Influences of initial velocity, diameter and Reynolds number on a circular turbulent air/air jet*

Mi Jian-Chun(米建春)[a)b)†] and Du Cheng(杜 诚)[a)b)]

[a)] *State Key Laboratory of Turbulence and Complex Systems, Peking University, Beijing* 100871, *China*

[b)] *Department of Energy and Resources Engineering, College of Engineering, Peking University, Beijing* 100871, *China*

(Received 20 April 2011; revised manuscript received 15 July 2011)

This paper assesses the suitability of the inflow Reynolds number defined by $Re_o \equiv U_o D/\nu$ (here U_o and D are respectively the initial jet velocity and diameter while ν is kinematic viscosity) for a round air/air jet. Specifically an experimental investigation is performed for the influences of U_o, D and Re_o on the mean-velocity decay and spread coefficients (K_u, K_r) in the far field of a circular air jet into air from a smoothly contracting nozzle. Present measurements agree well with those previously obtained under similar inflow conditions. The relations $K_u \propto U_o$ and $K_r \propto 1/U_o$ for $U_o < 5$ m/s appear to work, while each coefficient approaches asymptotically to a constant for $U_o > 6$ m/s, regardless of the magnitudes of Re_o and D. It is revealed that Re_o may not be an appropriate dimensionless parameter to characterize the entire flow of a free air/air jet. This paper is the first paper that has challenged the suitability of Re_o for turbulent free jets.

Keywords: turbulent jet, momentum conservation, Reynolds number

PACS: 47.27.–i, 47.27.wg **DOI:** 10.1088/1674-1056/20/12/124701

1. Introduction

Although the influence of Reynolds number on the development of a jet has been widely investigated,[1−21] this topic remains to be highly interesting and also challenging to the turbulence research community. This is mainly because the boundary-layer properties at the nozzle exit are known to depend significantly on the Reynolds number Re_o. It is difficult to separate the direct effect of Re_o on the development of a jet from those translated from its influence on the exit boundary layer. Zaman[1] found that the initial turbulence intensity and boundary-layer growth of a circular jet is a function of Re_o. His measurements showed that the boundary layer is initially laminar at $Re_o < 10^5$ but becomes nominally turbulent when $Re_o > 2.5 \times 10^5$. However, these critical values were determined for a particular nozzle of only one diameter and may not apply to various nozzles of different sizes or geometric profiles. Hence, Re_o exerts not only a direct influence on the turbulent mixing field, through the relative significance of viscous and inertial forces, but also an indirect effect via modifying the initial conditions (e.g., the boundary layer thickness).

There are several previous investigations of the effect of Re_o on a circular jet. Dimotakis *et al.*,[2] assessed the scalar-mixing fields of circular jets at $Re_o = 2.5 \times 10^3$ and 10^4 and found a transition in mixing behaviour for Re_o on the order of 10^4. Miller and Dimotakis[3] confirmed that the root-mean-squared (rms) scalar fluctuations decrease with the increase in Re_o and converge to an asymptotic state for $Re_o \geq 2.0 \times 10^4$ and a homogeneous, chaotic and well-mixed state. Likewise, Gilbrech[4] found that the asymptotic state of the mixing field occurs at $Re_o \approx 2.0 \times 10^4$ or beyond. Koochesfahani and Dimotakis[5] used scalar images of laser-induced fluorescence (LIF) at $Re_o = 1.75 \times 10^3$ and 2.30×10^4 and showed qualitatively a better-mixed state at the larger Re_o. Similarly, Michalke[6] found that the growth of instability waves in jet shear layers can be reduced dramatically when the Reynolds number is increased. Oosthuizen[7] measured the flow of a circular jet at a low Reynolds numbers and found that both the mean and turbulence fields depend strongly on Re_o for $Re_o < 10^4$. Bogey and Bailly[8] used large eddy simulations (LES) of a circular jet ($1.70 \times 10^3 \leq Re_o \leq 4.0 \times 10^5$) and showed that as Re_o is decreased, a jet develops more slowly within the potential core, but more rapidly fur-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10921202 and 11072005).

