Normal mode splitting and ground state cooling in a Fabry–Perot optical cavity and transmission line resonator*

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Optomechanical dynamics in two systems which are a transmission line resonator and Fabrya–Perot optical cavity via radiation–pressure are investigated by linearized quantum Langevin equation. We work in the resolved sideband regime where the oscillator resonance frequency exceeds the cavity linewidth. Normal mode splittings of the mechanical resonator as a pure result of the coupling interaction in the two optomechanical systems is studied, and we make a comparison of normal mode splitting of mechanical resonator between the two systems. In the optical cavity, the normal mode splitting of the movable mirror approaches the latest experiment very well. In addition, an approximation scheme is introduced to demonstrate the ground state cooling, and we make a comparison of cooling between the two systems dominated by two key factors, which are the initial bath temperature and the mechanical quality factor. Since both the normal mode splitting and cooling require working in the resolved sideband regime, whether the normal mode splitting influences the cooling of the mirror is considered. Considering the size of the mechanical resonator and precooling the system, the mechanical resonator in the transmission line resonator system is easier to achieve the ground state cooling than in optical cavity.

Keywords: optomechanical system, normal mode splitting, ground state cooling

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1. Introduction

Optomechanical systems are one of the most fascinating and rapidly growing fields in quantum optics and information technology.[1] Significant progress has been made in the investigation of optomechanics, such as squeezing,[2,3] ultrahigh precision displacement detection,[4,5] mass detection,[6] gravitational-wave detection,[7] and the transition between classical and quantum behaviour of a mechanical system.[8] However, many of the behaviours are masked by thermal fluctuations.[9−12] In deep studying optomechanical systems, the systems should be cooled to their ground states. Therefore, one needs to cool the mechanical resonators (MRs) down to ultralow temperatures to put them into their ground states. Recently, cooling the mechanical mirror far below the temperature of the environment is achieved.[13−17] For example, a temperature below 1 mK is necessary for cooling a 20-MHz mechanical resonator to its vibrational ground state.[15] Radiation pressure-induced dynamical back-action cooling is the primary cooling regime[9,18−22] which has been demonstrated in a variety of optomechanical systems.[13−15,22,23] On the experimental side, radiation pressure optomechanical cooling has been shown to be a potential scheme to approach the ground state.[13−15,22,24−26]

Dilution refrigerator technology[27] can precool the bath temperature. However, to reach temperatures below 1 mK is beyond its capability. In order to observe the quantized motion of an MR, one should cool the MR down to its ground state of vibration and detect the phonon number state. A quantum theory of cooling an MR by radiation pressure-induced dynamical back-action was developed.[19,20] To detect the phonon number state, a quantum description of cooling a micromechanical flexural oscillator by a one-dimensional transmission line resonator (TLR) via a force that resembles cavity radiation pressure has been proposed.[28] Both the two regimes can cool the mechanical resonator to its ground state. As we know, a quantum-mechanical description of the motion of the MR in an optomechanical system consisting of a mechanical resonator capacitively coupled to a supercon-

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ducting coplanar waveguide was designed, which focused on studying the entanglement between the MR and the TLR.[29] Yong *et al.*[28] have investigated the cooling of the MR in the optomechanical system of the TLR without considering the normal mode splittings of the MR. Here, the normal mode splitting of the MR in the TLR system is considered and we compare the normal mode splitting of the MR between the TLR and optical cavity optomechanical systems. In addition, we derive the final effective phonon number that demonstrates the ground state cooling using an approximate scheme and compare the cooling of the MR in the two optomechanical systems. Further, we analyse whether the normal mode splitting influences the cooling of the mirror and two key factors that dominate optomechanical cooling.

In this paper, the optomechanical dynamics in two optomechanical systems, which are the TLR and Fabry–Perot optical cavity, are investigated. The fluctuation spectra of the MR in the two optomechanical systems presenting normal mode splitting with the increase of the input laser power are studied, and we make a comparison of the normal mode splitting between the two systems. In the optical cavity, the normal mode splitting of the movable mirror approaches the experimental result very well. As there are no atoms or other materials in the optomechanical cavity, this normal mode splitting is purely a result of the interaction between the cavity field and the mechanical mirror induced by radiation pressure. Furthermore, the cooling effects of the MR in the two systems are discussed in detail by using quantum Langevin equations. We derive the final effective mean phonon number which effectively demonstrates the ground state cooling. Our results show that the initial temperature is low, which can be achieved using a dilution refrigerator, a significant cooling of the mirror can be obtained, and lower initial temperatures will generally lead to better cooling. Another key factor dominating optomechanical cooling is the mechanical quality factor Q, which is also discussed. The bigger mechanical quality factor results in the better cooling of the optomechanical mirror. Moreover, in order to compare our results in the optical cavity with those in the TLR, we suppose that the achievable temperature is 10 mK and the achievable values of the mechanical quality factor Q are 4×10^4, 10^5, 4×10^5, and 10^6, although in the paper[30] the temperature was 300 mK and the mechanical quality factor $Q = 6700$. The results show that the MR can be cooled close to its ground state in the two systems using resolved sideband cooling, which is possible within the limitations of a high-quality cavity. However, it seems that this limit is easier to reach in the microwave TLR than in the optical cavity. Considering the size of the mechanical resonator and precooling the system, the TLR easily realizes ground state cooling.

