A pure spin-current injector of semiconductor quantum dots with Andreev reflection and Rashba spin–orbit coupling*

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We propose a four-terminal device consisting of two parallel quantum dots with Rashba spin–orbit interaction (RSOI), coupled to two side superconductor leads and two common ferromagnetic leads, respectively. The two ferromagnetic leads and two quantum dots form a ring threaded by Aharonov–Bohm (AB) flux. This device possesses normal quasiparticle transmission between the two ferromagnetic leads, and normal and crossed Andreev reflections providing conductive holes. For the appropriate spin polarization of the ferromagnetic leads, RSOI and AB flux, the pure spin-up (or spin-down) current without net charge current in the right lead, which is due to the equal numbers of electrons and holes with the same spin-polarization moving along the same direction, can be obtained by adjusting the gate voltage, which may be used in practice as a pure spin-current injector.

Keywords: Rashba spin–orbit coupling, Andreev reflection, pure spin current without net charge current

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1. Introduction

One of the central issues in spintronics is how to control the spin degree of freedom to realize spin accumulation and spin transport in nanodevices.[1,2] Recently, much attention has been paid to the generation of pure spin current in mesoscopic systems. Various theoretical proposals have been put forward to obtain the pure spin current, such as the use of spin Hall effects,[3−6] optical spin-orientation by linearly polarized light,[7−10] adiabatic[11,12] or nonadiabatic[13−16] spin pumping in metals and semiconductors, and generation in three-terminal spin devices[17−22] or ballistic bulk graphene.[23] In spite of this existing work, any new suggestions to realize pure spin current are still of practical importance.

Andreev reflection is a dynamic process in which an electron, incident from a normal metal to a metal-superconductor interface with energy less than the superconductor energy gap (Δ), is converted into a Cooper pair in the superconductor, and thus a hole with opposite spin and velocity reflects back. In multiterminal systems, the two electrons in a Cooper pair from the same lead or different leads are distinguished as the normal and crossed Andreev reflection ones.[24] To date, many studies have been done on the transport properties of superconductor-coupled mesoscopic hybrid systems for both fundamental interest and potential applications in nanoelectronics.[25−37] Using the property of opposite charges carried by an electron and a hole, a pure spin-current injector can be realized by the equal numbers of electrons and holes with the same spin polarization. The ferromagnetic (FM) leads are needed for spin-polarization resolution, and the Rashba spin–orbit interaction (RSOI) in semiconductor quantum dots (QDs) gives rise to the spin-polarization mechanism. In a ring device, the quantum phase interference between the upper and lower arms, induced by threaded Aharonov–Bohm (AB) flux, can enhance or suppress the charge current to the right (outgoing) lead.

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2. The model and methods

In this paper we theoretically put forward a four-terminal hybrid mesoscopic device of pure spin-current injector, in which two parallel semiconductor QDs with respective side superconductor leads are coupled longitudinally to two common FM leads. The schematic diagram of the device under consideration is depicted in Fig. 1. In the QDs we consider the RSOI $H_{\rm R} = \alpha_{\rm R}(\sigma_z p_x - \sigma_{\rm x} p_z)/\hbar$ induced by the structural inversion asymmetry.[38,39] The interaction strength $\alpha_{\rm R}$ can be tuned in the experiment by an external electric field or a gate voltage. For the transport along the $\hat{x}$ axis direction, the first term of $H_{\rm R}$ leads to a spin precession, and its second quantization form gives rise to an extra spin-dependent phase factor in the hopping matrix element between the leads and the QD, while the second term causes the mixing between the different degenerate energy levels and an interlevel spin flip is induced in the corresponding second quantization form.[40,41] Here we neglect the interdot and intradot electron–electron interactions and assume that each QD has only one spin-degenerated energy level, which can be modified by a gate voltage coupled to the dots by gate capacitances. So only the effect of the extra spin-dependent phase $\sigma\varphi_{{\rm R}i}$ ($\sigma = 1, -1$; $i = 1, 2$) in the tunnel terms between the QDs and the right FM lead needs be considered. We suppose a bias to be added between the left and right FM leads, and keep zero chemical potentials in the right FM lead and the two superconductor leads. Thus, we can collect four kinds of quasiparticles (two spin polarizations of electron and hole) in the right FM lead from the normal quasiparticle transmission and the crossed Andreev reflection, respectively. The pure spin-up (spin-down) current without net charge current can be realized by adjusting the gate voltage. This pure spin-up (spin-down) current is formed by the spin-up (or spin-down) electron current and hole current flowing along the same direction, which is different from the usual pure spin current. The second-quantized Hamiltonian of the system with RSOI and threaded AB flux can be decomposed into five parts of the following form:

$$H = H_{\rm L} + H_{\rm R} + H_{\rm D} + H_{\rm S} + H_{\rm T}. \tag{1}$$

$H_{\rm L}$ and $H_{\rm R}$ represent the left and right FM leads, respectively, and they are expressed as