†Corresponding author. E-mail: jcmi@coe.pku.edu.cn

© 2011 Chinese Physical Society and IOP Publishing Ltd

ther downstream. Their jets achieved self-similarity closer to the exit plane at low values of Re_o, a finding which agrees well with Pitts[9] for turbulent circular jets measured for $Re_o = 3950$–11880.

In the self-similar region of the mean flow of a circular jet, the centreline velocity U_c and half-radius R_u, at which the local mean velocity is $U_c/2$, should vary with x respectively in the following fashions (e.g., Ref. [12]):

$$U_c/U_o = K_u D/(x - x_u), \tag{1}$$

$$R_u/D = K_r(x - x_r)/D. \tag{2}$$

Here x_u and x_r are the corresponding virtual origin locations for U_c and R_u (conceptually these should be identical, but this is rarely the case). Many previous investigations, e.g., Refs. [10]–[12], have validated these self-similar relations. Our measurements, to be reported later, suggest that they are valid in the present jets for $x/D \geq 10 \sim 20$.

Panchapakesan and Lumley[10] and Hussein *et al.*[11] indicated that at sufficiently high Reynolds numbers, the decay and spread rates (K_u, K_r) should be approximately independent of Re_o, but they did not suggest a critical Reynolds number, $Re_{\rm cr}$, above which this independence occurs. Based on the previous work, nevertheless, Pope[12] concluded in his text that "the mean velocity profile and the spreading rate are independent of Re_o" "in the self-similar region ($x/D > 30$) of a high-Reynolds number turbulent jet ($Re_o > 10^4$)". Some evidence might appear in Fig. 1 (but only when each dataset from a single nozzle is viewed separately), which shows previous results for K_u obtained, typically at $x/D > 20$, by various independent studies.

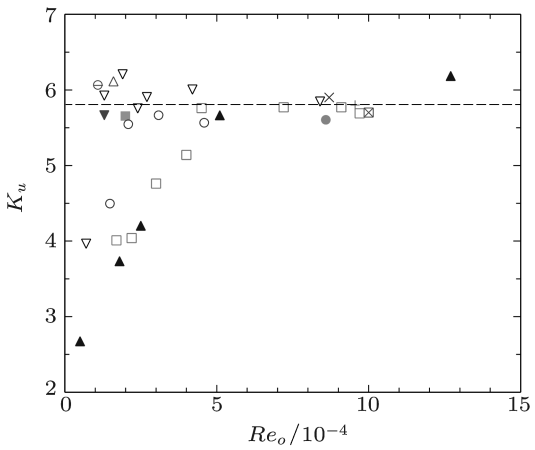


Fig. 1. The effect of the exit Reynolds number on the velocity decay coefficient K_u. Symbols: ∇, $D = 40$ mm;[13] ○, 80 mm;[13] □ and ▲, 150 mm;[13] others for eight data points.[10−12]

Despite a seemingly general consensus for the critical values claimed by Pope[12] and Dimoatkis *et al.*[2] among others, the measurements of Malmström *et al.*[13] and Mi *et al.*[14] are somewhat at odds on this conclusion, (see Fig. 1). Malmström *et al.* systematically measured the centerline velocity in low-speed (not necessarily low-Re_o) air jets issuing from a range of nozzle diameters. It can be deduced from Fig. 1 (or Fig. 6 in Ref. [13]) that as D increases, the critical Reynolds number $Re_{\rm cr}$ increases and does not appear to exhibit an asymptotic convergence based on the D variation. They further demonstrated that the mean velocity decay rate scales better with U_o than with Re_o. However, they admitted that the reason for the result "is not clear", which provides the motivation for the present study. Our objective for this paper is twofold: (i) to check by new experiments whether the observations of Malmström *et al.*[13] are valid; and (ii) to provide a fresh perspective on the influence of the Reynolds number on the far-field development of a circular jet.

2. Experimental details

The apparatus in which the present experiments were performed consisted of a fan, a settling chamber and a smooth contraction nozzle discharging into a large laboratory room. The settling chamber was 1 m long and 0.4 m in diameter. Fine mesh screens were placed inside the chamber to produce a uniform velocity profile and reduce the turbulence level. To fulfil the objectives of the study indicated above, we employed three nozzles of different exit diameters, i.e. $D = 10$, 20, and 50 mm. Note that the nozzle exit of $D = 20$ mm is flush with a flat plate of 200 mm in diameter whereas the other two are not. The exit turbulence level was less than 1.0% for the jets issuing from the three nozzles.

