Negative refractive index in a four-level atomic system*

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A closed four-level system in atomic vapour is proposed, which is made to possess left handedness by using the technique of quantum coherence. The density matrix method is utilized in view of the rotating-wave approximation and the effect of a local field in dense gas. The numerical simulation result shows that the negative permittivity and the negative permeability of the medium can be achieved simultaneously (i.e. the left handedness) in a wider frequency band under appropriate parameter conditions. Furthermore, when analysing the dispersion property of the left-handed material, we can find that the probe beam propagation can be controlled from superluminal to subluminal, or *vice versa* via changing the detuning of the probe field.

Keywords: quantum interference, electromagnetically induction, left-handed materials, negative refractive index

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1. Introduction

The propagation of an electromagnetic wave in matter is characterized by the frequency-dependent relative dielectric permittivity $\varepsilon_{\rm r}$ and magnetic permeability $\mu_{\rm r}$. Their product defines the index of refraction $\varepsilon_{\rm r}\cdot\mu_{\rm r}=n^2$. Naturally, the wave vector $\boldsymbol{K}$, the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{H}$ form a right-handed system in conventional materials and have both positive permittivity and permeability simultaneously. However, in 1968, Veselago[1] theoretically constructed an electromagnetic material in which the wave vector is opposite to the direction of energy propagation. Thus Veselago called such a material a "left-handed" material (LHM). An LHM has a negative refractive index when the permittivity and the permeability are negative simultaneously. It is a new kind of material that offers the possibility of molding the flow of light inside media and it has attracted considerable attention[2−13] because of its surprising counterintuitive electromagnetic and optical properties. It can be verified that left-handed media exhibit a number of peculiar electromagnetic and optical properties, such as reversals of Doppler shift, anomalous refraction, negative Goos–Haanchen shift,[5] amplification of evanescent waves,[7] reversed circular Brag phenomenon,[12] enhanced quantum interference, modified spontaneous emission rates, unusual photon tunneling effect,[6] subwave length focusing,[7,13] and so on.

LHMs have been realized by several approaches, including artificial composite meta-materials,[14−17] transmission line simulation,[18] photonic crystal structures,[11,19,20] chiral materials[21−23] and photonic resonant materials.[24−27] The first four methods require the careful fabrication of a spatially periodic structure. The last method is a quantum optical method in which the physical mechanism is the quantum interference and coherence that arise from the transition process in a multilevel atomic system. It was proposed first in a three-level medium[24] and requires a rigorous level condition. Thommen and Mandel[27] proposed enhancing the freedom of choice of levels and making the scheme much more applicable to a realistic system according to a coherent cross-coupling between electric and magnetic dipole transitions in a four-level scheme. Research on optical LHMs are under way.[28,29]

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In this paper, according to the theory of quantum coherence effect to realize left-handedness, we put forward a four-level dense atomic vapour scheme. In the system, we can dominate the relative permeability and relative permittivity by the electromagnetic interaction between a multi-level atom and a multi-mode optical field in our scheme, and the contribution of a Lorentz–Lorenz local field of the four-level dense atomic vapour should be considered. Under some appropriate conditions, the system shows negative permittivity and negative permeability simultaneously, thus the system realizes the left-handed effect in a wider frequency band and also shows novel dispersion properties in a certain probe frequency band.