$$H_{\rm L} = \sum_{k_{\rm L}\sigma} (\epsilon_{k_{\rm L}\sigma} - eV_1) a^{\dagger}_{k_{\rm L}\sigma} a_{k_{\rm L}\sigma}, \tag{2}$$

$$H_{\rm R} = \sum_{k_{\rm R}\sigma} (\epsilon_{k_{\rm R}\sigma} - eV_2) a^{\dagger}_{k_{\rm R}\sigma} a_{k_{\rm R}\sigma}, \tag{3}$$

where $\epsilon_{k_{\rm L(R)}\sigma} = \epsilon_{k_{\rm L(R)}} + (-1)^{\delta_{\sigma\downarrow}} \Lambda$ (Stoner model) is the spin-dependent energy of the electron in the left (right) FM lead, with the band spin splitting Λ, $\sigma = \uparrow$, $\downarrow$ and $\delta_{\uparrow\uparrow(\downarrow\downarrow)} = 1$; $\delta_{\uparrow\downarrow(\downarrow\uparrow)} = 0$.[42] $a^{\dagger}_{k_{\rm L}\sigma}$ ($a^{\dagger}_{k_{\rm R}\sigma}$) and $a_{k_{\rm L}\sigma}$ ($a_{k_{\rm R}\sigma}$) are the creation and annihilation operators for the left (right) lead, respectively. $H_{\rm S}$ is the Hamiltonian of the two superconductor leads,

$$\begin{aligned} H_{\rm S} = & \sum_{p_{\rm u}\sigma} \epsilon_{p_{\rm u}} b^{\dagger}_{p_{\rm u}\sigma} b_{p_{\rm u}\sigma} + \sum_{p_{\rm u}} (\Delta b^{\dagger}_{p_{\rm u}\uparrow} b^{\dagger}_{-p_{\rm u}\downarrow} + {\rm H.c.}) \\ & + \sum_{p_{\rm d}\sigma} \epsilon_{p_{\rm d}} b^{\dagger}_{p_{\rm d}\sigma} b_{p_{\rm d}\sigma} \\ & + \sum_{p_{\rm d}} (\Delta b^{\dagger}_{p_{\rm d}\uparrow} b^{\dagger}_{-p_{\rm d}\downarrow} + {\rm H.c.}), \end{aligned} \tag{4}$$

where $b^{\dagger}_{p_{\rm u}\sigma}$ ($b^{\dagger}_{p_{\rm d}\sigma}$) and $b_{p_{\rm u}\sigma}$ ($b_{p_{\rm d}\sigma}$) are the creation and the annihilation operators for the up (down) superconductor lead and Δ is the energy gap of both the up and the down superconductor leads for the two superconductor leads that are made from the same material. $H_{\rm D}$ in Eq. (1) describes the double QDs without interdot coupling, and is expressed as

$$H_{\rm D} = \sum_{\sigma, i=1,2} \epsilon_i d^{\dagger}_{i\sigma} d_{i\sigma}, \tag{5}$$

where $d^{\dagger}_{i\sigma}$ and $d_{i\sigma}$ are, respectively, the creation and the annihilation operators of the electron with energy ϵ_i and spin σ in dot i. $H_{\rm T}$ is the hopping term between leads and the QDs and given as

$$\begin{aligned} H_{\rm T} = & \sum_{k_{\rm L}\sigma} (L_1 \, {\rm e}^{-{\rm i}\,\phi/4} a^{\dagger}_{k_{\rm L}\sigma} d_{1\sigma} \\ & + L_2 \, {\rm e}^{{\rm i}\,\phi/4} a^{\dagger}_{k_{\rm L}\sigma} d_{2\sigma} + {\rm H.c.}) \\ & + \sum_{k_{\rm R}\sigma} (R_1 \, {\rm e}^{{\rm i}\,\phi/4} \, {\rm e}^{{\rm i}\,\sigma\varphi_{\rm R1}} a^{\dagger}_{k_{\rm R}\sigma} d_{1\sigma} \\ & + R_2 \, {\rm e}^{-{\rm i}\,\phi/4} \, {\rm e}^{{\rm i}\,\sigma\varphi_{\rm R2}} a^{\dagger}_{k_{\rm R}\sigma} d_{2\sigma} + {\rm H.c.}) \\ & + \sum_{p_{\rm u}\sigma} (M_1 b^{\dagger}_{p_{\rm u}\sigma} d_{1\sigma} + {\rm H.c.}) \\ & + \sum_{p_{\rm d}\sigma} (M_2 b^{\dagger}_{p_{\rm d}\sigma} d_{2\sigma} + {\rm H.c.}), \end{aligned} \tag{6}$$

where L_i (R_i) is the coupling strength between the i-th QD and the left (right) FM lead, M_1 (M_2) is the coupling strength between the QD1 (QD2) and the up (down) superconductor lead, and ϕ denotes the phase factor induced by the magnetic flux threading the AB ring. The RSOI induces the phase factor $-\varphi_{{\rm R}i} = k_{{\rm R}i} L_i = \alpha_{{\rm R}i} m^*_i L_i/\hbar^2$ ($i = 1, 2$), which may reach $\pi/2$ or larger experimentally. The spin polarization occurs when the magnetic flux threading the AB ring and the RSOI are both nonzero.[40,41] $\sigma = 1$ and -1

denote the spin-up and the spin-down of electrons, respectively.