The streamwise velocity U was obtained with a single hot-wire (5-μm tungsten; ~ 1 mm long) probe with an overheat ratio of 1.6. Calibrations of the probe were carefully performed at the nozzle exit plane or using a special nozzle for low speed cases, before and after each set of measurements. The signals from the anemometer circuit were offset, amplified and then digitized on a PC using a 12-bit A/D converter. The sampling frequency was set at 9.2 kHz and each record duration was in the range 40 s–50 s. The measurement uncertainty of the mean velocity was estimated to be within 1%. The crosswise measurements at a

number of x locations determined the centreline mean velocities (U_c), used to obtain the decay coefficient K_u and the half-radius R_u, a radial location at which the mean velocity $U = U_c/2$, which was used to obtain the spread rate K_r (Eq. (2) in Section 1).

3. Results

Figure 2 presents the Re_o dependence of the decay coefficient K_u for the present data (D = 10 mm and 50 mm) along with those of Malmström *et al.*[13] (D = 40, 80, and 150 mm) and of Nottage[15] (D = 150 mm). It is demonstrated that, as Re_o is increased from below 3000, K_u decreases until Re_o reaches the critical value ($Re_{\rm cr}$). However, it is also clearly evident that $Re_{\rm cr}$ increases as D is increased. This is shown in Fig. 3, which plots the values of $Re_{\rm cr}$, estimated from Fig. 2 (dashed vertical lines), as a function of D/D_r (taking the reference diameter D_r = 1 m). Although some scatter is apparent, as is expected in combined data from three different experiments, the six points conform reasonably well to the line of best fit, $Re_{\rm cr} \approx 3.85 \times 10^5 (D/D_r)$. This suggests that $Re_{\rm cr}$ occurs at a critical value of the exit velocity, $U_{\rm cr} \approx 3.85 \times 10^5 \nu/D_r \approx 5.8$ (m/s), independent of D. This value of $U_{\rm cr}$ agrees well with $U_{\rm cr} \approx 6$ m/s suggested earlier in Ref. [13]. (Note that the air viscosity was taken to be $\nu \approx 1.5 \times 10^{-5}$ m^2/s in all the cases, given that air at room temperature was used for all the six jets).

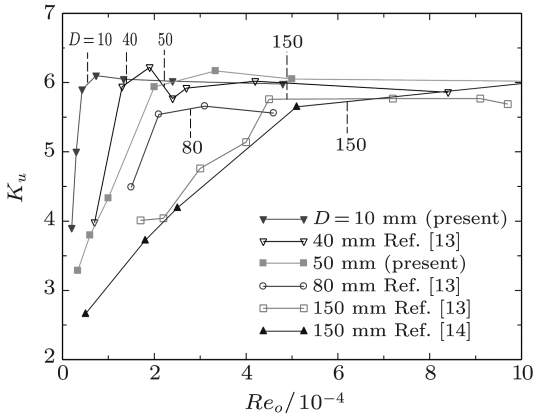


Fig. 2. The effect of Re_o on K_u.

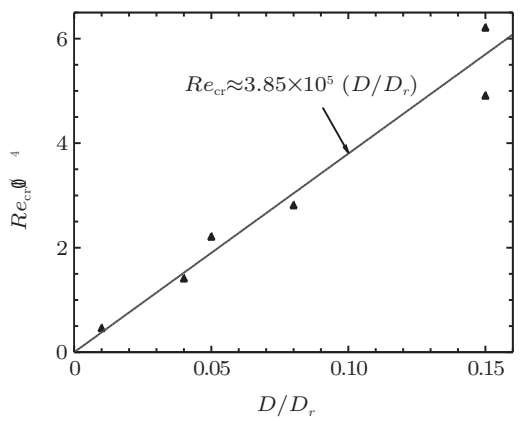


Fig. 3. The effect of D on $Re_{\rm cr}$. Reference diameter D_r = 1 m.

