymbol{p}^2/2m + \frac{\alpha}{\hbar}\boldsymbol{\sigma}\cdot(\hat{z}\times\boldsymbol{p}) + V(\boldsymbol{r}), \qquad (1)$$

where $\boldsymbol{p}$ is the 2D momentum operator, m is the effective mass of the electron in the ring, the second term on the right-hand side is the RSOI with α denoting its strength, $\boldsymbol{\sigma}$ is the Pauli operator, and $V(\boldsymbol{r})$ is the potential confining electron in the ring. One can see that

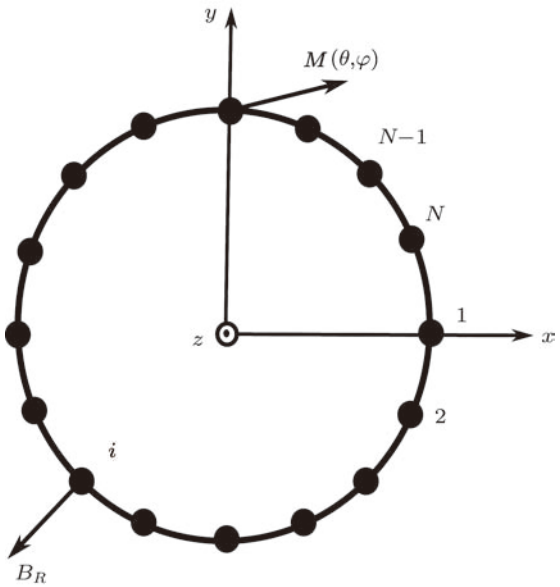


Fig. 1. (colour on line) Schematic representation of a uniformly magnetized lattice ring subjected to RSOI. The ring plane is in the xy plane and the uniform magnetization is represented by $\boldsymbol{M}(\theta,\phi)$. $\boldsymbol{B}_R$ is along the radial axis when the electron is circulating the ring in a clockwise direction.

there is a pseudomagnetic field $\boldsymbol{B}_R = (\hat{z} \times \boldsymbol{p})$ due to the RSOI, which is perpendicular to the direction of the electron momentum $\boldsymbol{p}$; $\hat{z}$ is the unit vector along z axis.

In the cylindrical coordinate, the above Hamiltonian can be to the lowest radial mode transformed into[35,36]

$$H = \frac{\hbar^2}{2mR^2}\left(-\mathrm{i}\frac{\partial}{\partial\varphi} + \frac{\beta}{2}\sigma_r\right)^2, \tag{2}$$

where $\sigma_r = \sigma_x\cos\varphi + \sigma_y\sin\varphi$ is defined as the radial spin operator, a tangential spin operator $\sigma_\varphi = \partial\sigma_r/\partial\varphi$ is implicit in the above equation, R is the perimeter of the ring and φ is a cylindrical coordinate variable. Although the ring Hamiltonian together with a uniform magnetization can be solved rigorously, in the partial magnetization case that is also worth studying we discretize the above Hamiltonian into a tight-binding one[37]

$$\begin{aligned} H \;=\; & \sum_{i=1,\sigma\sigma'}^{N} C^\dagger_{i\sigma}(\varepsilon_i\delta_{\sigma\sigma'} + \boldsymbol{\sigma}\cdot\boldsymbol{M}_i)C_{i\sigma'} \\ & - t\sum_{i\sigma}^{N}(C^\dagger_{i+1,\sigma}C_{i\sigma} + \text{c.c.}) \\ & - t_{\text{so}}\sum_{i,\sigma\sigma'}^{N}(C^\dagger_{i-1,\sigma}\,\mathrm{i}\,\sigma_r(\varphi_i)C_{i\sigma'} + \text{h.c.}), \end{aligned} \tag{3}$$

$$\varphi_i = \frac{2\pi(i-1/2)}{N}, \tag{4}$$

where the operator $C^\dagger_{i,\sigma}$ ($C_{i,\sigma}$) creates (annihilates) an electron with spin σ at the site i ($i = 1,\ldots,N$) of the ring; $\sigma_r(\varphi_i) = \sigma_x\cos\varphi_i + \sigma_y\sin\varphi_i$; N is the total number of lattice sites; $t = \hbar^2/2ma^2$ is the hopping energy between the two nearest neighbour sites, while $t_{\text{so}} = 2\alpha/a$ is the RSOI strength in the discrete version, with a being the lattice constant; σ and σ' are the spin indexes; ε_i is the on-site energy and $\varepsilon_i = 2t$ in the 1D ring; and $\boldsymbol{M}_i = M(\theta_i,\phi_i)$ is the magnetization in the unit of energy at each site i. The magnetization can be realized by the magnetic proximity effect through the location of a ferromagnetic insulator on the ring, and it is crucial to generate the PCC circulating in the ring. Certainly some magnetic atoms doped in the ring can also validate our model.[38]