Using the standard nonequilibrium Green's function techniques, the spin-up and the spin-down electron currents flowing through the right lead can be expressed in terms of the dot Green's functions as

$$I_{\mathrm{R}\sigma} = \frac{\mathrm{i}\,e}{\hbar}\int\frac{\mathrm{d}\varepsilon}{2\pi}\{\boldsymbol{\Gamma}^{\mathrm{R}}_{\sigma}(\varepsilon)[(\boldsymbol{G}^{\mathrm{r}}_{\sigma}(\varepsilon)-\boldsymbol{G}^{\mathrm{a}}_{\sigma}(\varepsilon))f_{\mathrm{R}}(\varepsilon+eV_2) + \boldsymbol{G}^{<}_{\sigma}(\varepsilon)]\}_{11+33}, \quad (\sigma=\uparrow,\downarrow), \tag{7}$$

where $\boldsymbol{G}^{\mathrm{r}}_{\sigma}(\varepsilon)$ and $\boldsymbol{G}^{<}_{\sigma}(\varepsilon)$ are the Fourier transformations of the retarded Green's functions $\boldsymbol{G}^{\mathrm{r}}_{\sigma}(t,t') = -\mathrm{i}\,\theta(t-t')\langle\{\boldsymbol{\Psi}(t),\boldsymbol{\Psi}^{\dagger}(t')\}\rangle$ and the lesser Green's functions $\boldsymbol{G}^{<}_{\sigma}(t,t') = \mathrm{i}\langle\boldsymbol{\Psi}^{\dagger}(t')\boldsymbol{\Psi}(t)\rangle$, respectively. Because of the existence of the superconductor lead, it is convenient to introduce the 4×4 Nambu representation with row vectors $\boldsymbol{\Psi}^{\dagger} = (d^{\dagger}_{1\uparrow}, d_{1\downarrow}, d^{\dagger}_{2\uparrow}, d_{2\downarrow})$, $(d^{\dagger}_{1\downarrow}, d_{1\uparrow}, d^{\dagger}_{2\downarrow}, d_{2\uparrow})$ for spin-up and spin-down electrons, respectively. In the Nambu representation, the retarded Green's function of the double QDs coupled with the leads can be calculated from the Dyson equation directly,

$$\boldsymbol{G}^{\mathrm{r}}_{\sigma}(\varepsilon) = \frac{1}{[\boldsymbol{g}^{\mathrm{r}}(\varepsilon)]^{-1} - \boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{L}\sigma}(\varepsilon) - \boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{R}\sigma}(\varepsilon) - \boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{u}}(\varepsilon) - \boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{d}}(\varepsilon)}, \tag{8}$$

where $\boldsymbol{g}^{\mathrm{r}}(\varepsilon)$ is the retarded Green's function of the isolated double QDs and can be expressed as

$$[\boldsymbol{g}^{\mathrm{r}}(\varepsilon)]^{-1} = \begin{pmatrix} \varepsilon-\epsilon_1+\mathrm{i}0^{+} & 0 & 0 & 0 \\ 0 & \varepsilon+\epsilon_1+\mathrm{i}0^{+} & 0 & 0 \\ 0 & 0 & \varepsilon-\epsilon_2+\mathrm{i}0^{+} & 0 \\ 0 & 0 & 0 & \varepsilon+\epsilon_2+\mathrm{i}0^{+} \end{pmatrix}; \tag{9}$$

$\boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{L}\sigma}(\varepsilon)$, $\boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{R}\sigma}(\varepsilon)$, $\boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{u}}(\varepsilon)$, and $\boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{d}}(\varepsilon)$ are the retarded self energies under the wide-bandwidth approximation and expressed as

$$\boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{L}\sigma}(\varepsilon) = -\frac{\mathrm{i}}{2}\begin{pmatrix} \Gamma^{\mathrm{L}}_{1\sigma} & 0 & \sqrt{\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{L}}_{2\sigma}}\,\mathrm{e}^{\mathrm{i}\,\phi/2} & 0 \\ 0 & \Gamma^{\mathrm{L}}_{1\sigma} & 0 & \sqrt{\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{L}}_{2\sigma}}\,\mathrm{e}^{-\mathrm{i}\,\phi/2} \\ \sqrt{\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{L}}_{2\sigma}}\,\mathrm{e}^{-\mathrm{i}\,\phi/2} & 0 & \Gamma^{\mathrm{L}}_{2\sigma} & 0 \\ 0 & \sqrt{\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{L}}_{2\sigma}}\,\mathrm{e}^{\mathrm{i}\,\phi/2} & 0 & \Gamma^{\mathrm{L}}_{2\sigma} \end{pmatrix}, \tag{10}$$

