Solitary excitations in one-dimensional ferromagnetic spin chains with biquadratic exchange interaction*

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(Received 15 April 2011; revised manuscript received 17 June 2011)

By using the traveling wave method, the solutions of the elliptic function wave and the solitary wave are obtained in a ferromagnetic spin chain with a biquadratic exchange interaction, a single ion anisotropic interaction and an anisotropic nearest neighbour interaction. The effects of the biquadratic exchange interaction and the single ion anisotropic interaction on the properties (width, peak and stability) of the soliton are investigated. It is also found that the effects vary with the strengths of these interactions.

Keywords: biquadratic exchange interaction, elliptic function wave, soliton

PACS: 05.45.Yv, 05.45.–a, 75.10.Hk **DOI:** 10.1088/1674-1056/20/12/120509

1. Introduction

The investigation of one-dimensional (1D) ferromagnetic spin chains which goes back to Bethe[1] has been studied in different aspects: theory, experiment, and numerical simulation. Great improvements have been gained.[2−23] In modern nonlinear science, the field of 1D ferromagnetic systems is wide-ranging and developing rapidly. Over the past several decades, much effort has been made to study the origin of the biquadratic exchange interaction.[24−28] These studies[29−31] have demonstrated that the biquadratic exchange interaction gives essential modifications to the thermodynamic properties of the ferromagnetic chains. However, previous research[32−34] does not take into account the biquadratic interaction. In the present paper, we consider the effects of the biquadratic exchange interaction and the single ion anisotropic interaction on the properties of the soliton in a discrete ferromagnetic chain by the traveling wave method.

In Section 2, the Hamiltonian of the system is introduced and the dynamical equation of the Bose operator is derived. The nonlinear Schrödinger equation for the probability amplitude is established by using the Glauber coherent state and the continuum limit. In Section 3, the nonlinear Schrödinger equation is solved by using the traveling wave method, and solutions of elliptic function wave and solitary wave are obtained. The effects of the biquadratic exchange interaction and the single ion anisotropic interaction on the properties (width, peak and stability) of the soliton are discussed in detail. Section 4 is devoted to the conclusion.

2. Hamiltonian and equation of model

The model is a discrete spin chain, and the Hamiltonian can be described as

$$\begin{aligned} H = & -\frac{J}{2}\sum_{l,\rho}(S_l^+ S_{l+\rho}^- + S_l^- S_{l+\rho}^+) \\ & - J'\sum_{l,\rho} S_l^z S_{l+\rho}^z - D\sum_l (S_l^z)^2 \\ & - K\sum_{l,\rho}(S_l^z)^2 (S_{l+\rho}^z)^2, \end{aligned} \tag{1}$$

where the sums runs over the lattice sites which are separated by lattice space a. The index ρ denotes the nearest neighbour of each lattice. In addition, S_l^δ $(\delta = x, y, z)$ is the δ component of the spin vector on site l, and $S_l^\pm = S_l^x \pm \mathrm{i}S_l^y$.

The first two terms in Hamiltonian (1) represent the nearest neighbour exchange energies. J and J' are exchange integrals and are positive. When $J' \neq J$, the

*Project supported by the National Natural Science Foundation of China (Grant No. 10874049) and the State Key Program for Basic Research of China (Grant No. 2007CB925204).

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ferromagnetic chain is anisotropic; only when $J' = J$, $D = K = 0$, will the ferromagnetic chain be isotropic. The third term is the single ion anisotropy energy that appears because of the crystalline field. D is the single ion anisotropic parameter. The fourth term represents the biquadratic exchange interaction. The parameter K is the strength of the biquadratic exchange interaction in the classical approximation. The necessity to include this term goes back to Anderson[24] who gave its interpretation in terms of a superexchange mechanism. Alder[35] held a discussion about the biquadratic exchange interaction based on the extensive review of experimental results, which proved the importance of this term in a variety of compounds. Kapor and Skrinjar[36] gave another interpretation of the biquadratic exchange interaction in terms of three-spin exchange interaction. In the ferromagnetic ground state, the parameter K must satisfy the following conditions: when $S = 1$, $0 < K < J/2$; and when $S > 1$, $-J/S(2S-3) < K < J(S+1)/S^2$.[37]