It is emphasized here that the system is in equilibrium and there is no applied bias. The spin-resolved charge current can be worked out from the Heisenberg equation of motion as $I_{i\sigma} = e\,\mathrm{d}\hat{n}_{i\sigma}/\mathrm{d}t = (e/\mathrm{i}\,\hbar)[\hat{n}_i, H]$, where $\hat{n}_{i\sigma}$ is the electron operator with spin σ at site i. In terms of the lesser Green's function $G^<$, it can be written as ($e = \hbar = 1$)

$$I_{\mathrm{i}\,\sigma}(t) = [\tilde{t}_{i,i-1}G^<_{i-1,i}(t,t) - G^<_{i,i-1}(t,t)\tilde{t}_{i-1,i}]_{\sigma\sigma}, \tag{5}$$

where $G^<(t,t')_{i\sigma,i'\sigma'} = \mathrm{i}\langle C^\dagger_{i'\sigma'}(t')C_{i\sigma}(t)\rangle$, $\langle\cdots\rangle$ denotes the quantum statistic average, $\tilde{t}_{i,i-1} = t\sigma_0 + \mathrm{i}\,t_{\text{so}}\sigma_r(\varphi_i) = \tilde{t}^\dagger_{i-1,i}$ is the 2×2 hopping matrix between the i-th and $(i-1)$-th site, and σ_0 is the unit matrix. In the 2D case the matrix dimension of $\tilde{t}$ is $2m\times2m$ with m being the transverse site number of the ring, and the current formula I_σ needs a trace over these transverse sites. In a steady state the current can be further reduced to

$$\begin{aligned} I_i \;=\; & \int\frac{\mathrm{d}E}{2\pi}\,\mathrm{Tr}[\sigma_0(\tilde{t}_{i,i-1}G^<(E)_{i-1,i} \\ & - G^<(E)_{i,i-1}\tilde{t}^\dagger_{i,i-1})]. \end{aligned} \tag{6}$$

By using the Keldysh equation at equilibrium

$$G^<(E) = (G^{\text{a}} - G^{\text{r}})f(E), \tag{7}$$

the spin-resolved current then relates to the retarded (G^{r}) and advanced ($G^{\text{a}} = (G^{\text{r}})^\dagger$) Green's function and reads

$$\begin{aligned} I_i \;=\; & \int\frac{\mathrm{d}E}{2\pi}2\,\mathrm{Re}\{\mathrm{Tr}[\sigma_0(\tilde{t}_{i,i-1}G^{\text{r}}(E)_{i-1,i} \\ & - G^{\text{r}}(E)_{i,i-1}\tilde{t}_{i-1,i})]\}f(E), \end{aligned} \tag{8}$$

where $f(E)$ is the Fermi–Dirac distribution function. The retarded Green's function could be worked out by direct matrix inverse

$$G^{\text{r}} = [E + \mathrm{i}\,0^+ - H]^{-1}. \tag{9}$$

To save computation time we employ a recursive method to derive the relevant quantities such as $G^{\text{r}}_{i,i-1}$ in Eq. (8), and the derivation is presented in detail in Appendix A. Following the same procedure, the ESC in the ring is given by

$$\begin{aligned} \boldsymbol{J}_i \;=\; & \int\frac{\mathrm{d}E}{2\pi}2\,\mathrm{Re}\{\mathrm{Tr}[\boldsymbol{\sigma}(\tilde{t}_{i,i-1}G^{\text{r}}(E)_{i-1,i} \\ & - G^{\text{r}}(E)_{i,i-1}\tilde{t}_{i-1,i})]\}f(E). \end{aligned} \tag{10}$$

This equation could actually be obtained by directly replacing σ_0 with $\boldsymbol{\sigma}$ in Eq. (8). The pure spin current $\boldsymbol{J}_i$ is a tensor in nature, and its spin polarization has three components in Cartesian coordinates. In the following discussion on ESC, we focus on the nonlocal σ_r spin current (J_r) and σ_z spin current (J_z), because the magnitudes of the σ_r and σ_z spin currents are "conserved" and independent of the coordinate variable φ in a uniform Rashba ring, whereas the tangential spin current σ_φ is local and varies with lattice site i.

Since the studied system is in equilibrium, the electrons in the full conduction band contribute to both the charge and spin currents, so the integrations in Eqs. (8) and (10) should be performed from the bottom of the energy band to the Fermi energy at the zero temperature. This case is the same as the usual PCC in a normal ring threaded by a magnetic flux. The PCC is a quantum mechanics effect and the electronic transport should remain coherent in the whole ring. It is therefore assumed that the perimeter of the ring R is less than the phase relaxation length of the electron and the localization length due to the static disorder. Certainly the spin is not a conserved quantity in the Rashba ring because of the RSOI, so the ESC may be different from site to site. In contrast the PCC is a conserved quantity and can be estimated at an arbitrary site in the ring.