$$\boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{R}\sigma}(\varepsilon) = -\frac{\mathrm{i}}{2}\boldsymbol{\Gamma}^{\mathrm{R}}_{\sigma} = -\frac{\mathrm{i}}{2}\begin{pmatrix} \Gamma^{\mathrm{R}}_{1\sigma} & 0 & R_{13} & 0 \\ 0 & \Gamma^{\mathrm{R}}_{1\sigma} & 0 & R_{24} \\ R_{31} & 0 & \Gamma^{\mathrm{R}}_{2\sigma} & 0 \\ 0 & R_{42} & 0 & \Gamma^{\mathrm{R}}_{2\sigma} \end{pmatrix}, \tag{11}$$

$$\boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{u}}(\varepsilon) = -\frac{\mathrm{i}}{2}\rho_{\mathrm{u}}(\varepsilon)\begin{pmatrix} \Gamma^{\mathrm{u}} & -\Delta\Gamma^{\mathrm{u}}/\varepsilon & 0 & 0 \\ -\Delta\Gamma^{\mathrm{u}}/\varepsilon & \Gamma^{\mathrm{u}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{12}$$

$$\boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{d}}(\varepsilon) = -\frac{\mathrm{i}}{2}\rho_{\mathrm{d}}(\varepsilon)\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \Gamma^{\mathrm{d}} & -\Delta\Gamma^{\mathrm{d}}/\varepsilon \\ 0 & 0 & -\Delta\Gamma^{\mathrm{d}}/\varepsilon & \Gamma^{\mathrm{d}} \end{pmatrix}, \tag{13}$$

with $R_{31} = R^{*}_{13} = R_{24} = R^{*}_{42} = \mathrm{e}^{\mathrm{i}(\phi+2\sigma(\varphi_{\mathrm{R}1}-\varphi_{\mathrm{R}2}))/2}$, $\Gamma^{\mathrm{L}}_{i\sigma}$ ($\Gamma^{\mathrm{R}}_{i\sigma}$) is the linewidth function describing the coupling between the dot i and the left (right) FM lead, Γ^{u} (Γ^{d}) describes the coupling between the QD1 (QD2) and the up (down) superconductor lead, $\rho_{\mathrm{u}}(\varepsilon)$ and $\rho_{\mathrm{d}}(\varepsilon)$ in the retarded self-energy of the superconductor leads are defined as

$$\rho_{\mathrm{u}}(\varepsilon) = \rho_{\mathrm{d}}(\varepsilon) = \begin{cases} \dfrac{|\varepsilon|}{\sqrt{\varepsilon^2-\Delta^2}}, & |\varepsilon|>\Delta, \\ \dfrac{\varepsilon}{\mathrm{i}\sqrt{\Delta^2-\varepsilon^2}}, & |\varepsilon|<\Delta. \end{cases} \tag{14}$$

For the lesser Green's function, we use the Keldysh equation $\boldsymbol{G}^{<}_{\sigma} = \boldsymbol{G}^{\mathrm{r}}_{\sigma}\boldsymbol{\Sigma}^{<}_{\sigma}\boldsymbol{G}^{\mathrm{a}}_{\sigma}$, where the lesser self-energy $\boldsymbol{\Sigma}^{<}_{\sigma} = \boldsymbol{\Sigma}^{<}_{\mathrm{L}\sigma} + \boldsymbol{\Sigma}^{<}_{\mathrm{R}\sigma} + \boldsymbol{\Sigma}^{<}_{\mathrm{u}} + \boldsymbol{\Sigma}^{<}_{\mathrm{d}}$ with

$$\boldsymbol{\Sigma}^{<}_{\mathrm{L}\sigma}(\varepsilon) = -2\boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{L}\sigma}(\varepsilon)\boldsymbol{F}_{\mathrm{L}}(V_1), \tag{15}$$

$$\boldsymbol{\Sigma}^{<}_{\mathrm{R}\sigma}(\varepsilon) = -2\boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{R}\sigma}(\varepsilon)\boldsymbol{F}_{\mathrm{R}}(V_2), \tag{16}$$

$$\boldsymbol{\Sigma}^{<}_{\mathrm{u}}(\varepsilon) = -2f_{\mathrm{u}}(\varepsilon)\theta(|\varepsilon|-\Delta)\boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{u}}(\varepsilon), \tag{17}$$

$$\boldsymbol{\Sigma}^{<}_{\mathrm{d}}(\varepsilon) = -2f_{\mathrm{d}}(\varepsilon)\theta(|\varepsilon|-\Delta)\boldsymbol{\Sigma}^{\mathrm{r}}_{\mathrm{d}}(\varepsilon), \tag{18}$$

and

$$\boldsymbol{F}_\alpha(V_i)=\begin{pmatrix} f_\alpha(\varepsilon+eV_i) & 0 & 0 & 0\\ 0 & f_\alpha(\varepsilon-eV_i) & 0 & 0\\ 0 & 0 & f_\alpha(\varepsilon+eV_i) & 0\\ 0 & 0 & 0 & f_\alpha(\varepsilon-eV_i)\end{pmatrix}\ (\alpha=\mathrm{L},\mathrm{R};\ i=1,2). \tag{19}$$