To assess the generality of this result more properly, all the available data of K_u are plotted against U_o/U_r in Fig. 4, where U_r = 6 m/s is the reference velocity. Very good agreement between all the datasets is evident, providing strong evidence of the dependence of K_u on U_o, rather than on Re_o. The asymptotic convergence of K_u appears to occur at a critical value of $U_{\rm cr} \approx 6$ m/s (refer to inset of Fig. 4). The evidence for the observation is very strong, because it is consistent for all 54 cases taken from the six independent investigations cited above — all from smooth contraction nozzles with or without a flat surface placed at the exit. There appear to be only two exceptions shown in the figure; i.e., the results obtained by Todde *et al.*[16] and ourselves for D = 20 mm. However, these exceptions can be explained very well. While we placed a flat dish of 200 mm at the nozzle exit, Todde *et al.*[16] employed a smooth contraction nozzle flush with a wall. Their centreline velocity measurements were performed in a jet at several very low exit velocities varying between U_o = 0.3 m/s and U_o = 2.5 m/s whereas ours were taken at U_o = 3 m/s–15 m/s. The corresponding data of K_u in Fig. 4, are obviously higher than those from the cases without exit surface, even at $U_o > 6$ m/s. Abdel–Rahman *et al.*[17] showed that the presence of an exit surface can reduce the jet entrainment of ambient fluid and thus the centreline velocity decay by about 20%. By taking this into account, Todde *et al.*'s data and our own for D = 20 mm in fact provide a good support for our above analysis regarding the dependence of K_u on U_o.

Two further interesting observations can be made from Fig. 4. Firstly, when $U_o \geq U_{\rm cr} \approx 6$ m/s, K_u asymptotically approaches to a value that is quite insensitive to the jet initial conditions, when taking into account experimental uncertainties. The low sensitivity is reflected in the narrow band of variation of the asymptotic values of K_u (5.6 to 6.2) even though the data were obtained in many independent investigations. That is, while these jets all issued from smooth

contraction nozzles, they were otherwise investigated under different initial and boundary conditions, e.g., different nozzles, different rooms and even different measurement techniques. It is therefore deduced that other initial conditions should not exert a strong influence on the self-similar mean-flow characteristics. The second significant, and perhaps surprising, observation is that for the low exit-velocity cases of $U_o < U_{\rm cr}$, the decay coefficient K_u is found to be linearly correlated (at least to the first-order accuracy) with U_o, regardless of Re_o or D. Taking together, these data imply that K_u depends primarily on U_o, not Re_o, for those jets issuing from smooth contraction nozzles, and any other dependencies are of the second order. This supports the earlier finding[13] that the mean velocity decay rate scales more appropriately with U_o than with Re_o.

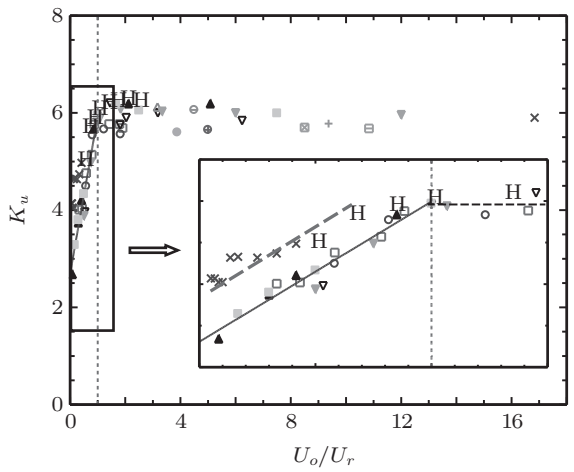


Fig. 4. Dependence of K_u on U_o. Symbols: H, present ($D = 20$ mm with a dish at the exit); ×, Todde *et al.*;[16] others (see Figs. 1 and 2). Reference velocity $U_r = 6$ m/s.