3. Numerical results and discussion

In this section we calculate both the PCC and ESC flowing in the magnetized Rashba ring according to the formulae introduced in the last section. Three cases of magnetization distribution in the ring are computed and discussed: zero magnetization, uniform magnetization and partial magnetization. The calculations are carried out at zero temperature and on the condition that the hopping energy is taken as the energy unit $t = 1$, the Fermi energy is set as $E_{\rm F} = 0.2t$ (in most situations other than those stated directly), and the Fermi-wave length is about $\lambda_{\rm F} = 15a$, so that the tight-binding approximation can be used to simulate a continuum model since the wavelength of the electron below $E_{\rm F}$ is much larger than the lattice constant a. In addition, the number of sites in the ring (N) is set as 200, i.e. $N = 200$. For an InGaAs-based 2DEG, the RSOI constant α can be controlled by a gate voltage with typical values[7] in a range of 0.5×10^{-11} eV·m–2.0×10^{-11} eV·m, and is estimated to be $t_{\rm so} \sim 0.01t$ or so in the lattice version. The magnetization M is set to have the same magnitude as the RSOI constant, but much less than the Fermi energy.

3.1. Zero magnetization

First we focus on the ESC in the Rashba ring without any magnetization. In Ref. [21] it was shown that a nonzero ESC is circulating in a hybrid ring that consists of two parts with and without RSOI, and the ESC was estimated in the non-RSOI part where the definition of the spin current is the absence of ambiguity. From the point of view of the pseudomagnetic field $\boldsymbol{B}_{\rm R}$, one can readily catch the ESC flowing in the ring. The exchange coupling between two pseudomagnetic fields due to the SOIs, which was shown in Ref. [22], leads to a nonzero ESC flowing in the system. In the Rashba ring the momentum-dependent $\boldsymbol{B}_{\rm R} = \hat{z} \times \boldsymbol{p}$ is along the radial axis as shown in Fig. 1. The electron at each site i is subjected to the different directional $\boldsymbol{B}_{\rm R}(i)$ fields, therefore the exchange coupling between pseudomagnetic fields ($\boldsymbol{B}_{\rm R}(i)$ fields) will result in a nonzero ESC in the ring, and its spin polarization should point along the z axis, $\boldsymbol{J}_i \sim \sum_{j \neq i} \boldsymbol{B}_{\rm R}(i) \times \boldsymbol{B}_{\rm R}(j)$. Another naive argument for the ESC flowing in the ring is that the local spin state of the electron is dependent on the momentum direction at each site i due to the RSOI. The electrons circulating in a clockwise direction have different spin states from those electrons circulating in the ring in an anticlockwise direction, so an ESC with spin polarization along the radial axis can also flow in the ring. The ESC is actually a pure spin current that consists of spin-up and the spin-down charge currents flowing oppositely with equal magnitudes.

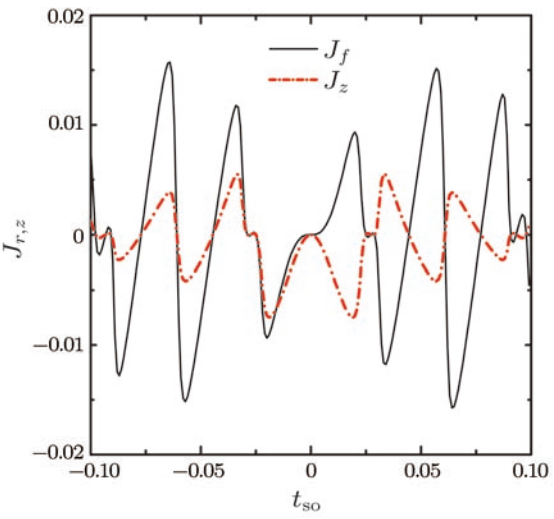


Fig. 2. (colour on line) ESC J_r and J_z, each as a function of RSOI constant $t_{\rm so}$ in the ring without any magnetization. In the calculations the parameters are set to be $E_{\rm F} = 0.2t$, $M = 0$ and $N = 200$.