Then the spin-dependent currents flowing through the right lead can be obtained by substituting the retarded Green's function $\boldsymbol{G}^{\mathrm{r}}_\sigma(\varepsilon)$ and the lesser Green's function $\boldsymbol{G}^{<}_\sigma(\varepsilon)$ into Eq. (7) as

$$I_{\mathrm{R}\sigma}=I_{\mathrm{NA}\sigma}+I_{\mathrm{CA}\sigma}+I_{\mathrm{RL}\sigma}+I_{\mathrm{Q}\sigma} \tag{20}$$

with

$$\begin{aligned} I_{\mathrm{NA}\sigma} &= \frac{e}{\hbar}\int\frac{\mathrm{d}\varepsilon}{2\pi}[f_{\mathrm{R}}(\varepsilon+eV_2)\\ &\quad -f_{\mathrm{R}}(\varepsilon-eV_2)]T_{\mathrm{NA}\sigma}, \qquad (21)\\ I_{\mathrm{CA}\sigma} &= \frac{e}{\hbar}\int\frac{\mathrm{d}\varepsilon}{2\pi}[f_{\mathrm{R}}(\varepsilon+eV_2)\\ &\quad -f_{\mathrm{L}}(\varepsilon-eV_1)]T_{\mathrm{CA}\sigma}, \qquad (22)\\ I_{\mathrm{RL}\sigma} &= \frac{e}{\hbar}\int\frac{\mathrm{d}\varepsilon}{2\pi}[f_{\mathrm{R}}(\varepsilon+eV_2)\\ &\quad -f_{\mathrm{L}}(\varepsilon+eV_1)]T_{\mathrm{RL}\sigma}, \qquad (23)\\ I_{\mathrm{Q}\sigma} &= \frac{e}{\hbar}\int\frac{\mathrm{d}\varepsilon}{2\pi}\{[f_{\mathrm{R}}(\varepsilon+eV_2)-f_{\mathrm{u}}(\varepsilon)]T_{\mathrm{Q}1\sigma}\\ &\quad +[f_{\mathrm{R}}(\varepsilon+eV_2)-f_{\mathrm{d}}(\varepsilon)]T_{\mathrm{Q}2\sigma}\}. \qquad (24) \end{aligned}$$

Now we focus only on the case of $|eV_1|<\Delta$ and keep zero chemical potentials in the right FM lead and the two superconductor leads in the zero-temperature limit, thus the transmission currents through the superconductor leads $I_{\mathrm{Q}1\sigma}$, $I_{\mathrm{Q}2\sigma}$, and the current $I_{\mathrm{NA}\sigma}$ induced by the normal Andreev reflection in the right lead vanish. Only the currents $I_{\mathrm{CA}\sigma}$ induced by the crossed Andreev reflection and normal quasiparticle transmission current $I_{\mathrm{RL}\sigma}$ flow through the right lead, and the corresponding transmission probabilities $T_{\mathrm{CA}\sigma}$ and $T_{\mathrm{RL}\sigma}$ can be obtained, respectively, as