2. Models and Hamiltonian

The first system that we consider is shown schematically in Fig. 1(a).[28] A mechanical resonator is fixed on both ends located in the centre of the transmission line resonator and is coupled capacitively to the central conductor of the TLR. The fundamental flexural mode of MR is described as a harmonic oscillator of frequency ω_b and effective mass m. The TLR is driven by an external microwave at a frequency ω_d and can be modeled as a single-mode LC resonator with frequency $\omega_a' = \sqrt{1/L_aC_a}$, where L_a and C_a are the inductance and the capacitance, respectively. Here, we will focus on the case of a micromechanical resonator where the condition $\omega_b \ll \omega_a'$ is satisfied. The Hamiltonian reads

$$H = \hbar\Delta_0 a^\dagger a + \frac{1}{2}(p^2/m + m\omega_b^2 x^2) - \frac{\hbar g_0}{2}(2a^\dagger a + 1)x + \hbar(\varepsilon a^\dagger + \varepsilon^* a). \quad (1)$$

The first term on the right-hand side in Eq. (1) is the energy of the TLR, where a ($a^\dagger$) is the lowering (rising) operator of the TLR mode satisfying the commutation relation $\left[a, a^\dagger\right] = 1$. $\Delta_0 = \omega_a - \omega_d$ is the detuning, and $\omega_a = \omega_a' + C_g^0 V_{\rm rms}^2/\hbar$ (where C_g^0 is the initial capacitance and $V_{\rm rms}$ is voltage) is the modified frequency of the TLR shifted by the coupling between the TLR and the MR. The second term gives the energy of the MR, where the position (momentum) operator x (p) of the MR satisfies $[x, p] = \mathrm{i}\hbar$. The third term comes from the coupling of the TLR to the MR, and $g^0 = C_g^0 V_{\rm rms}^2/(\hbar d)$ is a real coupling constant. The last term in Eq. (1) describes the input driving of the TLR by an external microwave field with the coupling strength $|\varepsilon| = 2\kappa P/\hbar\omega$,[13,31] where κ is the decay rate of the TLR and P is the input external microwave drive power.

The second optomechanical cavity investigated here is represented in Fig. 1(b),[9,30] which is driven by a laser with frequency ω_L. A single-mode optical cavity of length L, frequency ω_c, and linewidth κ is considered. A moving end mirror is modeled as a simple mechanical resonator with mass m and resonant frequency ω_m. Energy decay rate is γ_m which is associated with a thermal bath at a temperature T. In a

frame rotating at the laser frequency ω_L, the Hamiltonian for the system can be written as[20,32]

$$H = \hbar\Delta_0 c^\dagger c + \frac{1}{2}(p^2/m + m\omega_m^2 q^2) - \hbar g_0 c^\dagger c q + \mathrm{i}\,\hbar\varepsilon(c^\dagger - c). \quad (2)$$

the first term on the right-hand side in Eq. (2) is the energy of the cavity field. c and $c^\dagger$ are the annihilation operator and the creation operator for the cavity field, respectively, and $\Delta_0 = \omega_c - \omega_L$ is the laser detuning. The second term gives the energy of the mechanical mode, modeled as a harmonic oscillator at frequency ω_m and described by the dimensionless position and momentum operators q and p. The third term comes from the coupling of the movable mirror to the cavity field via radiation pressure and $g_0 = \omega_c/L$ is the optomechanical coupling constant between the cavity field and the movable mirror. The last term describes the coupling between the input laser field and the cavity field, where $\varepsilon = \sqrt{2\kappa P/\hbar\omega_L}$ is corresponding to the input laser power P. $\kappa = \pi c/(2FL)$ is the cavity decay rate due to the photon leakage through the fixed partially transmitting mirror, where F is cavity finesse.