Next, we assess whether similar observations can be made in relation to present measurements of the spread rate K_r. Indeed as demonstrated in Fig. 5, the present values of K_r are of a decreasing function of U_o for $U_o < U_{\rm cr}$ and converge asymptotically to a constant (K_{ro}) at higher U_o. The constant K_{ro} is approximately 0.097 for the cases of $D = 10$ mm and 50 mm, while it is 0.84 for $D = 20$ mm with an exit surface. The present value of $K_{ro} \approx 0.097$ for $U_o > U_{\rm cr}$ agrees well with the literature results as indicated by the averaged value (≈ 0.095), estimated from earlier work.[10,11] This finding again supports the second-order influences of other initial and boundary conditions on the far-field properties when $U_o > U_{\rm cr}$.

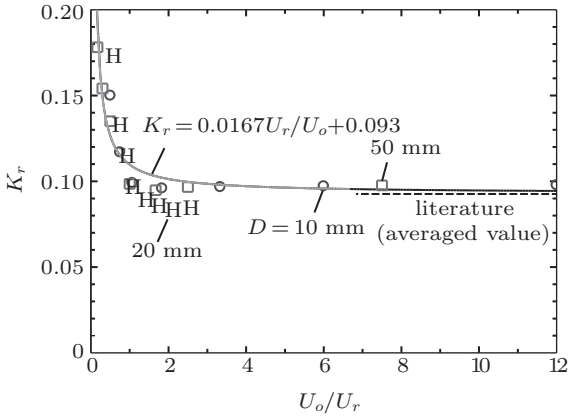


Fig. 5. The dependence of spread rate K_r on U_o. Reference velocity $U_r = 6$ m/s.

4. Discussion

First we seek an explanation for the dependence of K_u and K_r on U_o for $U_o < U_{\rm cr}$. In general, for low-Re_o flows, the range of turbulence-kinetic-energy (k) containing scales and that of the k-dissipation (ε) overlap so that $\varepsilon \sim \nu k/L^2$, where L is the characteristic length scale of the mean flow, e.g., R_u for circular jets, (see Ref. [18]). Similarity analysis for a circular jet[11] finds that, in the far-field, $K_r = {\rm d}R_u/{\rm d}x \sim \varepsilon R_u(k^2U_c)^{-1}$. It follows that $K_r \sim v/(U_cR_u)$. The momentum conservation requires that $(U_cR_u)^2 \sim (U_oD)^2$ or $U_cR_u \sim U_oD$. Also, it follows from Eqs. (1) and (2) that $K_rK_u \sim U_cR_u/(U_oD) \sim$ constant, assuming that $x_u = x_r$. We hence obtain that

$$K_u = C_1[(U_oD)/v] = C_1Re_o, \tag{3}$$

$$K_r = C_2[v/(U_oD)] = C_2/Re_o, \tag{4}$$

where C_1 and C_2 are independent of $Re_o = U_oD/v$ (but not necessarily independent of U_o and D). These results qualitatively corroborate those for the laminar jet, (see Schlichting[19]). It appears from Fig. 2 that equation (3) receives some support from all the data obtained at $U_o < 6$ m/s, though not obvious. However, more clearly from Figs. 4 and 5, equations (3) and (4) should reduce to

$$K_u = C_1'U_o, \tag{5}$$

$$K_r = C_2'/U_o, \tag{6}$$

where C_1' and C_2' are independent of all Re_o, U_o and D. In other words, equations (5) and (6) should be valid for $U_o < 6$ m/s when a circular air/air jet is issued from a smooth contraction nozzle of any diameter.

Next, we assess the above observations in the light of traditional knowledge. Consider the general equations governing momentum transport in turbulent flows. The equations of the mean momentum in a statistically steady, non-reacting turbulent flow may be written as (e.g., Ref. [12])

$$\overline{U}_i \frac{\partial \overline{U}_j}{\partial x_i} = \nu \frac{\partial}{\partial x_i}\left(\frac{\partial \overline{U}_j}{\partial x_i}\right) - \frac{\partial \overline{u_i u_j}}{\partial x_i} - \frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_j}, \qquad (7)$$