2. Model and density matrix equation

Now we discuss the coherent effect in a four-level V-type atomic system, as shown in Fig. 1.[30,31] It has a single excited state $|2\rangle$ and three closely spaced lower levels $|1\rangle$, $|3\rangle$, and $|4\rangle$. The spontaneous emission from the excited state $|2\rangle$ to the lower state $|1\rangle$ can strongly affect the neighbouring transition $|2\rangle \rightarrow |3\rangle$ ($|2\rangle \rightarrow |4\rangle$) and induce interference. In this case, level $|1\rangle$ and level $|2\rangle$ have opposite parities and $\boldsymbol{d}_{21} = \langle 2|\hat{\boldsymbol{d}}|1\rangle \neq 0$, where $\hat{\boldsymbol{d}}$ is the electric dipole operator. The two levels $|1\rangle$ and $|4\rangle$ have the same parities with $\boldsymbol{\mu}_{41} = \langle 4|\hat{\boldsymbol{\mu}}|1\rangle \neq 0$, where $\hat{\boldsymbol{\mu}}$ is the magnetic-dipole operator. A resonant coupling field and a weak probe field, respectively, with Rabi-frequencies $\Omega_{\rm c}$ and $\Omega_{\rm p}$, couple two lower meta-stable states $|1\rangle$ and $|3\rangle$ with upper level $|2\rangle$, forming a simple Λ-configuration. Meanwhile, the resulting dark state is coherently coupled to another meta-stable state $|4\rangle$ by a microwave or quasi-static coherent field with Rabi frequency $\Omega_{\rm d}$. We use $2\gamma_1$, $2\gamma_3$, $2\gamma_4$ to denote the spontaneous decay rates from level $|2\rangle$ to levels $|1\rangle$, $|3\rangle$, and $|4\rangle$, respectively, $\omega_{\rm p}$, $\omega_{\rm c}$, and $\omega_{\rm d}$ to represent carrier frequencies of the corresponding fields and $\Delta_1 = \omega_{21} - \omega_{\rm p}$ and $\Delta_3 = \omega_{34} - \omega_{\rm d}$ refer to the frequency detunings of the probe field and the coherent driving field, respectively.

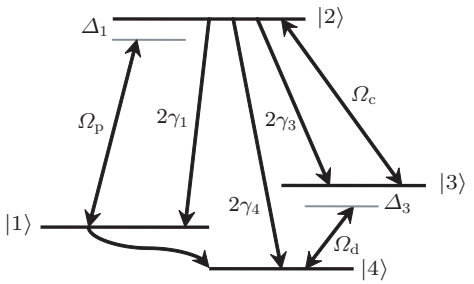


Fig. 1. The model of four-level V atom system interacting with three light fields.

Under rotating-wave and dipole approximations, the density matrix elements are given as follows:

$$
\begin{aligned}
\frac{\partial \rho_{11}}{\partial t} &= 2\gamma_{11}\rho_{22} - {\rm i}\,\Omega_{\rm p}(\rho_{12} - \rho_{21}),\\
\frac{\partial \rho_{12}}{\partial t} &= -(\gamma_1 + \gamma_3 + \gamma_4 - {\rm i}\,\Delta_1)\rho_{12}\\
&\quad - {\rm i}\,\Omega_{\rm c}\rho_{13} + {\rm i}\,\Omega_{\rm p}(\rho_{22} - \rho_{11}),\\
\frac{\partial \rho_{13}}{\partial t} &= {\rm i}\,\Delta_1\rho_{13} - {\rm i}\,\Omega_{\rm c}\rho_{12} - {\rm i}\,\Omega_{\rm d}\rho_{14} + {\rm i}\,\Omega_{\rm p}\rho_{23},\\
\frac{\partial \rho_{14}}{\partial t} &= {\rm i}\,\Delta_1\rho_{14} - {\rm i}\,\Omega_{\rm d}\rho_{13} + {\rm i}\,\Omega_{\rm p}\rho_{24},\\
\frac{\partial \rho_{22}}{\partial t} &= -(2\gamma_1 + 2\gamma_3 + 2\gamma_4)\rho_{22}\\
&\quad + {\rm i}\,\Omega_{\rm c}(\rho_{32} - \rho_{23}) + {\rm i}\,\Omega_{\rm p}(\rho_{12} - \rho_{21}),\\
\frac{\partial \rho_{23}}{\partial t} &= -(\gamma_1 + \gamma_3 + \gamma_4)\rho_{22}\\
&\quad + {\rm i}\,\Omega_{\rm c}(\rho_{33} - \rho_{22}) - {\rm i}\,\Omega_{\rm d}\rho_{24} + {\rm i}\,\Omega_{\rm p}\rho_{13},\\
\frac{\partial \rho_{24}}{\partial t} &= -(\gamma_1 + \gamma_3 + \gamma_4)\rho_{24}\\
&\quad + {\rm i}\,\Omega_{\rm c}\rho_{34} - {\rm i}\,\Omega_{\rm d}\rho_{23} + {\rm i}\,\Omega_{\rm p}\rho_{13},\\
\frac{\partial \rho_{33}}{\partial t} &= 2\gamma_3\rho_{22} + {\rm i}\,\Omega_{\rm c}(\rho_{23} - \rho_{32}) + {\rm i}\,\Omega_{\rm d}(\rho_{43} - \rho_{34}),\\
\frac{\partial \rho_{34}}{\partial t} &= {\rm i}\,\Omega_{\rm c}\rho_{24} + {\rm i}\,\Omega_{\rm d}(\rho_{44} - \rho_{33}),
\end{aligned}
\tag{1}
$$

where we have assumed a closed atomic system, i.e. $\sum_{i=1}^{4} \rho_{ii} = 1$.