$$\begin{aligned} T_{\mathrm{CA}\sigma} &= \Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{R}}_{1\sigma}|G^{\mathrm{r}}_{\sigma12}(\varepsilon)|^2+\Gamma^{\mathrm{L}}_{2\sigma}\Gamma^{\mathrm{R}}_{2\sigma}|G^{\mathrm{r}}_{\sigma34}(\varepsilon)|^2+\Gamma^{\mathrm{L}}_{2\sigma}\Gamma^{\mathrm{R}}_{1\sigma}|G^{\mathrm{r}}_{\sigma14}(\varepsilon)|^2+\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{R}}_{2\sigma}|G^{\mathrm{r}}_{\sigma32}(\varepsilon)|^2\\ &\quad +2\Gamma^{\mathrm{R}}_{2\sigma}\sqrt{\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{L}}_{2\sigma}}\mathrm{Re}[\,\mathrm{e}^{\mathrm{i}\,\phi/2}G^{\mathrm{r}*}_{\sigma32}(\varepsilon)G^{\mathrm{r}}_{\sigma34}(\varepsilon)]+2\Gamma^{\mathrm{R}}_{1\sigma}\sqrt{\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{L}}_{2\sigma}}\mathrm{Re}[\,\mathrm{e}^{\mathrm{i}\,\phi/2}G^{\mathrm{r}*}_{\sigma12}(\varepsilon)G^{\mathrm{r}}_{\sigma14}(\varepsilon)]\\ &\quad +2\Gamma^{\mathrm{L}}_{2\sigma}\sqrt{\Gamma^{\mathrm{R}}_{1\sigma}\Gamma^{\mathrm{R}}_{2\sigma}}\mathrm{Re}[\,\mathrm{e}^{\mathrm{i}\,\phi/2+\mathrm{i}\,\sigma(\varphi_{\mathrm{R}1}-\varphi_{\mathrm{R}2})}G^{\mathrm{r}*}_{\sigma34}(\varepsilon)G^{\mathrm{r}}_{\sigma14}(\varepsilon)]\\ &\quad +2\Gamma^{\mathrm{L}}_{1\sigma}\sqrt{\Gamma^{\mathrm{R}}_{1\sigma}\Gamma^{\mathrm{R}}_{2\sigma}}\mathrm{Re}[\,\mathrm{e}^{\mathrm{i}\,\phi/2+\mathrm{i}\,\sigma(\varphi_{\mathrm{R}1}-\varphi_{\mathrm{R}2})}G^{\mathrm{r}*}_{\sigma32}(\varepsilon)G^{\mathrm{r}}_{\sigma12}(\varepsilon)]\\ &\quad +2\sqrt{\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{L}}_{2\sigma}}\sqrt{\Gamma^{\mathrm{R}}_{1\sigma}\Gamma^{\mathrm{R}}_{2\sigma}}\mathrm{Re}[\,\mathrm{e}^{\mathrm{i}\,\sigma(\varphi_{\mathrm{R}1}-\varphi_{\mathrm{R}2})}G^{\mathrm{r}*}_{\sigma34}(\varepsilon)G^{\mathrm{r}}_{\sigma12}(\varepsilon)]\\ &\quad +2\sqrt{\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{L}}_{2\sigma}}\sqrt{\Gamma^{\mathrm{R}}_{1\sigma}\Gamma^{\mathrm{R}}_{2\sigma}}\mathrm{Re}[\,\mathrm{e}^{\mathrm{i}\,\phi+\mathrm{i}\,\sigma(\varphi_{\mathrm{R}1}-\varphi_{\mathrm{R}2})}G^{\mathrm{r}*}_{\sigma32}(\varepsilon)G^{\mathrm{r}}_{\sigma14}(\varepsilon)], \qquad (25)\\ T_{\mathrm{RL}\sigma} &= \Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{R}}_{1\sigma}|G^{\mathrm{r}}_{\sigma11}(\varepsilon)|^2+\Gamma^{\mathrm{L}}_{2\sigma}\Gamma^{\mathrm{R}}_{1\sigma}|G^{\mathrm{r}}_{\sigma13}(\varepsilon)|^2+\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{R}}_{2\sigma}|G^{\mathrm{r}}_{\sigma31}(\varepsilon)|^2+\Gamma^{\mathrm{L}}_{2\sigma}\Gamma^{\mathrm{R}}_{2\sigma}|G^{\mathrm{r}}_{\sigma33}(\varepsilon)|^2\\ &\quad +2\Gamma^{\mathrm{R}}_{1\sigma}\sqrt{\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{L}}_{2\sigma}}\mathrm{Re}[\,\mathrm{e}^{\mathrm{i}\,\phi/2}G^{\mathrm{r}*}_{\sigma13}(\varepsilon)G^{\mathrm{r}}_{\sigma11}(\varepsilon)]+2\Gamma^{\mathrm{R}}_{2\sigma}\sqrt{\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{L}}_{2\sigma}}\mathrm{Re}[\,\mathrm{e}^{\mathrm{i}\,\phi/2}G^{\mathrm{r}*}_{\sigma33}(\varepsilon)G^{\mathrm{r}}_{\sigma31}(\varepsilon)]\\ &\quad +2\Gamma^{\mathrm{L}}_{1\sigma}\sqrt{\Gamma^{\mathrm{R}}_{1\sigma}\Gamma^{\mathrm{R}}_{2\sigma}}\mathrm{Re}[\,\mathrm{e}^{\mathrm{i}\,\phi/2+\mathrm{i}\,\sigma(\varphi_{\mathrm{R}1}-\varphi_{\mathrm{R}2})}G^{\mathrm{r}*}_{\sigma31}(\varepsilon)G^{\mathrm{r}}_{\sigma11}(\varepsilon)]\\ &\quad +2\sqrt{\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{L}}_{2\sigma}}\sqrt{\Gamma^{\mathrm{R}}_{1\sigma}\Gamma^{\mathrm{R}}_{2\sigma}}\mathrm{Re}[\,\mathrm{e}^{\mathrm{i}\,\sigma(\varphi_{\mathrm{R}1}-\varphi_{\mathrm{R}2})}G^{\mathrm{r}*}_{\sigma31}(\varepsilon)G^{\mathrm{r}}_{\sigma13}(\varepsilon)]\\ &\quad +2\Gamma^{\mathrm{L}}_{2\sigma}\sqrt{\Gamma^{\mathrm{R}}_{1\sigma}\Gamma^{\mathrm{R}}_{2\sigma}}\mathrm{Re}[\,\mathrm{e}^{\mathrm{i}\,\phi/2+\mathrm{i}\,\sigma(\varphi_{\mathrm{R}1}-\varphi_{\mathrm{R}2})}G^{\mathrm{r}*}_{\sigma33}(\varepsilon)G^{\mathrm{r}}_{\sigma13}(\varepsilon)]\\ &\quad +2\sqrt{\Gamma^{\mathrm{L}}_{1\sigma}\Gamma^{\mathrm{L}}_{2\sigma}}\sqrt{\Gamma^{\mathrm{R}}_{1\sigma}\Gamma^{\mathrm{R}}_{2\sigma}}\mathrm{Re}[\,\mathrm{e}^{\mathrm{i}\,\phi+\mathrm{i}\,\sigma(\varphi_{\mathrm{R}1}-\varphi_{\mathrm{R}2})}G^{\mathrm{r}*}_{\sigma33}(\varepsilon)G^{\mathrm{r}}_{\sigma11}(\varepsilon)]. \qquad (26) \end{aligned}$$