using standard Cartesian tensor notation and summation on repeated indices (i or $j = 1, 2, 3$). In Eq. (7), $\overline{U}$ and $\overline{p}$ are the mean velocity and mean pressure, respectively; u is the velocity fluctuation relative to the mean value; ρ is density and ν is the kinematic viscosity (or momentum diffusivity). The over-bar denotes time averaging and the equations are subject to the assumption that ρ and ν are constant. Normalization of the equations by the reference scales of velocity (U_r) and length (L_r), which are constant for each particular flow, leads to the following non-dimensional form:

$$\overline{U}_i^* \frac{\partial \overline{U}_j^*}{\partial x_i^*} = \frac{1}{Re}\frac{\partial}{\partial x_i^*}\left(\frac{\partial \overline{U}_j^*}{\partial x_i^*}\right) - \frac{\partial \overline{u_i^* u_j^*}}{\partial x_i^*} - \frac{\partial \overline{p}^*}{\partial x_j^*}. \qquad (8)$$

In Eq. (8), $\overline{U}^* = \overline{U}/U_{\rm r}$, $u^* = u/U_{\rm r}$, $\overline{p}^* = \overline{p}/\rho U_{\rm r}^2$, and $x^* = x/L_r$; the Reynolds number is defined as $Re = U_r L_r/\nu$.

Equation (8) indicates that the mean velocity properties (normalized) of any two similar turbulent flows with the same boundary conditions (normalized) should be identical if their corresponding values of Re are equal, regardless of their spatial dimensions. The common assumption for a circular jet that $U_r = U_o$ and $L_r = D$ would imply that, if the initial and boundary conditions of turbulent circular jets are similar, the corresponding values of K_u and K_r should depend only on Re_o and collapse onto a single curve for various nozzles in different sizes. It is, however, clear that this assumption is contradicted by Figs. 1–5, at least, for air turbulent jets. Namely, the traditionally defined Reynolds number may not be appropriate to characterize the circular jet. Hence we must consider alternative assumptions. For instance, the other length scale at the exit of a smooth contraction nozzle is likely to be the boundary layer thickness δ. For a sufficiently large contraction ratio of similar profile, δ is expected to depend on U_o. This suggests that the δ-based Reynolds number Re_δ may be a more appropriate parameter for controlling the development of a jet, rather than the traditional Re_o. Nevertheless, this is proposed to be examined in future by experiments.

Another point is also worthy to making from Eq. (8). That is, for sufficiently-high-Re flows or when $Re > Re_{\rm cr}$, strict matching of Re is not necessary for nearly identical solutions of the momentum equations since in these cases the first term on the right-hand side of Eq. (8) becomes negligible and their mean velocity field (normalized) should thus be nearly independent of Re. This gains some support from the present and previous measurements, but is subjected to the measured jets issuing from the nozzles of identical diameter (see Fig. 2). Figure 3 demonstrates that $Re_{\rm cr}$ depends strongly upon the nozzle diameter D, although the jets may have nominally the similar initial and boundary conditions.

Furthemore, a sensible reason for the deduction that U_o controls the development of a jet, rather than Re_o, may be found by virtually considering two air jets under the same Reynolds number of, say, $Re_o = 10^5$ ($\gg 10^4$, the value recognized for turbulent jets, see, e.g., Ref. [12]) but issuing from nozzles of $D = 10$ mm and 10^8 mm (or even much greater!), respectively. The exit velocity U_o is approximately 150 m/s for the small jet, compared with only 1.5×10^{-5} m/s for the very large jet. It is readily apparent that the molecular diffusion cannot be considered to be negligible for the lateral spreading of the very large jet since U_o is extremely small, despite the high value of Re_o; by comparison, the molecular diffusion should certainly be ignored for the small jet. Hence, the molecular diffusion for the very large jet is supposed to make a much greater relative contribution to the global spread and decay, so yielding greater values of K_u and K_r.

5. Concluding remarks

The normalized mean flow properties of a turbulent round jet issuing, at least, from a smoothly contracting nozzle appear to scale primarily with the exit velocity U_o, rather than the exit Reynolds number Re_o. Furthermore, the parameter whose critical value determines the asymptotic rates of the jet spread and decay for varying nozzle dimension (i.e., D) appears to be U_o but not Re_o, the most likely choice by convention. As U_o increases from a value well below 5 m/s, the mean velocity decay coefficient K_u and the spread rate K_r vary with U_o approximately in the manner: $K_u \sim U_o$ and $K_r \sim U_o^{-1}$ for $U_o < 6$ m/s, and become asymptotically constant at $U_o > 6$ m/s.

