Novel delay-dependent stability criteria for neural networks with interval time-varying delay*

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The problem of delay-dependent asymptotic stability for neural networks with interval time-varying delay is investigated. Based on the idea of delay decomposition method, a new type of Lyapunov–Krasovskii functional is constructed. Several novel delay-dependent stability criteria are presented in terms of linear matrix inequality by using the Jensen integral inequality and a new convex combination technique. Numerical examples are given to demonstrate that the proposed method is effective and less conservative.

Keywords: neural networks, interval time-varying delay, delay-dependent stability, convex combination, linear matrix inequality

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1. Introduction

During the past decades, neural networks (NNs) have been extensively investigated because of their successful applications in various fields such as signal processing, pattern recognition, image processing, associative memory, and combinatorial optimization.[1] However, integration and communication delays are inevitably encountered in the implementation of artificial NNs, which may lead to instability and oscillation. Thus, the issue of stability analysis of NNs with time-delay has been received much attention in recent years. By employing the Lyapunov–Kraskovskii (LK) stability theorem, the derived stability criteria for delayed neural networks (DNNs) can be classified into two categories, namely, delay-independent ones[2−5] and delay-dependent ones.[6−28]

As is well known, since the delay information is contained in the delay-dependent condition, great efforts have been devoted to the delay-dependent stability analysis for DNNs. The main aim of delay-dependent criteria is to achieve a maximum admissible upper bound (MAUB) of the time delay such that the DNNs are asymptotically stable for any delay less than the MAUB. The larger the MAUB of the time delay, the less conservative the delay-dependent stability criterion will be.[22]

In Ref. [11], the delay-dependent stability criteria for DNNs were derived by introducing a new type of Lyapunov–Krasovskii functional (LKF). Based on a Newton–Leibniz formula, a new stability condition was developed in Ref. [12]. In Ref. [13], a novel delay-dependent stability criterion for DNNs was obtained by using the free weighting matrix method. The result in Ref. [13] was further improved in Ref. [14] by considering some useful terms which were ignored in Ref. [13] when estimating the upper bound on the derivative of the LKF. With the construction of a new LKF and some advanced techniques, a less conservative delay-dependent stability criterion was obtained by Shao.[15] In order to reduce the conservatism, some augmented LKFs or new techniques were proposed in Refs. [16]–[19]. Recently, some new LKFs based on the delay decomposition method were constructed to investigate the stability of NNs with delay,[20−24] and some less conservative delay-dependent stability criteria were derived.

However, in most of the existing results, the range of time-varying delay for DNNs is always assumed to be from 0 to an upper bound. Such a restriction is very conservative. In practice, the time-varying delay may be an interval delay, which implies that the lower bound of the delay is not restricted to being 0. In this case, the stability conditions for neural network with time-varying delay in Refs. [6]–[24] are not applicable because they do not take into account the information

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about the lower bound of delay. Therefore, it is of great importance to investigate the delay-dependent stability criteria of DNNs containing a nonzero lower bound of interval time-varying delay.

In Ref. [25], when estimating the upper bound of the derivative of a new type of LKF, the relationship between the time-varying delay and its lower and upper bounds was taken into account, and some delay-interval-dependent stability criteria for delayed neural networks were presented. In Ref. [26], a new type of LKF was introduced, and some new delay-dependent stability criteria were proposed by using the free-weighting matrix method. In Ref. [27], by constructing an augmented new type of LKF which contains some triple-integral terms, several improved delay-dependent stability conditions were derived. However, these mentioned results are still conservative to some extent, which leaves room for further improvement.

In this paper, the delay-dependent asymptotic stability for NNs with interval time-varying delay is investigated. In order to reduce the conservatism, based on the delay decomposition method, a new type of LKF is proposed. By employing the Jensen integral inequality and a new convex combination technique, some less conservative delay-dependent stability criteria are derived in terms of linear matrix inequality (LMI). Numerical examples are given to demonstrate the effectiveness of the presented method and to show that this method is less conservative than some existing ones.

Throughout this paper, the notion $\boldsymbol{P} > 0$ ($\boldsymbol{P} \geq 0$) means that $\boldsymbol{P}$ is symmetric and positive-definite (positive semi-definite); the superscript "T" represents the transpose; "$\boldsymbol{I}$" and "0" denote the identity and zero matrix with appropriate dimensions; $\mathrm{diag}\{l_1, l_2, \ldots, l_n\}$ refers to a block diagonal matrix; the symmetric terms in a symmetric matrix are denoted by $*$.

2. Problem formulation

Consider the following NNs with interval time-varying delay:

$$\dot{\boldsymbol{z}}(t) = -\boldsymbol{C}\boldsymbol{z}(t) + \boldsymbol{A}f(z(t)) + \boldsymbol{B}f(z(t - d(t))), \quad (1)$$

where $\boldsymbol{z}(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^{\mathrm{T}}$ is the neuron state vector, $f(z(t)) = [f_1(z_1(t)),\ f_2(z_2(t)), \ldots, f_n(z_n(t))]^{\mathrm{T}}$ denotes the neural activation function, $\boldsymbol{C} = \mathrm{diag}\{c_1, c_2, \ldots, c_n\}$ is a constant positive diagonal matrix, $\boldsymbol{A}$ and $\boldsymbol{B}$ are the connection weight matrix and the delayed connection weight matrix, respectively, $d(t)$ is a time-varying function satisfying

$$h_1 \leq d(t) \leq h_2 \quad (2)$$

and

$$\dot{d}(t) \leq \mu \quad (3)$$

with $h_2 > h_1 \geq 0$ and μ being a constant. It is assumed that each neuron activation function $f_i(\cdot)$ is bounded and satisfies the following condition:

$$0 \leq \frac{f_i(z_i)}{z_i} \leq k_i, \quad f_i(0) = 0, \quad \forall z_i \neq 0, \quad i = 1, 2, \ldots, n, \quad (4)$$

where k_i are positive constants.