3. Numerical results

Since we consider only a bias in the left FM lead and keep zero chemical potentials in the right FM lead and the two superconductor leads, there exist three kinds of conductive mechanisms in this device, i.e. normal quasiparticle transmission, normal Andreev reflection and crossed Andreev reflection, and their corresponding conductive routes are shown in Fig. 1. Spin-up and spin-down electrons can conduct

from the left FM lead to the right FM lead through the two QDs, which is the normal quasiparticle transmission. The spin-up and the spin-down electrons can also be converted into a Cooper pair tunneling into the superconductor lead by reflecting back a hole with opposite spin and velocity. The reflected holes can go back to the left FM lead or transmit into the right FM lead, which are called the normal Andreev reflection and the crossed Andreev reflection, respectively. Thus, only the currents $I_{\mathrm{CA}\sigma}$ and $I_{\mathrm{RL}\sigma}$ coming from the crossed Andreev reflection and the normal quasiparticle transmission are flowing through the right lead, and four kinds of quasiparticles transmit into the right FM lead, i.e. spin-up electron and hole, spin-down electron and hole. The electronic current flowing through the right lead is positive if it is formed by holes, and negative if it is formed by electrons.

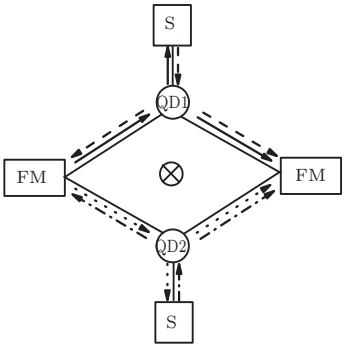


Fig. 1. Schematic diagram of the pure spin current injector of a four-terminal device consisting of two parallel quantum dots coupled, respectively, to two side superconductor leads and two common ferromagnetic leads. The added lines with arrowheads are the conductive routes of the electrons and holes. The solid, dashed, dotted and dash-dotted lines represent the routes of the spin-up electrons, spin-down holes, spin-down electrons and spin-up holes, respectively. For concision reasons, only half of the routes are given since the conductive routes are actually up-and-down symmetric.