The above behaviours have been found by the present study (and in Ref. [13]) to occur in circular

air/air jets issuing from smoothly contracting nozzles. Despite that there are surely some differences in contraction profile and also it is not yet to know whether the trend will extend to contractions of all profiles, the evidence is sufficient to conclude that it is inappropriate to characterize the entirely free jet flow primarily by the Reynolds number (Re_o) based on the exit diameter (D) and bulk mean velocity (U_o), i.e., $Re_o \equiv U_o D/\nu$. These behaviours are independent of D and thus challenge the suitability of Re_o for turbulent free jets.

Finally, there are a number of relevant fundamental questions for future to address. For example, why is the value of about 6 m/s critical for circular air-to-air jets from a smooth contraction nozzle? Does this observation apply generally for circular and also non-circular free jets? What is the appropriate way to define the Reynolds number for free air-to-air jets when considering the failure of the conventional definition? Does this apply for jets of other fluids (e.g., water, oil, ...)?

Acknowledgments

The author expresses gratitude to Professors G. J. Nathan (University of Adelaide, AU) and R. A. Antonia (University of Newcastle, AU) for their insightful comments on the first draft a few years ago.

References

[1] Zaman K B M Q 1985 *J. Fluid Mech.* **152** 83
[2] Dimotakis P E, Miake-Lye R C and Panpantoniou D A 1983 *Phys Fluids* **26** 3185
[3] Miller P L and Dimotakis P E 1991 *Phys. Fluids* A **3** 1156
[4] Gilbrech R J 1991 *An Experimental Investigation of Chemically Reacting Gas-Phase Turbulent Jets* Ph. D. thesis California Institute of Technology, California, USA
[5] Koochesfahani M M and Dimotakis P E 1986 *J. Fluid Mech.* **170** 83
[6] Michalke A 1984 *Prog. Aerosp. Sci.* **21** 159
[7] Oosthuizen P H *83-FE-36 of ASME Applied Mechanics, Bioengineering, and Fluids Engineering Conference* Houston/TX USA, June 20–22, 1983
[8] Bogey C and Bailly C 2006 *Phys. Fluids* **18** 065101
[9] Pitts W M 1990 *Expts. Fluids* **11** 135
[10] Panchapakesan N R and Lumley J L 1993 *J. Fluid Mech.* **246** 197
[11] Hussein H J, Capp S P and George W K 1994 *J. Fluid Mech.* **258** 31
[12] Pope S B 2000 *Turbulent Flows* (Cambridge: Cambridge University Press) p. 101
[13] Malmström T G, Kirkpatrick A T, Christensen B and Knappmiller K D 1997 *J. Fluid Mech.* **346** 363
[14] Mi J, Nathan G J and Zhou Y *The 22nd International Congress of Theoretical & Applied Mechanics* 25–29 August 2008, Adelaide, Australia
[15] Nottage H B 1951 *Report on Ventilation Jets in Room Air Distribution* Case Inst. of Technology, Cleveland, Ohio
[16] Todde V, Spazzini P and Sandberg M 2009 *Expt. Fluids* **47** 279
[17] Abdel-Rahman A A, Chakroun W and AI-Fahed S F 1997 *Mech. Res. Comm.* **24** 277
[18] Du C, Xu M and Mi J 2010 *Acta Phys. Sin.* **59** 6331 (in Chinese)
[19] Mi J and Feng B 2010 *Acta Phys. Sin.* **59** 4748 (in Chinese)
[20] Mi J, Feng B, Deo R C and Nathan G J 2009 *Acta Phys. Sin.* **58** 7756 (in Chinese)
[21] Mi J and Feng B 2011 *Chin. Phys.* B **20** 074701
[22] Tennekes H and Lumley J L 1972 *A First Course in Turbulence* (Cambridge: The MIT Press)
[23] Schlichting H 1979 *Boundary Layer Theory* 7th edn. (New York: Mc Graw-Hill)