For the spin operators, we use the Holstein–Primakoff transformation

$$\begin{aligned} S_l^+ &= (\sqrt{2S - a_l^\dagger a_l})a_l \approx \sqrt{2S}\left(1 - \frac{a_l^\dagger a_l}{4S}\right)a_l, \\ S_l^- &= a_l^\dagger\left(\sqrt{2S - a_l^\dagger a_l}\right) \approx \sqrt{2S}a_l^\dagger\left(1 - \frac{a_l^\dagger a_l}{4S}\right), \\ S_l^z &= S - a_l^\dagger a_l. \end{aligned} \tag{2}$$

Here, the quantity S represents the modulus of the atomic spin at each site, and $a_l^\dagger$ and a_l are the Bose creating operator and the destroying operator on the l-th site, respectively, satisfying the commutation relations

$$[a_l, a_{l'}^\dagger] = \delta_{ll'},\ [a_l, a_{l'}] = [a_l^\dagger, a_{l'}^\dagger] = 0. \tag{3}$$

Substituting Eq. (2) into Eq. (1) and keeping terms to the fourth order in the Bose operators, Hamiltonian (1) can be rewritten as

$$\begin{aligned} H =\ & H_0 + D(2S-1)\sum_l a_l^\dagger a_l - D\sum_l a_l^\dagger a_l^\dagger a_l a_l \\ & + S\sum_{l\delta}[J'(a_l^\dagger a_l + a_{l+\delta}^\dagger a_{l+\delta}) \\ & - J(a_l a_{l+\delta}^\dagger + a_l^\dagger a_{l+\delta})] - J'\sum_{l\delta} a_l^\dagger a_l a_{l+\delta}^\dagger a_{l+\delta} \\ & + \frac{J}{4}\sum_{l\delta}(a_l^\dagger a_{l+\delta}^\dagger a_{l+\delta}a_{l+\delta} + a_{l+\delta}^\dagger a_l^\dagger a_l a_l \\ & + a_l^\dagger a_l^\dagger a_l a_{l+\delta} + a_{l+\delta}^\dagger a_{l+\delta}^\dagger a_{l+\delta}a_l) \\ & + K\sum_{l\delta}[2S^3(a_{l+\delta}^\dagger a_{l+\delta} + a_l^\dagger a_l) \\ & - S^2(a_{l+\delta}^\dagger a_{l+\delta}a_{l+\delta}^\dagger a_{l+\delta} + 4a_l^\dagger a_l a_{l+\delta}^\dagger a_{l+\delta} \\ & + a_l^\dagger a_l a_l^\dagger a_l)], \end{aligned} \tag{4}$$

where N is the number of atoms in the chain, and $H_0 = -2J'NS^2 - 2KNS^4 - NDS^2$ is the energy of ground state.

The dynamical equation of the Bose operator is then obtained via the Heisenberg equation of motion

$$\begin{aligned} & \mathrm{i}\hbar\frac{\partial\, a_l}{\partial\, t} \\ =\ & [a_l, H] \\ =\ & [4(J'-J)S + (D + 4KS^2)(2S-1)]a_l \\ & + 2JS(2a_l - a_{l+1} - a_{l-1}) - 2Da_l^\dagger a_l a_l \\ & - 2J'\sum_\delta a_{l+\delta}^\dagger a_{l+\delta}a_l \\ & + \frac{J}{2}\sum_\delta(a_{l+\delta}^\dagger a_{l+\delta}a_{l+\delta} + a_{l+\delta}^\dagger a_l a_l + 2a_l^\dagger a_l a_{l+\delta}) \\ & - 8KS^2\sum_\delta a_{l+\delta}^\dagger a_{l+\delta}a_l - 8KS^2 a_l^\dagger a_l a_l. \end{aligned} \tag{5}$$