3. Electric and magnetic polarizations of the four-level atomic system

In the following, we will discuss the electric and magnetic responses of the medium to the probe field. In order to discuss how the detailed properties of the atomic transitions between the levels relate to the electric and the magnetic susceptibilities, one must make a distinction between macroscopic field and the microscopic local field acting on the atoms in the vapour. In a dilute vapour, there are few differences between the macroscopic fields and the local fields, and both fields act on any atoms (molecules or group of molecules).[32] However, in a dense medium with closely packed atoms (molecules), the polarization of neighbouring atoms (molecules) gives rise to an internal field at any given atom in addition to the average macroscopic fields so that the total field at the

atom is different from the macroscopic field.[30] In order to achieve negative permittivity and permeability, the chosen vapour with an atomic concentration of $N = 1 \times 10^{21}$ m^{-3} should be so dense that the local field effect, which results from the dipole–dipole interaction between neighbouring atoms, must be taken into consideration. In the following we first examine the atomic electric and magnetic polarizabilities and then consider the effect of local field correction on the electric and magnetic susceptibilities (and hence on permittivity and permeability) of the coherent vapour medium. With the formula of the atomic electric polarization $\gamma_{\rm e} = 2\boldsymbol{d}_{21}\rho_{12}/\varepsilon_0 E_{\rm P}$, where $E_{\rm P} = \hbar\Omega_{\rm p}/\boldsymbol{d}_{21}$ one can arrive at

$$\gamma_{\rm e} = \frac{2\boldsymbol{d}_{21}^2\rho_{12}}{\varepsilon_0\hbar\Omega_{\rm p}}. \tag{2}$$

In the same way, we use the formula of atomic magnetic polarization $\gamma_m = 2\mu_0\mu_{41}\rho_{14}B_{\rm P}$[32] and the relation between the microscopic local electric and magnetic fields $E_{\rm P}/B_{\rm P} = c$ to obtain the explicit expression for the atomic magnetic polarizability as

$$\gamma_{\rm m} = \frac{2c\mu_0\mu_{41}\boldsymbol{d}_{21}\rho_{14}}{\hbar\Omega_{\rm p}}, \tag{3}$$

where μ_0 is the permeability of vacuum and c is the speed of light in vacuum. In that case, we have obtained the microscopic physical quantity $\gamma_{\rm e,m}$. In order to achieve a significant magnetic response, it should be noted that the transition frequency between levels $|4\rangle$–$|3\rangle$ should be approximately equal to the frequency of the probe light.

However, what we are interested in is the macroscopic physical quantities such as the electric and the magnetic susceptibilities, i.e. electric permittivity and magnetic permeability. The Clausius–Mossotti relation between electric and magnetic susceptibilities can reveal the connection between the macroscopic and microscopic quantities. According to the Clausius–Mossotti relation,[32] one can obtain the electric susceptibility of the atomic vapour medium

$$\chi_{\rm e} = N\gamma_{\rm e}\cdot\left(1 - \frac{N\gamma_{\rm e}}{3}\right)^{-1}.$$

The relative electric permittivity of the atomic medium is

$$\varepsilon_{\rm r} = 1 + \chi_{\rm e} = \frac{1 + 2/3N\gamma_{\rm e}}{1 - 1/3N\gamma_{\rm e}}. \tag{4}$$

Meanwhile, the magnetic Clausius–Mossotti relation[33]

$$\gamma_{\rm m} = \frac{1}{N}\left(\frac{\mu_{\rm r} - 1}{\dfrac{2}{3} + \dfrac{\mu_{\rm r}}{3}}\right)$$

shows the connection between the macroscopic magnetic permeability $\mu_{\rm r}$ and the microscopic magnetic polarizations $\gamma_{\rm m}$. It followes that the relative magnetic permeability of the atomic vapour medium is

$$\mu_{\rm r} = \frac{1 + 2/3N\gamma_{\rm m}}{1 - 1/3N\gamma_{\rm m}}. \tag{5}$$

In left-handed materials, the expression of medium refractive index is defined as[1]