Without loss of generality, here we assume DNNs (1) to have a zero equilibrium point. After the transformation of the equilibrium point, error system (5) in Ref. [27] is equivalent to the given model (1) in this paper. Furthermore, the main results in Ref. [27] are based on system (5). Thus in this paper, we only give the system (1) and continue to investigate the stability problem directly. How to make the transformation of equilibrium point is omitted here, which is very simple and can be found in Ref. [27] and so on.

In addition, we need the following Jensen integral inequality which plays an important role in the derivation of the main results.

Lemma 1[29] For any positive symmetric constant matrix $\boldsymbol{M} > 0$, there exist scalars r_1 and r_2 satisfying $r_2 > r_1$ and a vector function $\omega : [r_1, r_2] \to R^n$ such that the integrations concerned are well defined, then

$$-(r_2 - r_1)\int_{t-r_2}^{t-r_1} \boldsymbol{\omega}^{\mathrm{T}}(s)\boldsymbol{M}\boldsymbol{\omega}(s)\mathrm{d}s \leq -\left(\int_{t-r_2}^{t-r_1} \boldsymbol{\omega}(s)\mathrm{d}s\right)^{\mathrm{T}} \boldsymbol{M}\left(\int_{t-r_2}^{t-r_1} \boldsymbol{\omega}(s)\mathrm{d}s\right). \quad (5)$$

3. Main results

In this section, we continue to study the delay-dependent stability criteria of NNs described by Eq. (1) with time-varying delay satisfying inequalities (2) and (3). First, we introduce a new type of LKF as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \quad (6)$$

where

$$V_1(t) = \boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{z}(t) + \int_{t-d(t)}^{t-h_1} \boldsymbol{z}^{\mathrm{T}}(s)\boldsymbol{R}_1\boldsymbol{z}(s)\mathrm{d}s + 2\lambda_i \sum_{i=1}^{n} \int_0^{z_i} f_i(s)\mathrm{d}s + \int_{t-d(t)}^{t} f^{\mathrm{T}}(\boldsymbol{z}(s))\boldsymbol{R}_2 f(\boldsymbol{z}(s))\mathrm{d}s,$$

$$V_2(t) = \int_{t-\frac{1}{2}h_1}^{t} \begin{bmatrix} \boldsymbol{z}(s) \\ \boldsymbol{z}(s-\frac{1}{2}h_1) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{Q}_1 & \boldsymbol{Q}_2 \\ * & \boldsymbol{Q}_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{z}(s) \\ \boldsymbol{z}\left(s-\frac{1}{2}h_1\right) \end{bmatrix} \mathrm{d}s$$
$$+ \int_{t-h_a}^{t-h_1} \begin{bmatrix} \boldsymbol{z}(s) \\ \boldsymbol{z}(s-\frac{1}{2}h_m) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{Q}_4 & \boldsymbol{Q}_5 \\ * & \boldsymbol{Q}_6 \end{bmatrix} \begin{bmatrix} \boldsymbol{z}(s) \\ \boldsymbol{z}(s-\frac{1}{2}h_m) \end{bmatrix} \mathrm{d}s$$
$$+ \int_{t-\frac{1}{2}h_2}^{t} \begin{bmatrix} \boldsymbol{z}(s) \\ \boldsymbol{z}(s-\frac{1}{2}h_2) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{Q}_7 & \boldsymbol{Q}_8 \\ * & \boldsymbol{Q}_9 \end{bmatrix} \begin{bmatrix} \boldsymbol{z}(s) \\ \boldsymbol{z}(s-\frac{1}{2}h_2) \end{bmatrix} \mathrm{d}s,$$

$$V_3(t) = \frac{1}{2}h_1 \int_{-\frac{1}{2}h_1}^{0} \int_{t+\theta}^{t} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_1\dot{\boldsymbol{z}}(s)\mathrm{d}s\mathrm{d}\theta + \frac{1}{2}h_1 \int_{-h_1}^{-h_1/2} \int_{t+\theta}^{t} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_2\dot{\boldsymbol{z}}(s)\mathrm{d}s\mathrm{d}\theta$$
$$+ \frac{1}{2}h_m \int_{-h_a}^{-h_1} \int_{t+\theta}^{t} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_3\dot{\boldsymbol{z}}(s)\mathrm{d}s\mathrm{d}\theta + \frac{1}{2}h_m \int_{-h_2}^{-h_a} \int_{t+\theta}^{t} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_4\dot{\boldsymbol{z}}(s)\mathrm{d}s\mathrm{d}\theta$$
$$+ \frac{1}{2}h_2 \int_{-h_2/2}^{0} \int_{t+\theta}^{t} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_5\dot{\boldsymbol{z}}(s)\mathrm{d}s\mathrm{d}\theta + \frac{1}{2}h_2 \int_{-h_2}^{-h_2/2} \int_{t+\theta}^{t} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_6\dot{\boldsymbol{z}}(s)\mathrm{d}s\mathrm{d}\theta,$$

with $\boldsymbol{P} > 0$, $\boldsymbol{R}_1 > 0$, $\boldsymbol{R}_2 > 0$, $\boldsymbol{Z}_i > 0$ $(i = 1, 2, \ldots, 6)$,

$$\begin{bmatrix} \boldsymbol{Q}_1 & \boldsymbol{Q}_2 \\ * & \boldsymbol{Q}_3 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \boldsymbol{Q}_4 & \boldsymbol{Q}_5 \\ * & \boldsymbol{Q}_6 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \boldsymbol{Q}_7 & \boldsymbol{Q}_8 \\ * & \boldsymbol{Q}_9 \end{bmatrix} \geq 0$$

of appropriate dimensions, $\lambda_i \geq 0$ $(i = 1, \ldots, n)$, $h_a = (h_1 + h_2)/2$, and $h_m = h_2 - h_1$.