The Glauber coherent state $|\alpha_l\rangle$ is adopted, which satisfies the relation

$$a_l|\alpha_l\rangle = \alpha_l|\alpha_l\rangle, \tag{6}$$

where $\alpha_l = \langle\alpha_l|a_l|\alpha_l\rangle$ is the probability amplitude of the particle at site l. Further, by using $|\psi\rangle = \prod|\alpha_l\rangle$ to describe the system state and using the continuum limit

$$\begin{aligned} \alpha_l(t) &= \alpha(x,t), \\ \alpha_{l\pm1}(t) &= \alpha(x,t) \pm \alpha_x(x,t) + \frac{1}{2}\alpha_{xx}(x,t) + \cdots \\ & \qquad (a = 1), \end{aligned} \tag{7}$$

equation (5) transforms into Eq. (8)

$$\begin{aligned} \mathrm{i}\hbar\frac{\partial\alpha}{\partial\, t} =\ & [4(J'-J)S + (D + 4KS^2)(2S-1)]\alpha \\ & - 2JS\alpha_{xx} - 2J'\alpha|\alpha|^2_{xx} \\ & - 4\left(J' - J + \frac{D}{2} + 6KS^2\right)\alpha|\alpha|^2. \end{aligned} \tag{8}$$

Assuming that the ferromagnetic chain forms a closed ring, therefore $\alpha_l(x,t)$ satisfies the periodic condition

$$\alpha_l(x,t) = \alpha_l(x + l, t), \tag{9}$$

where $l = Na$ is the perimeter of the ring.

We search for the solution of Eq. (8) in the form $\alpha_l(x,t) = \phi(\eta)\exp\mathrm{i}(kx - Et/\hbar)$, where $\phi(\eta)$ is a real function of η ($\eta = x - \nu\, t$) and ν is the velocity of the travelling wave. Then equation (8) becomes

$$k = \frac{\hbar\nu}{4SJ}, \tag{10}$$

$$
\begin{aligned}
&[2JS + 4J'\phi^2]\phi_{\eta\eta} + 4J'\phi\phi_\eta^2 \\
&+ 4(J' - J + \frac{D}{2} + 6KS^2)\phi^3 \\
&- [4(J' - J)S + (D + 4KS^2)(2S - 1) \\
&+ 2JSK^2 - E]\phi = 0. \qquad (11)
\end{aligned}
$$

Defining

$$
\begin{aligned}
\gamma &= \frac{2J'}{SJ}, \\
\mu &= \frac{J' - J + D/2 + 6KS^2}{SJ}, \\
\lambda &= \frac{4(J' - J)S + (D + 4KS^2)(2S - 1)}{2SJ} \\
&\quad + \frac{2JSK^2 - E}{2SJ}, \qquad (12)
\end{aligned}
$$

we obtain the following nonlinear Schrödinger equation:

$$
(1 + \gamma\phi^2)\phi_{\eta\eta} + \gamma\phi\phi_\eta^2 + 2\mu\phi^3 - \lambda\phi = 0. \qquad (13)
$$

Accordingly, the periodic condition equation (9) turns into

$$
\phi(\eta) = \phi(\eta + l). \qquad (14)
$$

3. Solutions of the equation with different interaction strengths

Because the solutions of Eq. (13) are related to the parameters γ, μ, and λ, the equation should be solved in different cases. The solving process of Eq. (13) in this paper is in the light of the process in Ref. [38].

3.1. Case of $\mu > 0$, $\gamma > 0$, $C > 0$

By multiplying both sides of Eq. (13) with $2\mathrm{d}\phi/\mathrm{d}\eta$ and integrating once, we have

$$
(1 + \gamma\phi^2)\left(\frac{\mathrm{d}\phi}{\mathrm{d}\eta}\right)^2 = \mu(\phi_1^2 - \phi^2)(\phi^2 - \phi_2^2), \qquad (15)
$$

where

$$
\phi_1^2 \geq \phi^2 \geq \phi_2^2, \quad \phi_1^2 + \phi_2^2 = \frac{\lambda}{\mu}, \quad \phi_1^2\phi_2^2 = \frac{C}{\mu} \qquad (16)
$$

with C being a constant.