$$n = -\sqrt{\varepsilon_{\rm r}\mu_{\rm r}}. \tag{6}$$

The expression of the absorption coefficient A in LHM is[34]

$$A = 2\pi\,{\rm Im}(-\sqrt{\varepsilon_{\rm r}\mu_{\rm r}}). \tag{7}$$

From the above-mentioned derivations, we can obtain the expressions for the electric permittivity and magnetic permeability of the coherent atomic vapour medium. In the following section, we will demonstrate that under the appropriate conditions, the permittivity and the permeability of the four-level coherent atomic vapour medium can be simultaneously negative in certain probe frequency ranges.

4. Numerical results and discussion

In numerical analysis, for simplicity, all the parameters are scaled by γ and we assume $\gamma_1 = 25\gamma$, $\gamma_3 = \gamma_4 = 10\gamma$, $\Delta_3 = 0$, $\Omega_{\rm c} = 50\gamma$, $\Omega_{\rm p} = 0.01\gamma$, $\Omega_{\rm d} = 25\gamma$, and $\gamma = 100$ MHz.

Figure 2 shows the dependences of the real parts of relative dielectric permittivity and the magnetic permeability on probe field detuning. It can be seen from Fig. 2(a) that the relative dielectric permittivity has a negative real part in the probe frequency detuning range $[-15\gamma, -3.2\gamma]$. It can also be seen from Fig. 2(b) that the real part of relative magnetic permeability is negative in the range $[-9.5\gamma, 15\gamma]$. So, the four-level coherent atomic vapour under consideration can exhibit simultaneously negative permittivity and permeability in the range $[-9.5\gamma, -3.2\gamma]$. Medium shows left-handed effect at the moment.

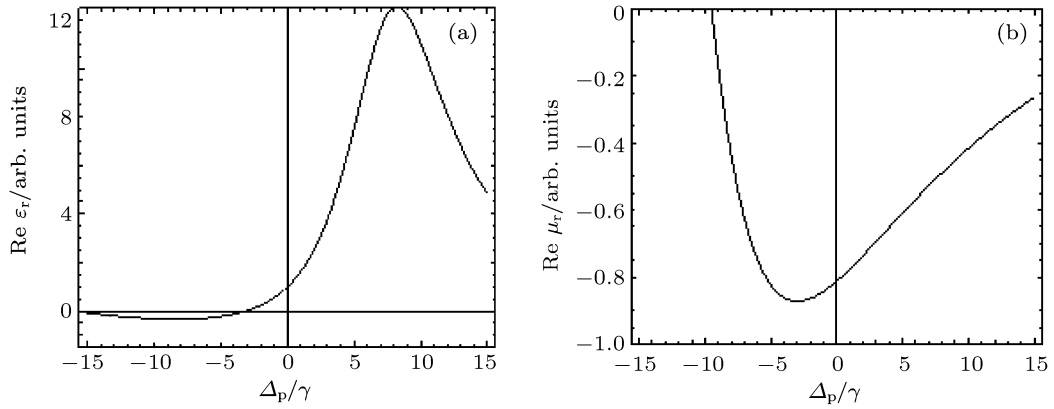


Fig. 2. Real parts of the relative permittivity ε_r (a) and relative permeability μ_r (b) versus the detuning of the probe field Δ_p.

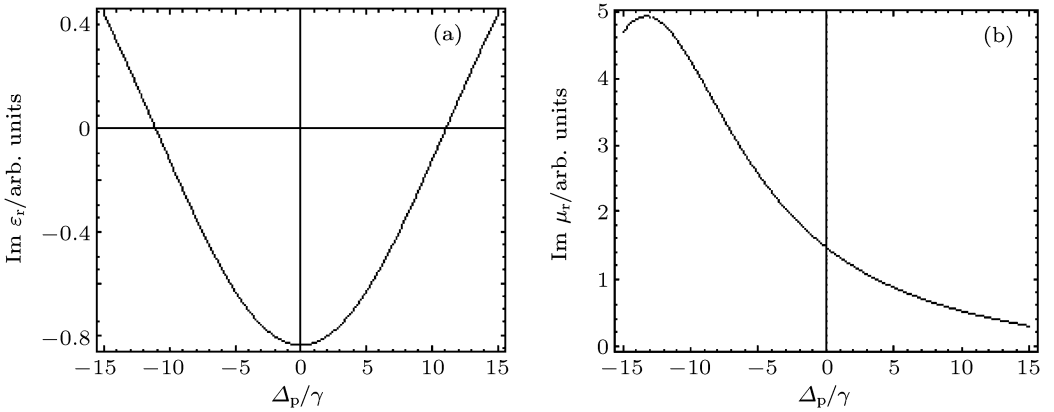


Fig. 3. Imaginary parts of the relative permittivity ε_r (a) and relative permeability μ_r (b) versus the detuning of the probe field Δ_p.