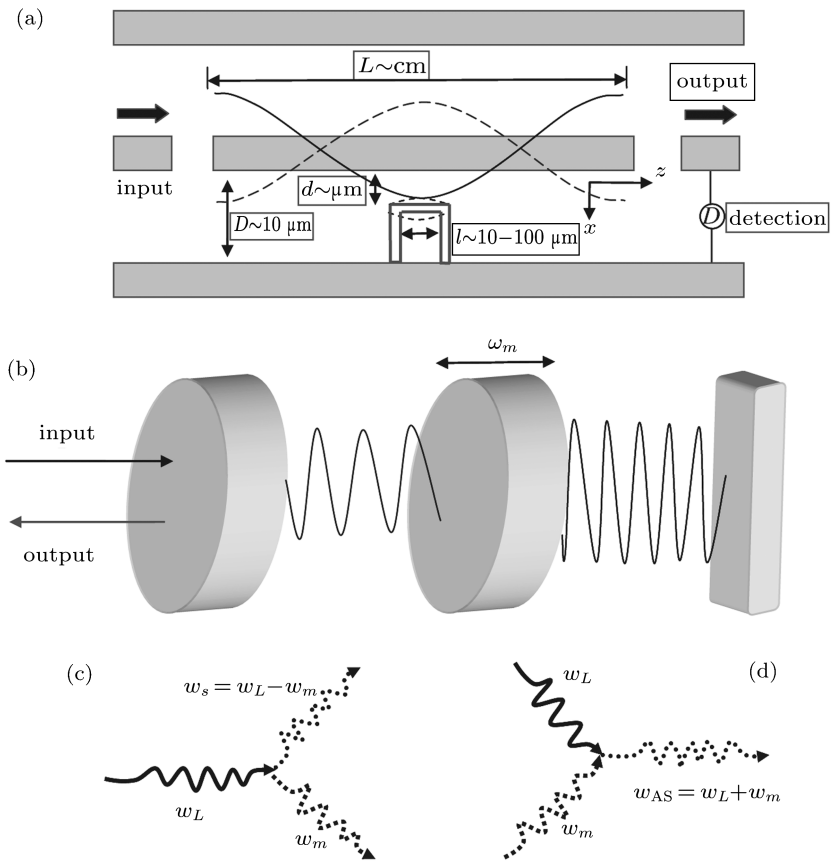


Fig. 1. Schematic diagrams of two optomechanical systems and two scattering processes. Panel (a) shows a mechanical resonator (MR) located at the centre of a one-dimensional transmission line resonator (TLR), where the external microwave drive field enters from the left and drives the TLR, and the signal at the output on the right end can be used to measure the motion of the MR via homodyne detection; panel (b) exhibits the cavity optomechanical interaction with a cavity field and a moveable mirror, where a laser with frequency entering the cavity through the partially transmitting mirror leads the moveable mirror to vibrate around its equilibrium position; panel (c) displays the Stokes scattering (ω_s) producing a phonon (ω_m); panel (d) indicates anti-stokes (ω_{AS}) progress annihilating a phonon ω_m.

The Hamiltonian of the two systems is considered in the condition of semiclassical theory and they have the same forms as shown in Eqs. (1) and (2). Thus, the optomechanical dynamics are investigated only in the optical cavity system as represented in Fig. 1(b). To discuss the fluctuation spectrum and the cooling of the optomechanics induced by radiation–pressure, the fluctuations of the system are considered and the linearized quantum Langevin equations are derived. Assuming that the nonlinearity is weak, we focus our attention on the dynamics of small fluctuations around the steady state of the system. Sep-

arating each operator into two parts, which are the steady-state mean value and a small fluctuation with zero mean value,[33,34] i.e., $q = q_s + \delta q$, $p = p_s + \delta p$, and $c = c_s + \delta c$, where

$$p_s = 0, \quad q_s = \hbar g_0 |c_s|^2 / m\omega_m^2,$$
$$c_s = \frac{\varepsilon}{\kappa + \mathrm{i}\,\Delta}, \quad \Delta = \Delta_0 - g_0 q_s, \tag{3}$$

the mean shift of the cavity frequency $g_0 q_s$ is determined by the laser input power P. From Eq. (3), we obtain the relation between the detuning Δ and input laser power, which reads

$$\Delta = \Delta_0 - P\left(\frac{2\kappa\omega_c}{m\omega_m^2 L^2}\frac{1}{|\kappa + \mathrm{i}\,\Delta|^2}\right), \tag{4}$$

which shows that the detuning Δ varies with input power P. Assuming that Δ is real and considering $\omega_c \simeq \omega_L$, the input power P and the detuning Δ have a one-to-one correspondence, which is used to discuss the normal mode splitting of the movable mirror.

Here, all the external parameters in the above equations must be chosen to satisfy the stability conditions which can be derived by using the Routh–Hurwitz criterion[35]

$$2\hbar g_0^2 \Delta c_s^2 (2\kappa + \gamma_m)^2 / m + \kappa\gamma_m\{\Delta^4 + 2\Delta^2[2\kappa^2 + \gamma_m^2 - 2\omega_m^2 + 2\kappa\gamma_m] + (\kappa^2 + \omega_m^2)(\kappa^2 + \omega_m^2 + 2\kappa\gamma_m) + \kappa^2\gamma_m^2\} > 0, \tag{5}$$

$$\omega_m^2(\kappa^2 + \Delta^2) - 2\hbar\frac{g_0^2 \Delta c_s^2}{m} > 0. \tag{6}$$

3. Fluctuation spectra of the mechanical resonators

In this section, we mainly derive the fluctuation spectrum of the movable micromechanical mirror. Using the Heisenberg equations of motion $\mathrm{i}\,\hbar(\partial\rho/\partial t) = [\,\rho, H]$ (where ρ represents operator) and adding the corresponding damping and noise terms, the linearized quantum Langevin equations can be obtained as follows:

$$\delta\dot{q} = \delta p/m, \tag{7}$$
$$\delta\dot{p} = \hbar g_0(c_s^*\delta c + c_s\delta c^\dagger) - m\omega_m^2\delta q - \gamma_m\delta p + \xi, \tag{8}$$
$$\delta\dot{c} = -(\kappa + \mathrm{i}\,\Delta)\delta c + \mathrm{i}\,g_0 c_s\delta q + \sqrt{2\kappa}\delta c_{\mathrm{in}}. \tag{9}$$

Here the input vacuum noise operator c_{in} with zero mean value has been introduced, which obeys the correlation function in the time domain[36] $\langle\delta c_{\mathrm{in}}(t)\delta c_{\mathrm{in}}^\dagger(t')\rangle = \delta(t - t')$, $\langle\delta c_{\mathrm{in}}(t)\delta c_{\mathrm{in}}(t')\rangle = \langle\delta c_{\mathrm{in}}^\dagger(t)\delta c_{\mathrm{in}}(t')\rangle = 0$. The mechanical mode is affected by a viscous force with damping rate γ_m and by a Brownian stochastic force with zero mean value $\xi(t)$, obeying the correlation function

$$\left\langle\xi(t)\xi(t')\right\rangle = \frac{\hbar\gamma_m}{2\pi}m\int\omega\ \mathrm{e}^{-\mathrm{i}\,\omega(t-t')} \times\left[1 + \coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}}T}\right)\right]\mathrm{d}\omega,$$

where k_{B} is the Boltzmann constant and T is the temperature of the reservoir of the movable mirror. The Brownian noise $\xi(t)$ is a non-Markovian, Gaussian quantum stochastic process.