In Fig. 2 the ESC J_r and J_z are plotted, each as a function of RSOI strength $t_{\rm so}$, and exhibit different symmetries of the RSOI. J_r is an odd function of $t_{\rm so}$, whereas J_z remains unchanged as the sign of $t_{\rm so}$ is reversed. As discussed above, the J_z stems from the exchange coupling between the local pseudomagnetic fields ($\boldsymbol{B}_{\rm R}(i)$ fields), thus only reversing $\boldsymbol{B}_{\rm R}(i)$ does

not change the sign of J_z. However, this is not the case for the J_r, which depends on the local $\boldsymbol{B}_{\rm R}$ directly, and moreover the spin polarization of J_r points along the direction of the $\boldsymbol{B}_{\rm R}$. As a result the J_r should change its sign upon reversing the $t_{\rm so}$ sign.

The oscillation of $J_{r,z}$ relates to the fact that the Fermi energy $E_{\rm F}$ can intersect the different electron levels with the increase of $t_{\rm so}$. The energy-resolved ESC $J_{r,z}(E)$ is presented in Fig. 3, and all the discrete energy levels below $E_{\rm F}$ contribute to the ESC. This case is the same as the usual PCC in a normal ring. The

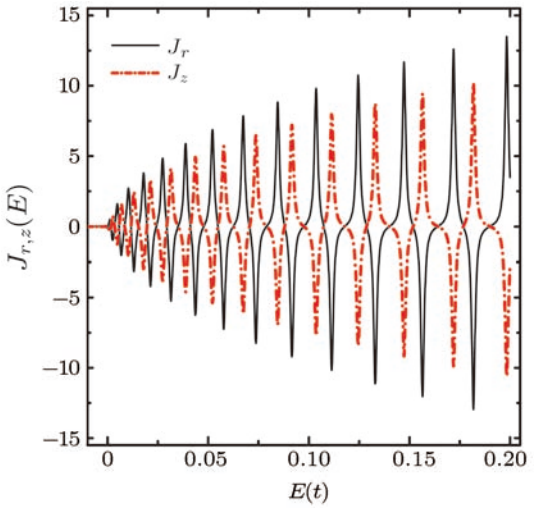


Fig. 3. (colour online) Energy-resolved ESC $J_r(E)$ and $J_z(E)$, each as a function of energy E in the ring without any magnetization. The parameters are set to be $t_{\rm so} = 0.02$, $M = 0$ and $N = 200$.

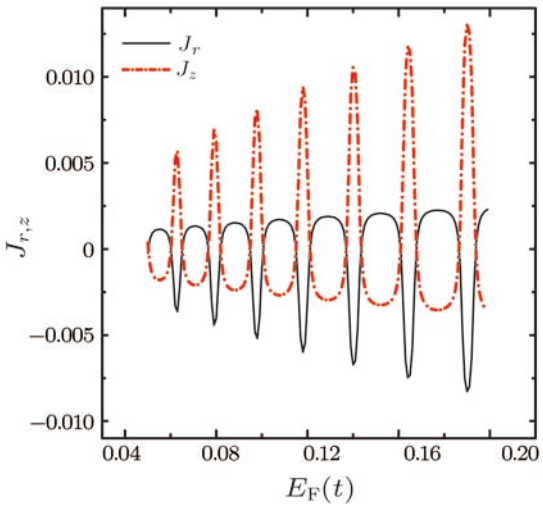


Fig. 4. (colour online) ESC J_r and J_z in the ring, each as a function of Fermi energy $E_{\rm F}$. The parameters are set to be $t_{\rm so} = 0.01$, $M = 0$ and $N = 200$.

two consecutive energy levels have opposite contributions to the charge current since their wavefunctions possess different parities. Therefore, the ESC is at the zero temperature mainly determined by a few levels just below $E_{\rm F}$. With the increase in RSOI strength $t_{\rm so}$, the energy levels go through $E_{\rm F}$ one by one, and thus the $J_{r,z}$ exhibits an oscillating behaviour as shown in Fig. 2. As a matter of fact the ESC also oscillates with the Fermi energy $E_{\rm F}$ of the ring for the same reason, and varies continuously from the positive to the negative value, which is shown in Fig. 4. Each oscillating period corresponds to one energy level going though the $E_{\rm F}$.

3.2. Uniform magnetization

In the above section we have shown that there is an ESC flowing in the Rashba ring and it is interpreted based on the pseudomagnetic field from the RSOI. Currently it is still a big challenge to detect the spin current, since it cannot be directly measured as a charge current. Hence the general method of detecting the spin current is to utilize the induced effect by the spin current, such as spin accumulation, charge current and voltage drop. The detection of the ESC becomes much more difficult because even its existence is still an open issue.[19,30] Some authors believe that the ESC comes from the incorrect definition of the spin current and is not a real spin transport, in other words it would disappear when the correct definition was found.