Remark 1 It is well known that it is difficult to derive a less conservative result by employing the identical LKF and the delay decomposition method is useful for the reduction of conservatism. For the purpose of saving the time consumed and simplifying the expressions, a new LKF in the form of expression (6) is constructed, where the intervals $[-h_1,\ 0]$, $[-h_1,\ -h_2]$, and $[-h_2,\ 0]$ are divided into two divisions, respectively. It is obviously that LKF (6) can make sufficient use of not only the information about the lower and upper bound of the delay but also some additional delayed state information, e.g., $\boldsymbol{z}(t-\frac{1}{2}h_1)$, $\boldsymbol{z}(t-h_a)$, and $\boldsymbol{z}(t-\frac{1}{2}h_2)$. At the same time, the information about the lower bound of the delay was used in LKF (6) by introducing the integral terms $\int_{t-d(t)}^{t-h_1} \boldsymbol{z}^{\mathrm{T}}(s)\boldsymbol{R}_1\boldsymbol{z}(s)\mathrm{d}s$. As discussed in Ref. [27], the lower bound of the delay used effectively is helpful to reduce the conservatism. Therefore, such an LKF is expected to be effective in the reduction of conservatism.

In the following, some less conservative delay-dependent asymptotic stability criteria are obtained and formulated in the form of LMI.

Theorem 1 For given scalars $h_2 > h_1 > 0$ and μ, system (1) is asymptotically stable for any time-varying delay satisfying inequalities (2) and (3) if there exist matrices $\boldsymbol{P} > 0$, $\boldsymbol{R}_1 > 0$, $\boldsymbol{R}_2 > 0$, $\boldsymbol{Z}_i > 0$ $(i = 1, 2, \ldots, 6)$,

$$\begin{bmatrix} \boldsymbol{Q}_1 & \boldsymbol{Q}_2 \\ * & \boldsymbol{Q}_3 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \boldsymbol{Q}_4 & \boldsymbol{Q}_5 \\ * & \boldsymbol{Q}_6 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \boldsymbol{Q}_7 & \boldsymbol{Q}_8 \\ * & \boldsymbol{Q}_9 \end{bmatrix} \geq 0,$$

$\boldsymbol{\Lambda} = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\} \geq 0$ and $\boldsymbol{W}_i = \mathrm{diag}\{\boldsymbol{W}_{1i}, \boldsymbol{W}_{2i}, \ldots, \boldsymbol{W}_{ni}\} \geq 0$ $(i = 1, 2)$, with appropriate dimensions such that the following LMIs hold:

$$\boldsymbol{\Sigma}_1 - 3\boldsymbol{\Pi}_1^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_1 - \boldsymbol{\Pi}_2^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_2 < 0, \tag{7}$$
$$\boldsymbol{\Sigma}_1 - \boldsymbol{\Pi}_1^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_1 - 3\boldsymbol{\Pi}_2^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_2 < 0, \tag{8}$$
$$\boldsymbol{\Sigma}_2 - 3\boldsymbol{\Pi}_3^{\mathrm{T}}\boldsymbol{Z}_4\boldsymbol{\Pi}_3 - \boldsymbol{\Pi}_4^{\mathrm{T}}\boldsymbol{Z}_4\boldsymbol{\Pi}_4 < 0, \tag{9}$$
$$\boldsymbol{\Sigma}_2 - \boldsymbol{\Pi}_3^{\mathrm{T}}\boldsymbol{Z}_4\boldsymbol{\Pi}_3 - 3\boldsymbol{\Pi}_4^{\mathrm{T}}\boldsymbol{Z}_4\boldsymbol{\Pi}_4 < 0, \tag{10}$$

where

$$\boldsymbol{\Sigma}_1 = \boldsymbol{\Omega} - \boldsymbol{\Gamma}_1^{\mathrm{T}}\boldsymbol{Z}_4\boldsymbol{\Gamma}_1 + \boldsymbol{E}^{\mathrm{T}}\boldsymbol{U}\boldsymbol{E},$$
$$\boldsymbol{\Sigma}_2 = \boldsymbol{\Omega} - \boldsymbol{\Gamma}_2^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Gamma}_2 + \boldsymbol{E}^{\mathrm{T}}\boldsymbol{U}\boldsymbol{E},$$
$$\boldsymbol{\Gamma}_1^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \boldsymbol{I} & 0 & -\boldsymbol{I} \end{bmatrix}^{\mathrm{T}},$$
$$\boldsymbol{\Gamma}_2^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \boldsymbol{I} & -\boldsymbol{I} & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$
$$\boldsymbol{\Pi}_1^{\mathrm{T}} = \begin{bmatrix} 0 & -\boldsymbol{I} & 0 & 0 & 0 & \boldsymbol{I} & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$
$$\boldsymbol{\Pi}_2^{\mathrm{T}} = \begin{bmatrix} 0 & \boldsymbol{I} & 0 & 0 & 0 & 0 & -\boldsymbol{I} & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$
$$\boldsymbol{\Pi}_3^{\mathrm{T}} = \begin{bmatrix} 0 & -\boldsymbol{I} & 0 & 0 & 0 & 0 & \boldsymbol{I} & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$
$$\boldsymbol{\Pi}_4^{\mathrm{T}} = \begin{bmatrix} 0 & \boldsymbol{I} & 0 & 0 & 0 & 0 & 0 & 0 & -\boldsymbol{I} \end{bmatrix}^{\mathrm{T}},$$
$$\boldsymbol{E}^{\mathrm{T}} = \begin{bmatrix} -C & 0 & A & B & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\boldsymbol{U}=\frac{1}{4}(h_1^2\boldsymbol{Z}_1+h_1^2\boldsymbol{Z}_2+h_m^2\boldsymbol{Z}_3+h_m^2\boldsymbol{Z}_4+h_2^2\boldsymbol{Z}_5+h_2^2\boldsymbol{Z}_6),$$