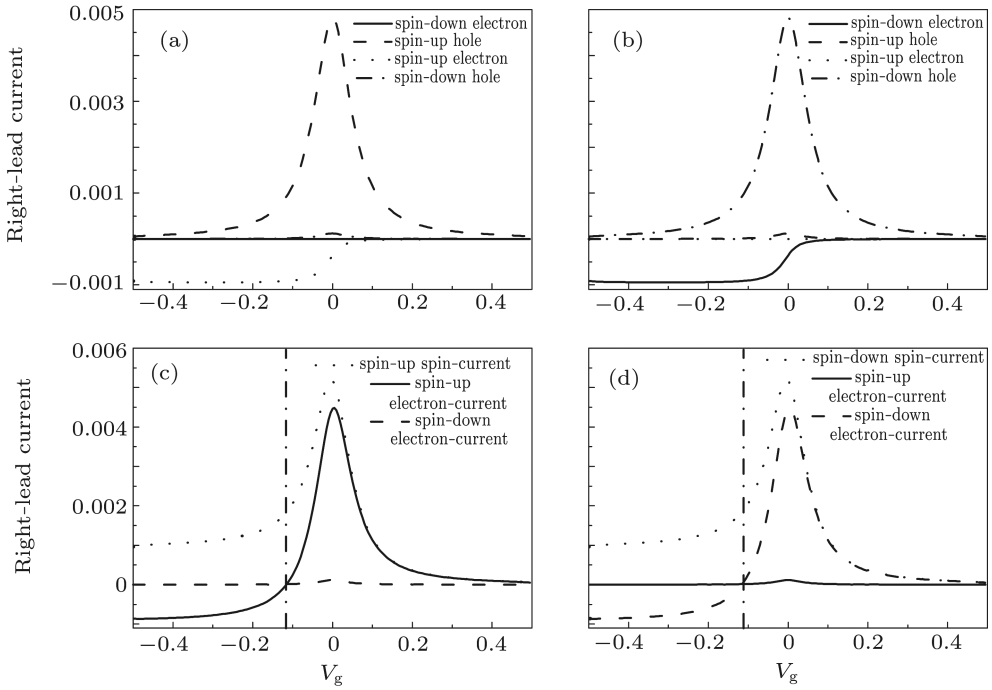


Fig. 2. The spin-polarization currents flowing through the right FM lead versus gate voltage V_g. The solid, dashed, dotted and dash-dotted lines in panels (a) and (b) represent the electronic currents formed by spin-down electrons, spin-up holes, spin-up electrons and spin-down holes, respectively. The solid and the dashed lines in panels (c) and (d) represent the electronic current formed by spin-up electrons and holes, and the electronic current formed by spin-down electrons and holes, respectively. The dotted line in panel (c) ((d)) represents the spin-up (spin-down) spin current formed by spin-up (spin-down) electrons and holes. The dash-dotted line in panel (c) ((d)) denotes the position where the pure spin-up (spin-down) current without net charge current occurs.

We now numerically analyse the spin-dependent currents flowing through the right FM lead (right-lead current). The ferromagnetisms of the leads are considered via defining the spin-dependent parameter $\Gamma^{\mathrm{L}}_{i\sigma} = \Gamma^{\mathrm{L}}_{i}[1 + (-1)^{\delta_{\sigma\downarrow}} p_{\mathrm{L}}]$ for the left FM lead and $\Gamma^{\mathrm{R}}_{i\sigma} = \Gamma^{\mathrm{R}}_{i}[1+\lambda(-1)^{\delta_{\sigma\downarrow}} p_{\mathrm{R}}]$ for the right FM lead ($\lambda = 1$ for the P alignment and $\lambda = -1$ for the AP alignment) respectively,[42] where p_{L} and p_{R} are the parameters

describing the degrees of spin polarization of the left and the right leads, respectively. Throughout the paper, we set $\Delta = 1$ as the energy unit and consider only the symmetric and weak coupling between dots and leads by setting $\Gamma_1^{\rm L} = \Gamma_2^{\rm L} = \Gamma_1^{\rm R} = \Gamma_2^{\rm R} = 0.02$, $\Gamma^{\rm u} = \Gamma^{\rm d} = 0.08$. In the following calculation, we make the chemical potential in the left lead higher than every level in the two QDs ($\epsilon_1 = \epsilon_2 = 0$). The spin-dependent right-lead currents versus gate voltage $V_{\rm g}$ added at two QDs are shown in Fig. 2. One can find from Fig. 2(a) (Fig. 2(b)) that for $p_{\rm L} = p_{\rm R} = 0.95$ (-0.95), $\phi = \pi/2$ ($3\pi/2$) and the RSOI phase factor $\varphi_{\rm R} = \varphi_{\rm R1} - \varphi_{\rm R2} = \pi/2$, the electric currents formed by spin-down (spin-up) electrons and holes almost vanish, i.e. the spin-down (spin-up) electrons and holes cannot be conducted into the right lead, while only the electric currents formed by spin-up (spin-down) electrons and holes are existent. Since only spin-up (spin-down) electrons and holes can conduct into the right lead, we can collect the spin-up (spin-down) spin current (the current formed by the total number of right-going spin-up (spin-down) electrons and holes) in the right lead; see Fig. 2(c) (Fig. 2(d)). Moreover, when the number of spin-up (spin-down) electrons is equal to that of the holes, the pure spin-up (spin-down) current without net charge current can be obtained in the right lead when the gate voltage is at a certain value, for $p_{\rm L} = p_{\rm R} = 0.95$ (-0.95), $\phi = \pi/2$ ($3\pi/2$) and the RSOI phase factor $\varphi_{\rm R} = \varphi_{\rm R1} - \varphi_{\rm R2} = \pi/2$; see Fig. 2(c) (Fig. 2(d)). The positions of the dash-dotted vertical lines in Figs. 2(c) and (d) indicate the gate voltage at which the total electric currents vanish and the pure spin current appears. This device may be utilized as a pure spin-current injector.

4. Summary

In summary, we theoretically put forward a four-terminal device of pure spin-current injector, in which three kinds of conductive mechanisms exist: normal quasiparticle transmission, normal Andreev reflection and crossed Andreev reflection. When a bias voltage is applied between the left and the right FM leads, and the chemical potentials in the right FM lead and in the two superconductor leads are kept at zero, the spin-up electrons and holes and the spin-down electrons and holes can be collected in the right FM lead. For the appropriate spin polarizations of FM leads, RSOI and AB flux, one can obtain the pure spin-up (or pure spin-down) current without net charge current in the right lead by adjusting the gate voltage. The pure spin current, unlike the usual pure spin current which is due to the spin-up electron moving along the opposite direction to the spin-down electron, is formed by electrons and holes with the same spin-polarization moving along the same direction. This device may be experimentally utilized as a pure spin-current injector.

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