Fourier transforming each operator in Eqs. (7)–(9) and solving it in the frequency domain, the position and momentum fluctuations of the movable mirror can be obtained as

$$\delta q(\omega) = \frac{C(\omega)\delta c_{\mathrm{in}}^\dagger(\omega) + C^*(-\omega)\delta c_{\mathrm{in}}(\omega) + [\Delta^2 + (\kappa - \mathrm{i}\,\omega)^2]\xi(\omega)}{B(\omega)}, \tag{10}$$

$$\delta p(\omega) = -\,\mathrm{i}\,m\omega\delta q(\omega), \tag{11}$$

where $B(\omega) = -2\hbar g_0^2\,|c_s|^2\,\Delta + m(\omega_m^2 - \omega^2 + \mathrm{i}\,\gamma_m\omega)[(\kappa + \mathrm{i}\,\omega)^2 + \Delta^2]$ and $C(\omega) = \mathrm{i}\,\hbar c_s\sqrt{2\kappa}g_0(\omega - \mathrm{i}\,\kappa + \Delta)$. The fluctuation spectra of position and momentum are defined by[25,31]

$$\langle\delta q(\omega)\delta q(\Omega)\rangle = 2\pi S_q(\omega)\delta(\omega + \Omega),$$
$$\langle\delta p(\omega)\delta p(\Omega)\rangle = 2\pi S_p(\omega)\delta(\omega + \Omega). \tag{12}$$

In order to calculate the spectrum of the movable mirror, the correlation functions of the noise sources are given in frequency domain as[34,37]

$$\langle\delta c_{\mathrm{in}}(\omega)\delta c_{\mathrm{in}}^\dagger(-\Omega)\rangle = 2\pi\delta(\omega + \Omega), \tag{13}$$
$$\langle\xi(\omega)\xi(\Omega)\rangle = 2\pi\hbar\gamma_m m\omega\left[1 + \coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}}T}\right)\right]\delta(\omega + \Omega). \tag{14}$$

According to Eqs. (10)–(14), the spectra of the mirror can be written as

$$S_q(\omega) = |\chi_{\mathrm{eff}}(\omega)|^2\,[S_{\mathrm{th}}(\omega) + S_{\mathrm{rp}}(\omega)],$$
$$S_p(\omega) = m^2\omega^2 S_q(\omega), \tag{15}$$

where $S_{\rm th}(\omega) = \hbar\gamma_m m\omega[1 + \coth(\hbar\omega/2k_{\rm B}T)]$ is the thermal noise spectrum due to the Brownian motion of the mirror, and

$$S_{\rm rp}(\omega) = \frac{\hbar m[(\omega - \Delta)^2 + \kappa^2]}{2\Delta} \Gamma_{\rm rp}(\omega)$$

is the radiation pressure noise spectrum due to the coupling between the cavity field and the movable micromirror. Here,

$$\Gamma_{\rm rp}(\omega) = \frac{4\hbar\Delta g_0^2 \left|c_s\right|^2 \kappa}{m\left|\Delta^2 + (\kappa + {\rm i}\,\omega)^2\right|^2}$$

is the additional damping term resulting from the optomechanical coupling. From Eq. (15), we find that the spectra of the mirror include two terms which are the thermal noise of the mechanical oscillator and radiation pressure noise.

The effective susceptibility is defined as $\chi_{\rm eff}(\omega) = [\Delta^2 + (\kappa + {\rm i}\,\omega)^2]/B(\omega)$, after simplifying it, we have

$$\chi_{\rm eff}(\omega) = \frac{1}{m\{[\omega_m^{\rm eff}(\omega)]^2 - \omega^2 + {\rm i}\,\omega[\gamma_m^{\rm eff}(\omega)]\}}, \quad (16)$$

where the effective frequency of the mirror is

$$\omega_m^{\rm eff}(\omega) = \sqrt{\omega_m^2 - \frac{(\Delta^2 + \kappa^2 - \omega^2)\Gamma_{\rm rp}(\omega)}{2\kappa}},$$

and the effective damping rate is $\gamma_m^{\rm eff}(\omega) = \gamma_m + \Gamma_{\rm rp}(\omega)$. The modification of the mechanical frequency $\omega_m^{\rm eff}(\omega)$ induced by radiation pressure is the so-called "optical spring effect". Optical spring effect may lead to significant frequency shifts in the case of low-frequency oscillators.

4. Cooling of the mechanical resonator

Using the fluctuation spectra of the mirror as given in Eq. (15), the final mean phonon number in the steady state can be defined as[27,30]

$$n_m^f = \frac{\left\langle \delta p^2 \right\rangle}{2\hbar m\omega_m} + \frac{m\omega_m}{2\hbar} \left\langle \delta q^2 \right\rangle - 1/2, \quad (17)$$

where the variances of position and momentum are $\left\langle \delta q^2 \right\rangle = (1/2\pi)\int_{-\infty}^{\infty} S_q(\omega)\,{\rm d}\omega$ and $\left\langle \delta p^2 \right\rangle = (1/2\pi)\int_{-\infty}^{\infty} S_p(\omega)\,{\rm d}\omega$. The final effective mean phonon number of the micro-mirror can be calculated directly by evaluating the integral. However, the resulting expressions are rather cumbersome and do not offer the necessary physical insight into the coupling between the cavity field and the mirror, and the key factors which dominate the cooling of the mirror. Consequently, Eq. (17) can be derived using the approximation scheme. We consider the regime where the effective mechanical damping rate is less than the mechanical frequency and the cavity decay rate ($\left|\gamma_m^{\rm eff}(\omega)\right| < \omega_m$, $\left|\gamma_m^{\rm eff}(\omega)\right| < \kappa$).[17,18] Then one can obtain an approximate expression for the variance