$$\boldsymbol{\Omega}=\begin{bmatrix}
\boldsymbol{\Omega}_{11} & 0 & \boldsymbol{\Omega}_{13} & \boldsymbol{\Omega}_{14} & \boldsymbol{\Omega}_{15} & 0 & 0 & \boldsymbol{\Omega}_{18} & 0\\
* & \boldsymbol{\Omega}_{22} & 0 & \boldsymbol{\Omega}_{24} & 0 & 0 & 0 & 0 & 0\\
* & * & \boldsymbol{\Omega}_{33} & \boldsymbol{\Omega}_{34} & 0 & 0 & 0 & 0 & 0\\
* & * & * & \boldsymbol{\Omega}_{44} & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & \boldsymbol{\Omega}_{55} & \boldsymbol{\Omega}_{56} & 0 & 0 & 0\\
* & * & * & * & * & \boldsymbol{\Omega}_{66} & \boldsymbol{\Omega}_{67} & 0 & 0\\
* & * & * & * & * & * & \boldsymbol{\Omega}_{77} & 0 & \boldsymbol{\Omega}_{79}\\
* & * & * & * & * & * & * & \boldsymbol{\Omega}_{88} & \boldsymbol{\Omega}_{89}\\
* & * & * & * & * & * & * & * & \boldsymbol{\Omega}_{99}
\end{bmatrix},$$

$$\begin{aligned}
&\boldsymbol{\Omega}_{11}=-\boldsymbol{P}\boldsymbol{C}-\boldsymbol{C}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{Q}_1+\boldsymbol{Q}_7-\boldsymbol{Z}_1-\boldsymbol{Z}_5,\\
&\boldsymbol{\Omega}_{13}=K\boldsymbol{W}_1+\boldsymbol{P}\boldsymbol{A}-\boldsymbol{C}^{\mathrm{T}}\boldsymbol{\Lambda},\quad \boldsymbol{\Omega}_{14}=\boldsymbol{P}\boldsymbol{B},\\
&\boldsymbol{\Omega}_{15}=\boldsymbol{Q}_2+\boldsymbol{Z}_1,\quad \boldsymbol{\Omega}_{18}=\boldsymbol{Q}_8+\boldsymbol{Z}_5,\\
&\boldsymbol{\Omega}_{22}=-(1-\mu)\boldsymbol{R}_1,\quad \boldsymbol{\Omega}_{24}=\boldsymbol{K}\boldsymbol{W}_2,\\
&\boldsymbol{\Omega}_{33}=-2\boldsymbol{W}_1+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{\Lambda}+\boldsymbol{\Lambda}A+\boldsymbol{R}_2,\\
&\boldsymbol{\Omega}_{34}=\boldsymbol{\Lambda}\boldsymbol{B},\quad \boldsymbol{\Omega}_{44}=-(1-\mu)\boldsymbol{R}_2-2\boldsymbol{W}_2,\\
&\boldsymbol{\Omega}_{55}=\boldsymbol{Q}_3-\boldsymbol{Q}_1-\boldsymbol{Z}_1-\boldsymbol{Z}_2,\quad \boldsymbol{\Omega}_{56}=-\boldsymbol{Q}_2+\boldsymbol{Z}_2,\\
&\boldsymbol{\Omega}_{66}=\boldsymbol{R}_1-\boldsymbol{Q}_3+\boldsymbol{Q}_4-\boldsymbol{Z}_2,\\
&\boldsymbol{\Omega}_{67}=\boldsymbol{Q}_5,\quad \boldsymbol{\Omega}_{77}=-\boldsymbol{Q}_4+\boldsymbol{Q}_6,\quad \boldsymbol{\Omega}_{79}=-\boldsymbol{Q}_5,\\
&\boldsymbol{\Omega}_{88}=\boldsymbol{Q}_9-\boldsymbol{Q}_7-\boldsymbol{Z}_5-\boldsymbol{Z}_6,\\
&\boldsymbol{\Omega}_{89}=-\boldsymbol{Q}_8+\boldsymbol{Z}_6,\quad \boldsymbol{\Omega}_{99}=-\boldsymbol{Q}_6-\boldsymbol{Q}_9-\boldsymbol{Z}_6,\\
&\boldsymbol{K}=\mathrm{diag}\{k_1,k_2,\ldots,k_n\}.
\end{aligned}$$

Proof Taking the time derivative of $V(t)$ in Eq. (6) along the trajectories of Eq. (1) yields

$$\dot{V}(t)=\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t), \tag{11}$$