In this section we will show that the ESC flowing in the Rashba ring can be converted into the PCC

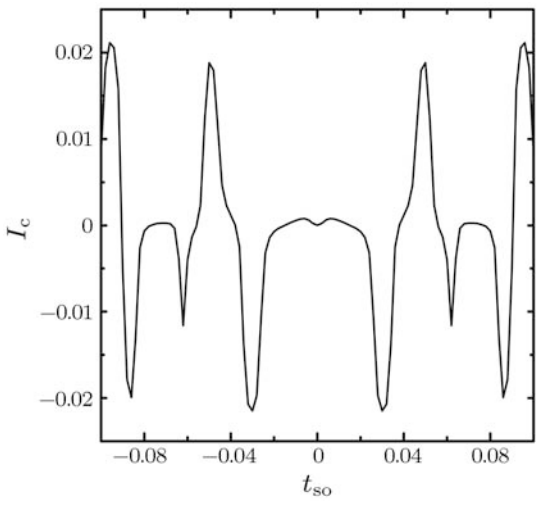


Fig. 5. Charge current $I_{\rm c}$ in the magnetized ring as a function of RSOI constant $t_{\rm so}$. The parameters are set to be $E_F = 0.2t$, $M = 0.02$, $N = 200$ and $\theta = 0$.

by introducing the magnetization $\boldsymbol{M}$ into the system, because the time reversal symmetry of the system

is broken due to the magnetization. Specifically the antisymmetry of two oppositely flowing spin-resolved charge currents with equal magnitudes of the ESC is broken, owing to the spin splitting from $\boldsymbol{M}$, so that a charge current can form. It is seen that there should be no charge current in a normal ring with only magnetization. In Fig. 5 the PCC (I_c) is presented as a function of the RSOI constant t_{so}. At $t_{so} = 0$ the current vanishes, i.e. $I_c = 0$. It is seen that the PCC is an even function of t_{so} like spin current J_z in Fig. 2, which indicates that the PCC may result from the J_z but not the J_r.

As discussed above, the J_z originates from the exchange coupling between local pseudomagnetic fields ($\boldsymbol{B}_R(i)$ fields), and as a result J_z remains unchanged by the reversing sign of t_{so}. The J_z is composed of two equal spin-resolved charge currents (say, $I_\uparrow = I_\downarrow$) flowing oppositely, however the equality is no longer valid when magnetization $\boldsymbol{M}$ is introduced and the velocities of the spin-up and the spin-down electrons are unequal for a given energy. Therefore, the optimal direction of $\boldsymbol{M}$ will be along the z axis if only J_z is decisive to the converted charge current I_c. Figure 6 shows the current versus polar angle θ of $\boldsymbol{M}$. The I_c values arrive at their maximum values as $\boldsymbol{M}$ is along the z axis, i.e. when $\theta = 0$ and π; whereas in the inset of Fig. 6 the PCC is independent of azimuthal angle ϕ, which further implies that the spin current J_r should not contribute to the I_c. Simply speaking,

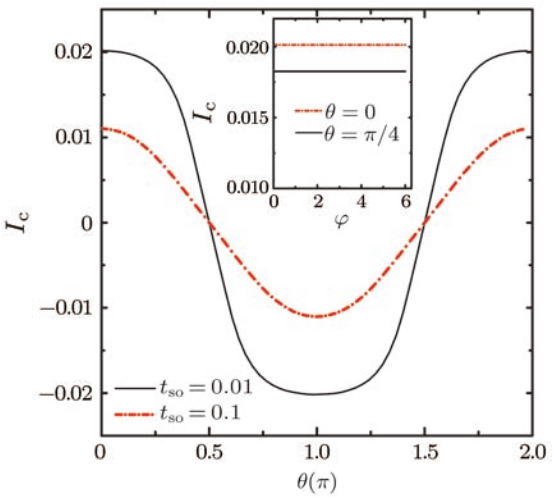


Fig. 6. (colour online) Charge current I_c in the magnetized ring as a function of polar angle θ with $\phi = 0$. The curves for I_c versus ϕ are plotted in the inset for different θ values. The parameters are set to be $E_F = 0.2t$, $M = 0.01$, $t_{so} = 0.01$, and $N = 200$.

the spin polarization of J_r is along the radial axis and the summation of the J_r on each lattice site will lead to a vanished polarization, and thus no charge current can be found. The system keeps $SO(2)$ symmetry with the uniform $\boldsymbol{M}$ considered in the Rashba ring, and even if the planar magnetization ($M \sin\theta$) can transform the local $J_r(i)$ into a local charge current, the summation over all sites will lead to zero charge current since there must be an inverse result at site $N/2 + i$ with respect to that at site i.