$$\left\langle \delta q^2 \right\rangle \approx \frac{S'_{\rm th}(\omega_m) + S'_{\rm rp}(\omega_m)}{2m^2\omega_m^2\left|\gamma_m^{\rm eff}(\omega_m)\right|}, \quad (18)$$

where the effective thermal noise spectrum $S'_{\rm th}(\omega) = \hbar\gamma_m m\omega\coth(\hbar\omega/2k_{\rm B}T)$ and the induced noise spectrum

$$S'_{\rm rp}(\omega) = \frac{\hbar m(\omega^2 + \Delta^2 + \kappa^2)}{2\Delta}\Gamma_{\rm rp}(\omega)$$

are the approximations of $S_{\rm th}(\omega)$ and $S_{\rm rp}(\omega)$, respectively. Similarly, one can obtain

$$\left\langle \delta p^2 \right\rangle = (m\omega_m)^2 \left\langle \delta q^2 \right\rangle. \quad (19)$$

The induced noise spectrum $S_{\rm rp}(\omega)$ heats the motion of the mirror. However, when the effective damping rate is enhanced ($\left|\gamma_m^{\rm eff}(\omega)\right| > \gamma_m$), the mechanical motion will be cooling and the cooling effect will dominate the heating effect. Actually, this is right when the significantly increased effective damping rate is positive for positive detuning, and in this paper we just consider such a condition. According to Eqs. (17)–(19), one has

$$n_m^f = \frac{\gamma_m n_m + \Gamma_{\rm rp}(\omega_m) n_{mc}}{\gamma_m + \Gamma_{\rm rp}(\omega_m)}, \quad (20)$$

where $n_{mc} = \kappa^2/(16\omega_m^2)$[20] is the induced mean phonon number due to the coupling between the mirror and the cavity, and $n_m = 1/(\exp(\hbar\omega_m/k_{\rm B}T) - 1)$ is the initial mean thermal excitation phonon number of the mirror.

5. Numerical results and discussion

We use parameters from a recent experiment on the observation of the normal-mode splitting in the fluctuation spectrum[30] and cool a micromechanical resonator to the ground state in one transmission line resonator system.[28] The parameters are listed in Tables 1 and 2. In the simplest case, two independent harmonic oscillators coupled via an additional joint spring will behave as a pair of uncoupled oscillators (the so-called normal modes) with shifted resonance frequencies compared with the individual resonators. Considering the case of resonators with equal bare frequencies, when the coupling is sufficiently strong, the

two normal modes will split. The normal mode splitting is not a novel phenomenon and it has been observed in both quantum and classical physics, which stems from the coupling of two degenerate modes with energy exchange taking place on a time scale faster than the decoherence of each mode. The vacuum Rabi splitting is an important example of the mode splitting which is due to strong interaction between the atoms and the cavity mode. The normal mode splitting also arises from photon–phonon interaction or artificial atoms in circuit quantum electrodynamics.[38,39] All these normal mode splittings correspond to a split in the energy spectrum of the coupled two-mode system which is obtained via linear response. However, in optomechanical system, normal mode splitting involves driving two parametrically coupled nondegenerate modes out of equilibrium. Consequently, the splitting is studied from the fluctuation spectrum rather than appearing directly in the cavity transmission.[40] This is the reason why the small fluctuation of the movable mirror is considered when we derive the fluctuation spectrum. In fact, the fluctuation spectrum of the optical cavity field has the same normal mode splitting behaviour as that of the mechanical oscillator. Here, we consider only the fluctuation spectrum of the mechanical oscillator.

Table 1. Transmission line resonator parameters cited from Ref. [28].

Parameter	Symbol	Value
External microwave driving transmission line resonator	ω_d/Hz	10^{10}
Frequency of the mechanical resonator	ω_b/Hz	4×10^6
Modified frequency of the TLR	ω_a/Hz	$2 \times 10^4 \omega_b$
Effective mass of MR	m/ng	15
Decay rate of the TLR	κ/Hz	$0.1\omega_b$
Damping rate of the MR	γ_b/Hz	$0.25 \times 10^{-4} \omega_b$
Input driving microwave	ε/Hz	$2.5 \times 10^3 \omega_b$
Real coupling constant	g_0	$3 \times 10^5 \omega_b \sqrt{n\omega_b/\hbar}$
Initial temperature	T/mK	100, 30, 10
Mechanical quality factor	$Q_b = \omega_b/\gamma_b$	4×10^4, 10^5, 4×10^5, 10^6

Table 2. Optical cavity parameters from Ref. [30].