where

$$\begin{aligned}
\dot{V}_1(t)\le{}& 2\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{z}}(t)+z^{\mathrm{T}}(t-h_1)\boldsymbol{R}_1\boldsymbol{z}(t-h_1)-(1-\mu)\boldsymbol{z}^{\mathrm{T}}(t-d(t))\boldsymbol{R}_1\boldsymbol{z}(t-d(t))\\
&+2f^{\mathrm{T}}(\boldsymbol{z}(t))\Lambda\dot{\boldsymbol{z}}(t)+f^{\mathrm{T}}(\boldsymbol{z}(t))\boldsymbol{R}_2f(\boldsymbol{z}(t))-(1-\mu)f^{\mathrm{T}}(\boldsymbol{z}(t-d(t)))\boldsymbol{R}_2f(\boldsymbol{z}(t-d(t))),\\
\dot{V}_2(t)={}&\begin{bmatrix}\boldsymbol{z}(t)\\ \boldsymbol{z}(t-\frac{1}{2}h_1)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{Q}_1 & \boldsymbol{Q}_2\\ * & \boldsymbol{Q}_3\end{bmatrix}\begin{bmatrix}\boldsymbol{z}(t)\\ \boldsymbol{z}(t-\frac{1}{2}h_1)\end{bmatrix}-\begin{bmatrix}\boldsymbol{z}(t-\frac{1}{2}h_1)\\ \boldsymbol{z}(t-h_1)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{Q}_1 & \boldsymbol{Q}_2\\ * & \boldsymbol{Q}_3\end{bmatrix}\begin{bmatrix}\boldsymbol{z}(t-\frac{1}{2}h_1)\\ \boldsymbol{z}(t-h_1)\end{bmatrix}\\
&+\begin{bmatrix}\boldsymbol{z}(t-h_1)\\ \boldsymbol{z}(t-h_a)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{Q}_4 & \boldsymbol{Q}_5\\ * & \boldsymbol{Q}_6\end{bmatrix}\begin{bmatrix}\boldsymbol{z}(t-h_1)\\ \boldsymbol{z}(t-h_a)\end{bmatrix}-\begin{bmatrix}\boldsymbol{z}(t-h_a)\\ \boldsymbol{z}(t-h_2)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{Q}_4 & \boldsymbol{Q}_5\\ * & \boldsymbol{Q}_6\end{bmatrix}\begin{bmatrix}\boldsymbol{z}(t-h_a)\\ \boldsymbol{z}(t-h_2)\end{bmatrix}\\
&+\begin{bmatrix}\boldsymbol{z}(t)\\ \boldsymbol{z}(t-\frac{1}{2}h_2)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{Q}_7 & \boldsymbol{Q}_8\\ * & \boldsymbol{Q}_9\end{bmatrix}\begin{bmatrix}\boldsymbol{z}(t)\\ \boldsymbol{z}(t-\frac{1}{2}h_1)\end{bmatrix}-\begin{bmatrix}\boldsymbol{z}(t-\frac{1}{2}h_2)\\ \boldsymbol{z}(t-h_2)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{Q}_7 & \boldsymbol{Q}_8\\ * & \boldsymbol{Q}_9\end{bmatrix}\begin{bmatrix}\boldsymbol{z}(t-\frac{1}{2}h_2)\\ \boldsymbol{z}(t-h_2)\end{bmatrix},\\
\dot{V}_3(t)={}&\dot{\boldsymbol{z}}^{\mathrm{T}}(t)U\dot{\boldsymbol{z}}(t)-\frac{1}{2}h_1\int_{t-\frac{1}{2}h_1}^{t}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_1\dot{\boldsymbol{z}}(s)\mathrm{d}s-\frac{1}{2}h_1\int_{t-h_1}^{t-h_1/2}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_2\dot{\boldsymbol{z}}(s)\mathrm{d}s-\frac{1}{2}h_m\int_{t-h_a}^{t-h_1}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_3\dot{\boldsymbol{z}}(s)\mathrm{d}s\\
&-\frac{1}{2}h_m\int_{t-h_2}^{t-h_a}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_4\dot{\boldsymbol{z}}(s)\mathrm{d}s-\frac{1}{2}h_2\int_{t-\frac{1}{2}h_2}^{t}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_5\dot{\boldsymbol{z}}(s)\mathrm{d}s-\frac{1}{2}h_2\int_{t-h_2}^{t-\frac{1}{2}h_2}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_6\dot{\boldsymbol{z}}(s)\mathrm{d}s.
\end{aligned}$$

Using Lemma 1, the following inequalities hold:

$$-\frac{1}{2}h_1\int_{t-h_1/}^{t}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_1\dot{\boldsymbol{z}}(s)\mathrm{d}s\le-\left(\boldsymbol{z}(t)-\boldsymbol{z}\left(t-\frac{1}{2}h_1\right)\right)^{\mathrm{T}}\boldsymbol{Z}_1\left(\boldsymbol{z}(t)-\boldsymbol{z}\left(t-\frac{1}{2}h_1\right)\right), \tag{12}$$

$$-\frac{1}{2}h_1\int_{t-h_1}^{t-h_1/2}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_2\dot{\boldsymbol{z}}(s)\mathrm{d}s\le-\left(\boldsymbol{z}\left(t-\frac{1}{2}h_1\right)-\boldsymbol{z}(t-h_1)\right)^{\mathrm{T}}\boldsymbol{Z}_2\left(\boldsymbol{z}\left(t-\frac{1}{2}h_1\right)-\boldsymbol{z}(t-h_1)\right), \tag{13}$$

$$-\frac{1}{2}h_2\int_{t-h_2/2}^{t}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_1\dot{\boldsymbol{z}}(s)\mathrm{d}s\le-\left(\boldsymbol{z}(t)-\boldsymbol{z}\left(t-\frac{1}{2}h_2\right)\right)^{\mathrm{T}}\boldsymbol{Z}_5\left(\boldsymbol{z}(t)-\boldsymbol{z}\left(t-\frac{1}{2}h_2\right)\right), \tag{14}$$

$$-\frac{1}{2}h_1\int_{t-h_2}^{t-h_2/2}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_6\dot{\boldsymbol{z}}(s)\mathrm{d}s\le-\left(\boldsymbol{z}\left(t-\frac{1}{2}h_2\right)-\boldsymbol{z}(t-h_2)\right)^{\mathrm{T}}\boldsymbol{Z}_6\left(\boldsymbol{z}\left(t-\frac{1}{2}h_2\right)-\boldsymbol{z}(t-h_2)\right). \quad (15)$$

Depending on whether the time-varying delay $d(t)$ belongs to the interval $h_1\le d(t)\le h_a$ or $h_a\le d(t)\le h_2$, we will prove the result in two cases.

Case 1 $h_1\le d(t)\le h_a$.