The PCC induced by the interplay between the J_z and $\boldsymbol{M}$ is also oscillatory with the increase in Fermi energy, as shown in Fig. 7(a), which is the same as the case of usual PCC generated by the magnetic flux. Moreover, the I_c will change its sign as the E_F goes through the electron energy level one by one. The magnitude of I_c increases with E_F because the velocity of the electron increases with the energy and a few levels just below E_F dominate the charge current. In Fig. 7(b) the I_c varies with the magnetization value M, and as $M = 0$ the I_c also vanishes. In contrast to t_{so}, the reversing of $\boldsymbol{M}$ can lead the sign of I_c to change because the magnitude difference between $I_\uparrow$ and $I_\downarrow$ in the J_z due to $\boldsymbol{M}$ will inverse too. Therefore the

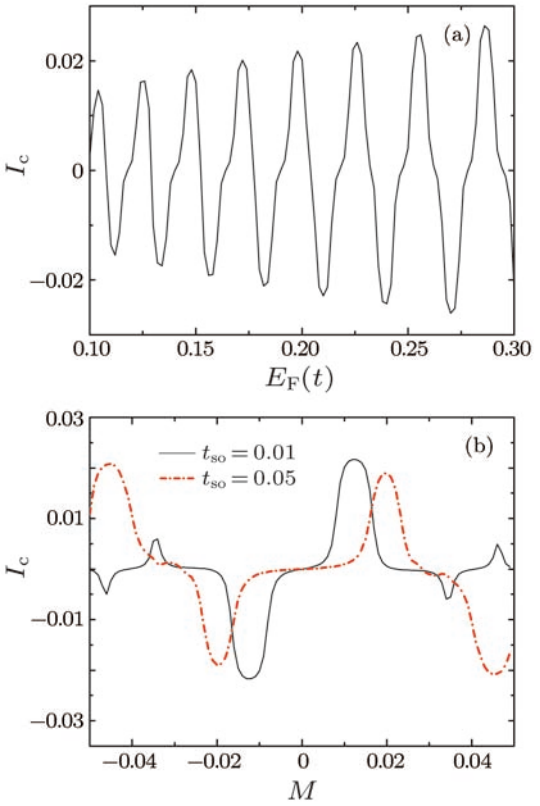


Fig. 7. (colour online) Curves for charge current I_c versus Fermi energy E_F (a) and magnetization M (b). The parameters are set to be $t_{so} = 0.01$, $\theta = 0$, $M = 0.01$ in (a), and $E_F = 0.2t$ in (b).

PCC is an odd function of $\boldsymbol{M}$. Certainly the PCC

is also oscillatory with the increase of M, which can affect the energy level of the electron in the ring.

3.3. Partial magnetization

We have discussed the uniform magnetization in the Rashba ring giving rise to a nonzero PCC. The physics reason is ascribed to the broken time reversal symmetry of the system as well as the existence of the ESC. This point implies that even partial or inhomogenous magnetization in the ring can also bring the same result, that a charge current can circulate in the ring steadily.

We now consider a hybrid Rashba ring in which only one part is magnetized, while the other is unmagnetized (zero magnetization). The length of the latter is set to be L_0 and $L_0 < R$. Figure 8 shows the curves for I_c versus L_0, and they decrease with the increase in nonmagnetization length. This indicates that partial magnetization can indeed lead to a nonzero charge current, although the magnitude of the PCC is smaller than that in the uniform $\boldsymbol{M}$ case. The clear distinction of the partial magnetization from the uniform $\boldsymbol{M}$ is that the system does not have the $SO(2)$ rotational symmetry at all, which must lead to different results from those in last section.

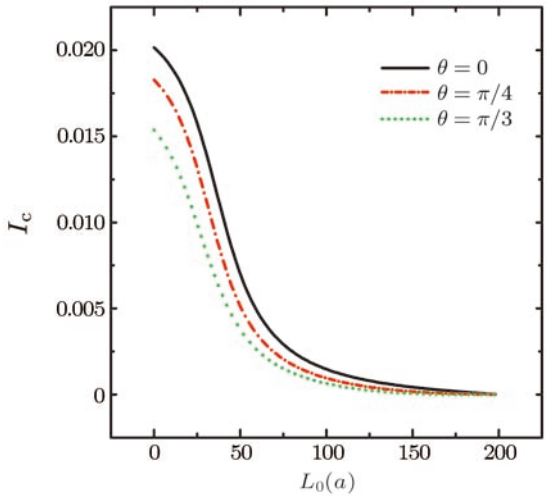


Fig. 8. (colour online) Curves for charge current I_c versus nonmagnetization length $L_0 a$ for the different θ values. The parameters are set to be $t_{so} = 0.01$, $\phi = 0$ and $M = 0.01$.