Parameter	Symbol	Value
Wavelength of the laser	λ/nm	1064
Cavity length	L/mm	25
Effective mass of the movable mirror	m/ng	145
Cavity frequency	ω_c/Hz	10^{15}
Input laser power	P/mW	6.9
Cavity decay rate	κ/Hz	$0.227\omega_m$
Initial temperature	T/mK	300, 100, 30, 10
Mechanical quality factor	Q	6700, 10^4, 4×10^4, 10^5, 5×10^5
Energy decay rate	γ_m/Hz	$2\pi \times 140$

The scaled spectra of normal mode splitting at positive detuning versus frequency are shown in Fig. 2. The spectra of the MR in the TLR have the same form of the movable mirror as that shown in Eq. (10) which are influenced by the thermal noise spectrum due to the Brownian motion of the MR and the induced noise spectrum due to the capacitive coupling to the driven TLR. The TLR, whose frequency is determined by its overall capacitance and inductance, acts as a cavity. The beam is placed near the middle of the TLR and is capacitively coupled to the TLR. When the mechanical beam is displaced, the overall capacitance of the TLR changes, thereby the resonant frequency of the TLR also changes. Since the TLR is driven by an external microwave power P with $\varepsilon = \sqrt{2\kappa P/\hbar\omega_d}$, then, with the increase of the external microwave power, the coupling between the TLR and MR will be enhanced, thereby the normal mode splitting of the MR also increases, as shown in Fig. 2(a). However, in an optical cavity, the radiation pressure force acting on the movable mirror leads to the micro-mirror oscillating around its equilibrium position, which is like a two-level atom inside an optical cavity. In the strong coupling regime, an atom in an optical cavity

can lead to Rabi-splitting,[41] and a three-level atomic system under the pump field and the probe field can show dynamic Stark splitting due to the action of the pump light. This corresponds to another famous phenomenon called electromagnetically induced transparency.[42] Owing to there being no atoms or other materials present in the optomechanical cavity, this normal mode splitting in the optomechanical system is purely the result of the interaction between the cavity field and the mechanical mirror. In Fig. 2(b), for small optical pump power, the splitting cannot

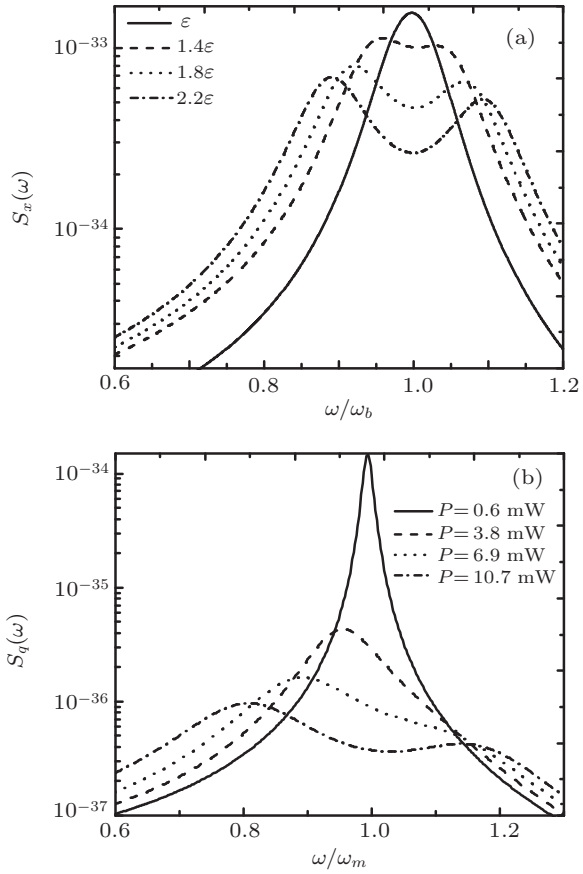


Fig. 2. Scaled spectra of the mechanical resonator in the two optomechanical systems. Panel (a) shows the curves for scaled spectrum $S_x(\omega)$ versus normalized frequency ω/ω_b in transmission line resonator; panel (b) exhibits the curves for scaled spectrum $S_q(\omega)$ versus normalized frequency ω/ω_m in optical cavity. All plots are shown on a logarithmic scale. Other parameters are listed in Tables 1 and 2.

be resolved, i.e., only one peak appears in the spectrum. However, when the pump laser is turned on, it will split into two sideband peaks, which is unambiguous evidence for entering the strong coupling regime.

Although both the TLR and the optical cavity systems can be observed by the normal mode splitting of the mechanical vibrator, if we consider trapping atoms or other materials in the optomechanical system to explore the normal mode splitting or other novel physical phenomenon, an optical cavity such as the Fabry–Perot cavity is a good option. Here, we should indicate that the effective detuning is $\Delta \simeq 1.02\omega_m$ in a recent experiment on the observation of the normal-mode splitting in the fluctuation spectrum.[30] From Eq. (4) we know that the values of the effective detuning are different for different input laser powers. We calculate the four values of the effective detuning of the four input laser power: $P = 0.6$ mW, $\Delta = 1.08897336\omega_m$; $P = 3.8$ mW, $\Delta = 1.07\omega_m$; $P = 6.9$ mW, $\Delta = 1.05\omega_m$; $P = 10.7$ mW, $\Delta = 1.0235\omega_m$. Comparing our results with the experiment data, we find that our results approach the experimental results very well.[30] However, when we just consider the effective detuning $\Delta \simeq 1.02\omega_m$, the results are a little different from the experimental ones. In addition, the order of magnitude of the spectrum of the mechanical oscillator is different from that in the recent experiment,[30] the reason is that the form of the Hamiltonian in this paper is different to m that in the experiment, and there are some transformations in the two forms of Hamiltonian. After transforming the Hamiltonian to the uniform form, we find that our results coincide with the experimental ones. Although normal mode splitting alone does not establish a proof for coherent dynamics, i.e. quantum interference effects, still, the demonstration of normal mode splitting is a necessary condition for future quantum experiments.