Using Lemma 1, one can obtain

$$\begin{aligned}
&-\frac{1}{2}h_m\int_{t-h_2}^{t-h_a}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_4\dot{\boldsymbol{z}}(s)\mathrm{d}s\\
&\le-(\boldsymbol{z}(t-h_a)-\boldsymbol{z}(t-h_2))^{\mathrm{T}}\\
&\quad\times\boldsymbol{Z}_4(\boldsymbol{z}(t-h_a)-\boldsymbol{z}(t-h_2)), \quad (16)\\
&-\frac{1}{2}h_m\int_{t-h_a}^{t-h_1}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_3\dot{\boldsymbol{z}}(s)\mathrm{d}s\\
&=-(h_a-h_1)\int_{t-d(t)}^{t-h_1}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_3\dot{\boldsymbol{z}}(s)\mathrm{d}s\\
&\quad-(h_a-h_1)\int_{t-h_a}^{t-d(t)}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_3\dot{\boldsymbol{z}}(s)\mathrm{d}s\\
&\le-\frac{(h_a-h_1)}{d(t)-h_1}(\boldsymbol{z}(t-h_1)-\boldsymbol{z}(t-d(t)))^{\mathrm{T}}\\
&\quad\times\boldsymbol{Z}_3(\boldsymbol{z}(t-h_1)-\boldsymbol{z}(t-d(t)))\\
&\quad-\frac{(h_a-h_1)}{h_a-d(t)}(\boldsymbol{z}(t-d(t))-\boldsymbol{z}(t-h_a))^{\mathrm{T}}\\
&\quad\times\boldsymbol{Z}_3(\boldsymbol{z}(t-d(t))-\boldsymbol{z}(t-h_a)). \quad (17)
\end{aligned}$$

In the following, in order to estimate the upper bound of $-\frac{1}{2}h_m\int_{t-h_a}^{t-h_1}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_3\dot{\boldsymbol{z}}(s)\mathrm{d}s$ more tightly, the idea of the delay partition method[30] will be employed. For simplicity, we divide the interval $[h_1,\ h_a]$ into two subintervals, i.e., $[h_1,\ \frac{1}{2}(h_a+h_1)]$ and $[\ \frac{1}{2}(h_a+h_1),\ h_a]$. For the subinterval of $h_1\le d(t)\le\frac{1}{2}(h_a+h_1)$, similar to that in Ref. [30], it is easy to have the following inequality:

$$\frac{h_a-h_1}{d(t)-h_1}+\frac{h_a-h_1}{h_a-d(t)}\ge 4. \quad (18)$$

According to the above inequality, we can obtain

$$\begin{aligned}
&-\frac{1}{2}h_m\int_{t-h_a}^{t-h_1}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_3\dot{\boldsymbol{z}}(s)\mathrm{d}s\\
&\le-(4-\frac{h_a-h_1}{h_a-d(t)})(\boldsymbol{z}(t-h_1)-\boldsymbol{z}(t-d(t)))^{\mathrm{T}}\\
&\quad\times\boldsymbol{Z}_3(\boldsymbol{z}(t-h_1)-\boldsymbol{z}(t-d(t)))\\
&\quad-\frac{h_a-h_1}{h_a-d(t)}(\boldsymbol{z}(t-d(t))-\boldsymbol{z}(t-h_a))^{\mathrm{T}}\\
&\quad\times\boldsymbol{Z}_3(\boldsymbol{z}(t-d(t))-\boldsymbol{z}(t-h_a)). \quad (19)
\end{aligned}$$

From inequality (4), the following inequality holds for any $\boldsymbol{W}_i=\mathrm{diag}\{\boldsymbol{W}_{1i},\boldsymbol{W}_{2i},\ldots,\boldsymbol{W}_{ni}\}\ge 0$ $(i=1,2)$:

$$\begin{aligned}
&2\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{K}\boldsymbol{W}_1f(\boldsymbol{z}(t))-2f^{\mathrm{T}}(\boldsymbol{z}(t))\boldsymbol{W}_1f(\boldsymbol{z}(t))\\
&\quad+2\boldsymbol{z}^{\mathrm{T}}(t-d(t))\boldsymbol{K}\boldsymbol{W}_2f(\boldsymbol{z}(t-d(t)))\\
&\quad-2f^{\mathrm{T}}(\boldsymbol{z}(t-d(t)))\boldsymbol{W}_2f(\boldsymbol{z}(t-d(t)))\ge 0. \quad (20)
\end{aligned}$$

Hence, according to expressions (11)–(16), (19), and (20), we have

$$\begin{aligned}
\dot{V}(t)\le\xi^{\mathrm{T}}(t)(&\boldsymbol{\Sigma}_1-\left(4-\frac{h_a-h_1}{h_a-d(t)}\right)\boldsymbol{\Pi}_1^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_1\\
&-\frac{h_a-h_1}{h_a-d(t)}\boldsymbol{\Pi}_2^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_2)\boldsymbol{\xi}(t), \quad (21)
\end{aligned}$$

where $\xi(t)=[\boldsymbol{z}^{\mathrm{T}}(t)\quad\boldsymbol{z}^{\mathrm{T}}(t-d(t))\quad f^{\mathrm{T}}(\boldsymbol{z}(t))\quad f^{\mathrm{T}}(\boldsymbol{z}(t-d(t)))\quad\boldsymbol{z}^{\mathrm{T}}(t-\frac{1}{2}h_1)\quad\boldsymbol{z}^{\mathrm{T}}(t-h_1)\quad\boldsymbol{z}^{\mathrm{T}}(t-h_a)\quad\boldsymbol{z}^{\mathrm{T}}(t-\frac{1}{2}h_2)\quad\boldsymbol{z}^{\mathrm{T}}(t-h_2)]^{\mathrm{T}}$.