Figure 9 shows that the I_c values vary with $\boldsymbol{M}$ direction (θ, ϕ). Figure 9(a) shows that $\theta = 0$ and π are not the optimal angles for the PCC, while the maximums slightly deviate from these two values for different values of azimuthal angle ϕ, which is very different from the results shown in Fig. 6 where the uniform $\boldsymbol{M}$ is considered and the maximums of I_c are at $\theta = 0$ and π. It is seen in Fig. 9(b) that each I_c varies with ϕ and is nonzero even at $\theta = \pi/2$, i.e. the $\boldsymbol{M}$ in the xy plane can also lead to the nonzero charge current.

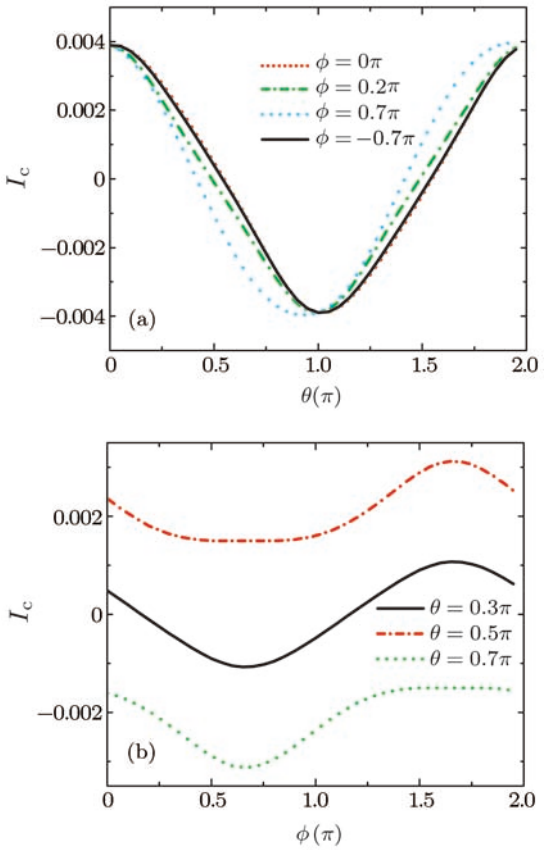


Fig. 9. (colour online) Curves for charge current I_c versus angles θ (a) and ϕ (b) for the partially magnetized ring. The parameters are set to be $t_{so} = 0.01$, $M = 0.01$, and $L_0 = Na/4$.

As discussed earlier, the planar polarization J_r in the uniform $\boldsymbol{M}$ case cannot give rise to the persistent charge current, even if the planar magnetization $M \sin\theta$ can break the equality between two oppositely flowing spin-resolved charge currents of the local $J_r(i)$, the summation over all sites will not bring in any charge current due to the rotational symmetry $SO(2)$ of the ring. However, for the partial magnetization case, such a symmetry is broken so that the summation over the local broken $J_r(i)$ can cause the nonzero PCC. In Fig. 9 the magnetization $M \cos\theta$ out of the ring plane still dominates the charge current. Obviously the larger the nonmagnetization length L_0, the more important the planar magnetization $M \sin\theta$ is.

In fact even if the magnetization $\boldsymbol{M}(i)$ at each site is different both in magnitude and direction, the nonzero charge current can also form unless the $\boldsymbol{M}(i)$

is sufficiently random, so there is no average magnetization in the ring. The PCC in the normal ring generated by the magnetic flux has already been observed experimentally by measuring the magnetization response, so it is believed that the PCC in the magnetized Rashba ring predicted in this work should be able to be measured in principle. The difficulty may come from the stray field introduced by magnetization and from the weak signal of the charge current. It is worth pointing out that the magnetization introduced in the other RSOI media does not necessarily lead to the automatic transformation of the ESC into the charge current. The charge current cannot flow in an open system. The Rashba ring is a perfectly closed system, and so the converted PCC from the ESC can circulate in the ring.

4. Summary and conclusion

We show that there is a measurable persistent charge current in a magnetized Rashba ring because of the interplay between the magnetization and the RSOI, neither of which alone can lead to the charge current. The physics origin is that the introduced magnetization in the ring breaks the time reversal symmetry of the system and transforms the original ESC due to the RSOI into the charge current. It is also shown that partial or inhomogenous magnetization can result in the charge current. The magnetization perpendicular to the ring plane dominates the PCC, especially for the uniform magnetization case. Our results clearly demonstrate that an ESC flowing in the Rashba ring does indeed exist and it is a real spin transport. The ESC can be estimated through the measurable persistent charge current.

Appendix A: Recursive iteration to calculate the lattice Green's function of the ring

In this appendix we present a recursive iteration method to calculate the retarded Green's function in the 1D ring instead of the direct matrix inverse, which may be very time-consuming, and moreover the calculation of the PCC needs integration over the full energy band. This case is different from the nonequilibrium current driven by the external electric field/voltage, which is determined by the electrons around the Fermi energy. Although the 1D ring is taken into account, the following procedure is applicable to the 2D ring as well.