In optomechanical systems, it has been shown that ground state cooling is possible only in the resolved sideband regime where the mechanical resonance frequency exceeds the bandwidth of the driving resonator.[19,20] Since this type of normal mode splitting in optomechanical cavity occurs during resolved sideband cooling, whether the normal mode splitting affects cooling in the resolved sideband regime should be considered. As discussed in the back-action optomechanical cooling schemes in optical cavities, the redshifted sideband leads to the anti-Stokes scattering $(\omega_L + \omega_m)$ which cools the mechanical oscillator mode by taking away a phonon of the mirror.[9] However, the blueshifted motional sideband results in Stokes scattering $(\omega_L - \omega_m)$ which heats the mirror by creating a phonon. The two progresses are shown in Figs. 1(c) and 1(d). When effective detuning $\Delta > 0$, the cavity field of the optomechanical interacts with the anti-Stokes sideband $(\omega_L + \omega_m)$ more than with the Stokes sideband $(\omega_L - \omega_m)$ and cooling will occur. These two processes lead to the mechanical oscillator oscillating around its equilibrium position, thereby changing the

amplitude of the cavity field. This is the reason why the fluctuations of the mechanical oscillator and the cavity field show normal mode splitting. Meanwhile, this is also the physical reason why the positive effective detuning ($\Delta > 0$) will lead to cooling. In the high-quality cavity limit $\kappa < \omega_m$, the anti-Stokes sideband is resolved and the corresponding cooling process is prominent. Especially, for the optimal effective detuning $\Delta = \omega_m$, the frequency of the optomechanical cavity is resonant with that of the anti-Stokes sideband, which will lead to optimal cooling. The normal mode splitting investigated above and the optomechanical cooling in the strong coupling optomechanical cavity are discussed at the optimal effective detuning $\Delta \simeq \omega_m$. The bath temperature can be lowered by using a current cryogenic technique. Here, the optomechanical system cooling combining resolved-sideband cooling with cryogenic precooling[27] is discussed. There are two key parameter factors that dominate the cooling of the micro-mirror. The first is the initial temperature of the thermal bath and the second is the quality factor of the mechanical oscillator.

The curves for the final mean phonon number as a function of effective detuning Δ with different initial temperatures are plotted in Fig. 3. Figure 3(a) shows the curves for the final mean phonon number of the MR in a TLR optomechanical system. For the initial temperatures $T = 100$ mK, 30 mK, and 10 mK, the final mean phonon numbers are $n_b^f \simeq 1.6$, $n_b^f \simeq 0.5$ and $n_b^f \simeq 0.16$, respectively, and the other parameters are listed in Table 1. It is obvious that a significant cooling of the MR is obtained and lower initial temperatures will generally lead to better cooling. For an initial temperature $T = 10$ mK, which can be realized experimentally by using a dilution refrigerator, the MR can be cooled close to the ground state because the final mean phonon number $n_b^f \simeq 0.16$. In order to compare the results in the TLR system, the final mean phonon numbers of the movable micromirror in optical cavity are also shown in Fig. 3(b). In an optomechanical cavity, the thermal bath temperature is $T = 300$ mK which comes from a report,[30] and we consider that there are another three initial temperatures $T = 100$ mK, 30 mK, and 10 mK, which can be realized experimentally by using a dilution refrigerator. The other parameters are listed in Table 2. When the initial temperature are $T = 300$ mK, 100 mK, 30 mK, and 10 mK, the final mean phonon numbers are $n_m^f = 3.643$, 1.226, 0.3765 and 0.1339, respectively. In order to reach ground state cooling, the final mean phonon number should be $n_m^f < 1$. The results show that only when the initial temperature is less than 100 mK can the ground-state cooling be realized and will the lower initial temperature generally lead to better cooling. Comparing the results of the two systems, it seems that the cooling of the optical cavity is better that than of the TLR. However, it is easier to precool the TLR than the optical cavity to a much low initial temperature by using standard dilution refrigeration techniques.

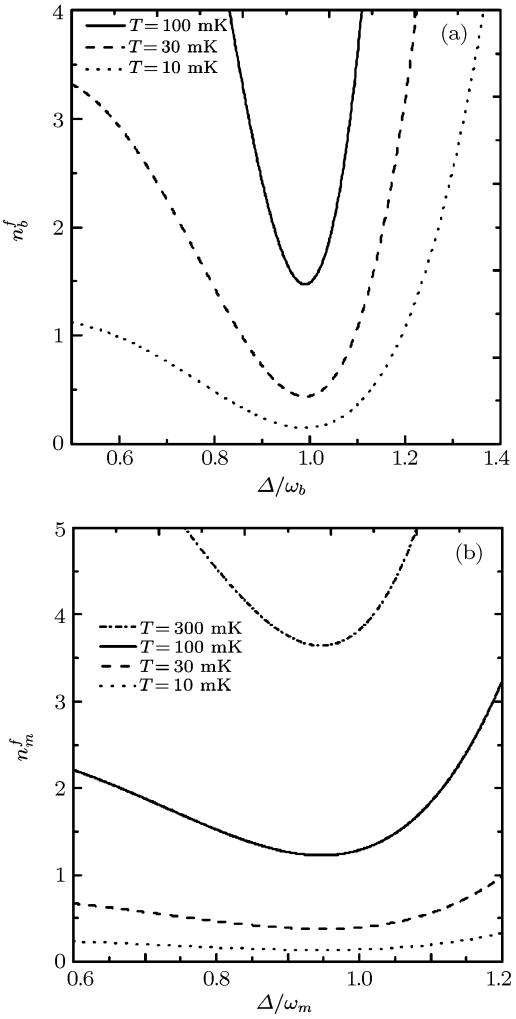


Fig. 3. Curves for final mean phonon number versus the effective detuning Δ for different initial temperatures of the two systems. Panel (a) gives the final mean phonon number of mechanical resonator in a transmission line resonator system with the initial temperature decreasing from $T = 100$ mK to $T = 10$ mK; panel (b) shows the curves for the final mean phonon number of the mirror in an optical cavity with initial temperature decreasing from $T = 300$ mK to $T = 10$ mK. Other parameters are listed in Tables 1 and 2.