Since

$$\boldsymbol{\Sigma}_1-\left(4-\frac{h_a-h_1}{h_a-d(t)}\right)\boldsymbol{\Pi}_1^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_1-\frac{h_a-h_1}{h_a-d(t)}\boldsymbol{\Pi}_2^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_2$$

is a convex combination of the matrices $\boldsymbol{\Pi}_1^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_1$ and $\boldsymbol{\Pi}_2^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_2$,

$$\begin{aligned}
\boldsymbol{\Sigma}_1\ &-\ \left(4-\frac{h_a-h_1}{h_a-d(t)}\right)\boldsymbol{\Pi}_1^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_1\\
&-\ \frac{h_a-h_1}{h_a-d(t)}\boldsymbol{\Pi}_2^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_2<0
\end{aligned}$$

holds if and only if

$$\boldsymbol{\Sigma}_1-3\boldsymbol{\Pi}_1^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_1-\boldsymbol{\Pi}_2^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_2<0, \quad (22)$$

and

$$\boldsymbol{\Sigma}_1-2\boldsymbol{\Pi}_1^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_1-2\boldsymbol{\Pi}_2^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_2<0. \quad (23)$$

If inequalities (22) and (23) hold, then we have $\dot{V}(t)<0$ and system (1) is asymptotically stable according to LK stability theory.

On the other hand, for the subinterval of $\frac{1}{2}(h_a+h_1)\le d(t)\le h_a$, we have

$$\begin{aligned}
&-\frac{1}{2}h_m\int_{t-h_a}^{t-h_1}\dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_3\dot{\boldsymbol{z}}(s)\mathrm{d}s\\
&\le-\frac{h_a-h_1}{d(t)-h_1}(\boldsymbol{z}(t-h_1)-\boldsymbol{z}(t-d(t)))^{\mathrm{T}}\\
&\quad\times\boldsymbol{Z}_3(\boldsymbol{z}(t-h_1)-\boldsymbol{z}(t-d(t)))\\
&\quad-(4-\frac{h_a-h_1}{d(t)-h_1})(\boldsymbol{z}(t-d(t))-\boldsymbol{z}(t-h_a))^{\mathrm{T}}\\
&\quad\times\boldsymbol{Z}_3(\boldsymbol{z}(t-d(t))-\boldsymbol{z}(t-h_a)). \quad (24)
\end{aligned}$$

Thus, $\dot{V}(t)<0$ holds if inequalities (23) and

$$\boldsymbol{\Sigma}_1-\boldsymbol{\Pi}_1^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_1-3\boldsymbol{\Pi}_2^{\mathrm{T}}\boldsymbol{Z}_3\boldsymbol{\Pi}_2<0 \quad (25)$$

are satisfied. Clearly, inequality (23) holds if inequalities (22) and (25) are true. In addition, notice that inequality (22) is equivalent to inequality (7) and inequality (25) is equivalent to inequality (8). So $\dot{V}(t) < 0$ holds for $h_1 \leq d(t) \leq h_a$ when inequalities (7) and (8) are satisfied.

Case 2 $h_a \leq d(t) \leq h_2$.

For this case, the treatment process is similar to that in Case 1. It can be proved that $\dot{V}(t) < 0$ holds if and only if

$$\boldsymbol{\Sigma}_2 - 3\boldsymbol{\Pi}_3^{\mathrm{T}}\boldsymbol{Z}_4\boldsymbol{\Pi}_3 - \boldsymbol{\Pi}_4^{\mathrm{T}}\boldsymbol{Z}_4\boldsymbol{\Pi}_4 < 0, \tag{26}$$

and

$$\boldsymbol{\Sigma}_2 - \boldsymbol{\Pi}_3^{\mathrm{T}}\boldsymbol{Z}_4\boldsymbol{\Pi}_3 - 3\boldsymbol{\Pi}_4^{\mathrm{T}}\boldsymbol{Z}_4\boldsymbol{\Pi}_4 < 0. \tag{27}$$

And also, notice that inequality (26) is equivalent to inequality (9) and inequality (27) is equivalent to inequality (10). So, $\dot{V}(t) < 0$ holds for $h_a \leq d(t) \leq h_2$ when equalities (9) and (10) are satisfied.

Thus, the proof is completed.

Remark 2 In Refs. [26] and [27], the inequality

$$-\frac{h_2 - h_1}{d(t) - h_1} - \frac{h_2 - h_1}{h_2 - d(t)} \leq -3$$

was actually utilized when $h_1 \leq d(t) \leq h_2$. However, while estimating the upper bound of $-\frac{1}{2}h_m \int_{t-h_a}^{t-h_1} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_3\dot{\boldsymbol{z}}(s)\mathrm{d}s$, the delay partition method and a new inequality

$$-\frac{h_a - h_1}{d(t) - h_1} - \frac{h_a - h_1}{h_a - d(t)} \leq -4$$

were used when $h_1 \leq d(t) \leq h_a$. To this point, the upper bound of $\dot{V}(t)$ can be estimated more tightly. Then, combining a new convex combination technique, several novel stability conditions are presented in terms of solutions to a set of LMIs. The obtained conditions are less conservative than existing results, which will be illustrated through some numerical examples.

Remark 3 The method proposed in this paper involves neither any free weighting matrices nor any useful terms that are neglected when estimating the cross-product terms, so it has much fewer decision variables, thereby reducing the complexity in computational demand.

In fact, Theorem 1 provides the criterion for DNNs with a differentiable function $d(t)$ satisfying $\dot{d}(t) < \mu$ and $h_2 > h_1 > 0$, where μ is a given constant. If the time-varying delay $d(t)$ is not differentiable or μ is unknown, by choosing $\boldsymbol{R}_1 = 0$ and $\boldsymbol{R}_2 = 0$, a rate-independent criterion can be derived from Theorem 1.