Figure 3 shows the dependences of the imaginary parts of relative dielectric permittivity and magnetic permeability on probe field detuning. An absorption peak appears in the resonance region (see Fig. 3(a)).

Figure 4(a) shows that the medium absorption coefficient is negative in the probe frequency detuning range and it can be found that the absorption is small in a certain range near the resonance region, especially for the case of $\Delta_p = -4.9\gamma$, where the absorption is zero. In our scheme, the absorption is reduced, even to zero. The main applied limitation of the left-handed materials is the large amounts of dissipation and absorption.[31] Particularly, the resolution of a perfect lens[4] obviously decreases because of the absorption. Maybe our scheme has potential applications in high resolution imaging and beam refocusing.

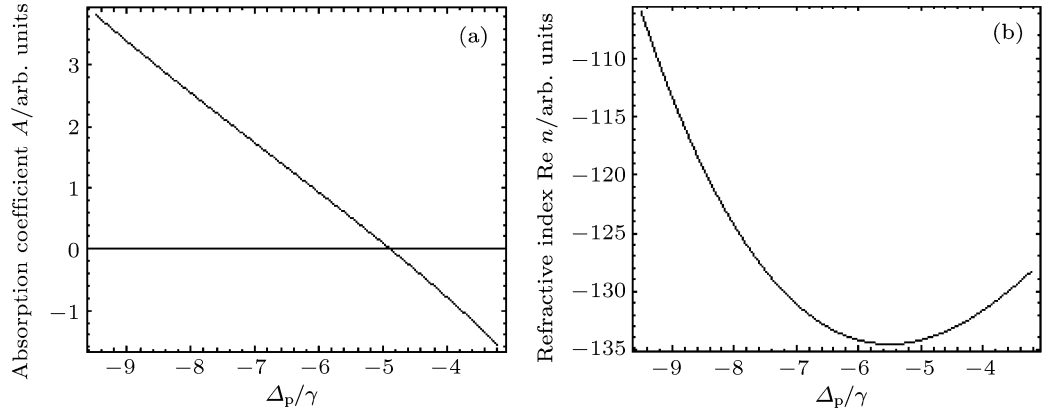


Fig. 4. Refractive index (a) and absorption coefficient (b) versus the detuning of the probe field Δ_p (all parameters are the same as those in Fig. 2).

Figure 4(b) shows the dispersion properties of the vapour medium. It exhibits different dispersive properties on its two sides, namely, normal dispersion in the range $[-5.4\gamma, -3.2\gamma]$ and anomalous dispersion $[-9.5\gamma, -5.4\gamma]$ of the probe frequency detuning. According to the group velocity definition $v_{\rm g} = c/\,{\rm Re}[(n+\omega {\rm d}n/{\rm d}\omega)]$,[35] the superluminal propagation will occur in $[-9.5\gamma, -5.4\gamma]$. Therefore, maybe we can manipulate the probe beam propagation from superluminal to subluminal or *vice versa* in this left-handed material.

5. Conclusion

A quantum system with an interaction between a closed V-type four-level dense atomic vapour and multi-mode light fields is adopted, which is made to possess left handedness by the technique of quantum coherence. The negative permittivity and the negative permeability of the medium can be achieved simultaneously in a wider frequency band under the appropriate parameter conditions than that in Ref. [27]. We also discussed how the medium affects light absorption and gain. The gain property of the LHM may be a scheme to solve the main applied limitation to the LHM because of the dissipation and absorption, especially in a perfect lens.[7] It may have potential applications in improving perfect lens resolution,[7,36] beam focusing,[37] and so on. Furthermore, by analysing the dispersion property of the LHM, we can also manipulate the probe beam propagation from superluminal to subluminal or *vice versa* in this left-handed material.

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