Figure 4 shows the curves of the final effective mean phonon number versus the mechanical quality factor. Figure 4(a) shows the curves for the

final effective mean phonon number versus the effective detuning, when the quality factors of MR are $Q_b(=\omega_b/\gamma_b) = 4\times10^4$, 10^5, 4×10^5, and 10^6, the corresponding minimal phonon numbers are $n_b^f = 0.16$, 0.06, 0.02, and 0.01. The other parameters

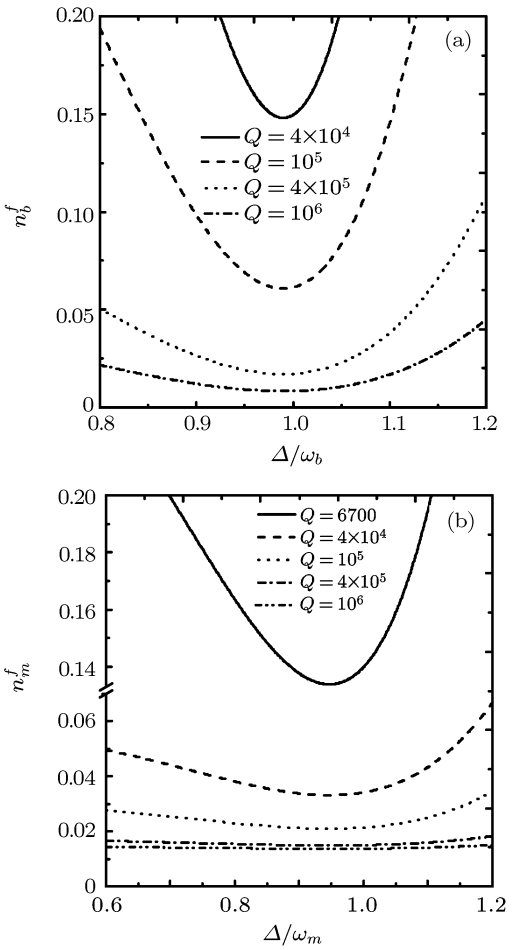


Fig. 4. Curves for final effective mean phonon number versus Δ for different quality factors of the two systems, with initial temperature $T = 10$ mK; panel (a) displays the variations of final effective mean phonon number with Δ at the quality factor $Q = 4\times10^4$, 10^5, 4×10^5, and 10^6 in transmission line resonator system; panel (b) gives the variations of final effective mean phonon number with Δ at the quality factor $Q = 6700$, 4×10^4, 10^5, 4×10^5, and 10^6 in optical cavity. Other parameters are listed in Tables 1 and 2.

are listed in Table 1. One can find that the cooling is better for a higher quality factor of the MR. While, in the optical cavity, the quality factor is $Q = \omega_m/\gamma_m = 6700$ and the initial temperature of the mirror is $T = 300$ mK, which is cited from Ref. [30]. In the present work, we consider the same condition as that of the TLR, i.e., the initial temperature $T = 10$ mK and the other four quality factors $Q = 4\times10^4$, 10^5, 4×10^5, and 10^6, which can be obtained by experiment in the Fabry–Perot cavity.[43] The other parameters are listed in Table 2. The mean phonon numbers are $n_m^f = 0.1339$, 0.03317, 0.021, 0.01491, and 0.0137, for $Q = 4\times10^4$, 10^5, 4×10^5, and 10^6, respectively, and which are plotted in Fig. 4(b). Our results show that a higher quality factor micro-mirror can easily approach ground-state cooling at the initial temperature of the mirror $T = 10$ mK. In fact, when the initial temperature of the mirror is $T = 300$ mK, once the quality factor of the mirror is large enough ($Q > 10^4$), the ground state can also be achieved. However, quality factors $Q_a = \omega_a/\kappa$ of the transmission line resonator is easy to achieve while the typical optical cavity quality factor is difficult.

6. Conclusion

We compare the optomechanical dynamics in two systems, which are the transmission line resonator and the Fabry–Perot optical cavity in detail. The normal mode splitting of the mechanical resonator is investigated,and we compare the normal-mode splitting of the mechanical resonator in a transmission line resonator optomechanical system and that in a Fabry–Perot optical cavity. In addition, the methods to cool the two systems dominated by two key factors, which are the initial temperature and the quality factor, are also compared. The discussion is based on a linearized quantum Langevin equation. The optimal cooling condition is required in a resolved sideband regime ($\kappa \ll \omega_m$), and normal mode splitting influences the cooling of the mechanical resonator in an optomechanical system. The results show that the mirror can be cooled close to its ground state with low initial temperature and high mechanical quality factor. Considering the size of the mechanical resonator and precooling the system, the mechanical resonator in transmission line resonator system is easier to achieve ground state cooling than in optical cavity.

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