Corollary 1 For given scalars $h_2 > h_1 > 0$, system (1) is asymptotically stable for any time-varying delay satisfying inequality (2) if there exist matrices $\boldsymbol{P} > 0$, $\boldsymbol{Z}_i > 0$ $(i = 1, 2, \ldots, 6)$,

$$\begin{bmatrix} \boldsymbol{Q}_1 & \boldsymbol{Q}_2 \\ * & \boldsymbol{Q}_3 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \boldsymbol{Q}_4 & \boldsymbol{Q}_5 \\ * & \boldsymbol{Q}_6 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \boldsymbol{Q}_7 & \boldsymbol{Q}_8 \\ * & \boldsymbol{Q}_9 \end{bmatrix} \geq 0,$$

$\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\} \geq 0$, and $\boldsymbol{W}_i = \mathrm{diag}\{\boldsymbol{W}_{1i}, \boldsymbol{W}_{2i}, \ldots, \boldsymbol{W}_{ni}\} \geq 0$ $(i = 1, 2)$ with appropriate dimensions such that the following LMIs (7)–(10) hold with $\boldsymbol{R}_1 = 0$ and $\boldsymbol{R}_2 = 0$.

Remark 4 When $h_1 = 0$ and $h_2 > 0$, the interval time-varying delay reduces to routine delay, the corresponding LKF can be chosen as follows:

$$\begin{aligned} V(t) = {} & \boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{z}(t) + \int_{t-d(t)}^{t} \boldsymbol{z}^{\mathrm{T}}(s)\boldsymbol{R}_1\boldsymbol{z}(s)\mathrm{d}s \\ & + 2\lambda_i \sum_{i=1}^{n} \int_0^{z_i} f_i(s)\mathrm{d}s \\ & + \int_{t-d(t)}^{t} f^{\mathrm{T}}(\boldsymbol{z}(s))\boldsymbol{R}_2 f(\boldsymbol{z}(s))\mathrm{d}s \\ & + \int_{t-h_2}^{t} \begin{bmatrix} \boldsymbol{z}(s) \\ z(s - \frac{1}{2}h_2) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{Q}_7 & \boldsymbol{Q}_8 \\ * & \boldsymbol{Q}_9 \end{bmatrix} \\ & \times \begin{bmatrix} \boldsymbol{z}(s) \\ z(s - \frac{1}{2}h_2) \end{bmatrix} \mathrm{d}s \\ & + \frac{1}{2}h_2 \int_{-\frac{1}{2}h_2}^{0} \int_{t+\theta}^{t} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_5\dot{\boldsymbol{z}}(s)\mathrm{d}s\mathrm{d}\theta \\ & + \frac{1}{2}h_2 \int_{-h_2}^{-\frac{1}{2}h_2} \int_{t+\theta}^{t} \dot{\boldsymbol{z}}^{\mathrm{T}}(s)\boldsymbol{Z}_6\dot{\boldsymbol{z}}(s)\mathrm{d}s\mathrm{d}\theta. \end{aligned} \tag{28}$$

Similar to the proof of Theorem 1, we can easily derive the result. Due to the limitation of the space, it is omitted here.

4. Numerical examples

In this section, we will give two examples to demonstrate that the proposed delay-dependent stability criteria are effective and less conservative.

Example 1 Consider the DNNs as those in Refs. [25]–[27] with

$$\boldsymbol{C} = \mathrm{diag}\{1.2769, 0.6231, 0.9230, 0.4480\},$$

$$\boldsymbol{A} = \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9634 & -0.5015 \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix},$$

$k_1 = 0.1137, \quad k_2 = 0.1279,$

$k_3 = 0.7994, \quad \text{and} \quad k_4 = 0.2368.$

The objective of this example is to achieve the MAUB of h_2 for given lower bound h_1 and various μ values such that the DNNs are asymptotically stable. The corresponding upper bounds on h_2 are listed in Table 1. According to Table 1, it can be shown that the results obtained in this paper are less conservative than those in Refs. [25]–[27].

Table 1. MAUB of h_2 for various values of h_1 and μ for example 1.

h_1	Methods	$\mu = 0.5$	$\mu = 0.9$	Unknown μ
1	He *et al.*[25]	2.5802	2.2736	2.2393
	Tian *et al.*[26]	2.7299	2.3811	2.3114
	Sun *et al.*[27]	2.6869	2.3924	2.3540
	Our result	3.0015	2.7111	2.7034
2	He *et al.*[25]	2.7500	2.6408	2.6299
	Tian *et al.*[26]	2.8606	2.7273	2.6950
	Sun *et al.*[27]	2.8475	2.7375	2.7190
	Our result	3.1843	3.0952	3.0927

Example 2 Consider the DNNs as those in Refs. [25] and [27] with

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{A} = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1.5 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} -2 & 0.5 \\ 0.5 & -2 \end{bmatrix},$$

$k_1 = 0.4, \quad \text{and} \quad k_2 = 0.8.$

The purpose of this example is to obtain the MAUB of h_2 for various values of h_1 and μ. The comparisons between Theorem 1 and Corollary 1 in this paper and those in Refs. [25] and [27] are listed in Table 2. This example shows that the method proposed in this paper yields results that are less conservative than those in Refs. [25] and [27].

Table 2. MAUB of h_2 for various values of h_1 and μ for example 2.

h_1	Methods	$\mu = 0.8$	$\mu = 0.9$	Unknown μ
0.5	He *et al.*[25]	1.3566	1.1689	1.0263
	Sun *et al.*[27]	1.3600	1.1786	1.0437
	Our result	1.4882	1.2674	1.1083
0.75	He *et al.*[25]	1.3856	1.2110	1.0803
	Sun *et al.*[27]	1.3900	1.2241	1.0972
	Our result	1.5096	1.3130	1.1976
1	He *et al.*[25]	1.4578	1.2887	1.1641
	Sun *et al.*[27]	1.4692	1.2948	1.1774
	Our result	1.5819	1.4308	1.3309

5. Conclusion

In this paper, we discussed the stability problem of NNs with interval time-varying delay. By employing a novel LKF, some less conservative delay-dependent stability conditions are derived using the Jensen integral inequality and a new convex combination technique. Numerical examples show the improvement over some existing results.

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