The retarded Green's functions, $G^{\rm r}_{i,i-1}$, $G^{\rm r}_{i-1,i}$ in Eq. (8) are our concerns. The Hamiltonian for the ring in the matrix version is given by

$$\mathscr{H} = \begin{pmatrix} H_{i,i} & \tilde{t}_{i,i+1} & \cdots & 0 \\ \tilde{t}_{i+1,i} & H_{i+1,i+1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & H_{i-1+N,i-1+N} \end{pmatrix} + \begin{pmatrix} 0 & 0 & \cdots & \tilde{t}_{i,i+N-1} \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{t}_{i+N-1,i} & 0 & \cdots & 0 \end{pmatrix}, \tag{A1}$$

where the first term is the tridiagonal matrix and dictates a 1D discrete chain, while the second term connects the beginning and the ending of the 1D chain to form a whole ring Hamiltonian. The diagonal block $H_{i,i}$ is the energy matrix of the i-th site, and $\tilde{t}_{i-1,i}$ is the hopping matrix between two consecutive sites. We need to keep in mind the periodic repetition of the quantities in the ring, i.e. $O(i) = O(i+N)$, where O denotes a physical quantity and N is the total site number of the lattice ring.

To solve the Green's function $G^{\rm r}_{i,i-1}$, we first work out the Green's functions $G^{\rm r}_{i,i-1+N}(c)$ and $G^{\rm r}_{i-1+N,i-1+N}(c)$ in the 1D chain by using the standard recursive method,[39]

$$G^{\rm r}_{i,i}(c, i-i) = [(E + {\rm i}\,0^+)\sigma_0 - H_{i,i}]^{-1}, \tag{A2}$$

$$\begin{aligned} & G^{\rm r}_{i+j,i+j}(c, i \to i+j) \\ &= [(E + {\rm i}\,0^+)\sigma_0 - \tilde{t}_{i+j,i+j-1} G^{\rm r}_{i+j-1,i+j-1} \\ &\quad \times (c, i \to i+j-1)\tilde{t}_{i+j-1,i+j}]^{-1}, \end{aligned} \tag{A3}$$

$$\begin{aligned} & G^{\rm r}_{i,i+j}(c, i \to i+j) \\ &= G^{\rm r}_{i,i+j-1}(c, i \to i+j-1)\tilde{t}_{i+j-1,i+j} \\ &\quad \times G^{\rm r}_{i+j,i+j}(c, i \to i+j). \end{aligned} \tag{A4}$$

Here the augment $(c, i \to i+j)$ in $G^{\rm r}(c, i \to i+j)$ denotes the 1D chain system, the sites are labeled from i to $i+j$ and the total site number of this 1D chain is $j+1$. The similar Green's functions $G^{\rm r}_{i-1+N,i}(c)$ and $G^{\rm r}_{i,i}(c)$ of the 1D chain can also be worked out using the same procedure.

We proceed to solve the Green's function $G^{\rm r}_{i,i-1}$ of the whole ring by use of the usual Dyson equation, $G^{\rm r} = G^{\rm r}_0 + G^{\rm r}_0 V G^{\rm r}$, where $G^{\rm r}_0$ is the Green's function

of the 1D chain, while V represents the second term of Eq. (A1). After a direct algebra, we can obtain

$$G^{\rm r}_{i,i-1} = G^{\rm r}_{i,i-1}(c) + G^{\rm r}_{i,i}(c)\tilde{t}_{i,i-1}G^{\rm r}_{i-1,i-1} + G^{\rm r}_{i,i-1}(c)\tilde{t}_{i-1,i}G^{\rm r}_{i,i-1}, \quad \text{(A5)}$$

$$G^{\rm r}_{i-1,i-1} = G^{\rm r}_{i-1,i-1}(c) + G^{\rm r}_{i-1,i-1}(c)\tilde{t}_{i-1,i}G^{\rm r}_{i,i-1} + G^{\rm r}_{i-1,i}(c)\tilde{t}_{i,i-1}G^{\rm r}_{i-1,i-1}. \quad \text{(A6)}$$

Here the repetitive index $i = N + i$ is used in the derivation, such as $G^{\rm r}_{i,i+N-1} = G^{\rm r}_{i,i-1}$. Substituting Eq. (A6) into Eq. (A5), the Green's function $G^{\rm r}_{i,i-1}$ can be worked out. Similarly, exchanging the indices i and $i-1$ in the final formula of $G^{\rm r}_{i,i-1}$, one can obtain $G^{\rm r}_{i-1,i}$. Afterwards, we can calculate the PCC in Eq. (8) and ESC in Eq. (10) flowing in the ring. It is noted that the recursive method presented here is also applicable to the finite width ring.

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