Integrating Eq. (15) once again and through a lengthy calculation, we can obtain the exact elliptic function wave

$$
\begin{aligned}
&\frac{1}{\sqrt{\phi_1^2(1 + \gamma\phi_2^2)}}\mathrm{dn}^{-1}\left(\sqrt{\frac{1/\phi_1^2 + \gamma}{1/\phi_2^2 + \gamma}}, q\right) \\
&+ \frac{1}{\sqrt{\phi_1^2(1 + \gamma\phi_2^2)}}\left[\left(\phi_1^2 + \frac{1}{\gamma}\right)\Pi(\tau, b, q) - \frac{1}{\gamma}F(\tau, q)\right] \\
&= \sqrt{\mu}(\eta - \eta_0), \qquad (17)
\end{aligned}
$$

where $\mathrm{dn}(x, q)$ is the third kind of elliptic function, $F(\tau, q)$ and $\Pi(\tau, b, q)$ are the first and third kind of elliptic integration, respectively. They are defined as follows:

$$
F(\tau, q) = \int_0^\tau \frac{\mathrm{d}\theta}{\sqrt{1 - q^2\sin^2\theta}}, \qquad (18)
$$

$$
\Pi(\tau, b, q) = \int_0^\tau \frac{\mathrm{d}\theta}{(1 + b\sin^2\theta)\sqrt{1 - q^2\sin^2\theta}}, \qquad (19)
$$

where

$$
\begin{aligned}
q^2 &= \frac{\phi_1^2 - \phi_2^2}{\phi_1^2(1 + \gamma\phi_2^2)}, \\
\tau &= \arcsin\sqrt{\frac{(1 + \gamma\phi_2^2)(\phi_1^2 - \phi^2)}{(1 + \gamma\phi^2)(\phi_1^2 - \phi_2^2)}}, \\
b &= -\frac{\gamma(\phi_1^2 - \phi_2^2)}{1 + \gamma\phi_2^2}. \qquad (20)
\end{aligned}
$$

And $\eta_0 = x_0 - \nu t_0$ is another constant in which x_0 is the location of the maximum value of the elliptic function wave at the time t_0. The parameter q is the modulus of the elliptic function and elliptic integrations given above, and $0 < q^2 < 1$.

When the chain is dilute, the probability amplitude of the particle at every site l is very small, i.e., $\phi^2 \ll 1$, which is required to obtain Hamiltonian (4). Therefore, the parameter τ can approximately be viewed as a constant, and then the integrations in Eqs. (18) and (19) can also be treated as a constant. So the second term in Eq. (17) can be incorporated into η_0. Finally we derive the approximate elliptic function wave

$$
\begin{aligned}
&\phi(\eta) \\
&= \left\{\left(\frac{1}{\phi_1^2} + \gamma\right)\mathrm{dn}^{-2}\right. \\
&\quad \left.\times \left[\frac{1}{q}\sqrt{\mu(\phi_1^2 - \phi_2^2)}(\eta - \eta_0), q\right] - \gamma\right\}^{-1/2}. \qquad (21)
\end{aligned}
$$

The periodic condition in Eq. (14) leads to

$$
\frac{1}{q}\sqrt{\mu(\phi_1^2 - \phi_2^2)}l = 2K(q), \qquad (22)
$$

where $2K(q)$ is the periodicity of elliptic function $\mathrm{dn}(x, q)$.

From Eqs. (20) and (22), we have

$$
\phi_{1,2}^2 = \frac{1}{2\gamma}[-1 \pm \gamma Bq^2 + \sqrt{(1 - \gamma Bq^2)^2 + 4\gamma B}\,], \qquad (23)
$$

where

$$B = \frac{4K^2(q)}{\mu l^2}. \tag{24}$$

From Eqs. (12), (16), and (23), we can derive the excitation energy of elliptic function wave

$$E = E_0 + \frac{1}{2}m\nu^2, \tag{25}$$

where

$$E_0 = E_{\rm g} - E_{\rm b}$$

is the rest energy,

$$m = \frac{\hbar^2}{4SJ} \tag{26}$$

is the effective mass,

$$E_{\rm g} = (D + 4KS^2)(2S-1) + 4S(J'-J) \tag{27}$$

is the energy gap, and

$$E_{\rm b} = \frac{SJ}{J'}(J' - J + \frac{D}{2} + 6KS^2) \times \left[\sqrt{(1-\gamma Bq^2)^2 + 4\gamma B} - 1\right] \tag{28}$$

is the binding energy that can be considered as a kind of energy lowered by the nonlinear interactions.

From the expressions of $E_{\rm b}$ and $E_{\rm g}$, it is obvious that the biquadratic exchange interaction and the single ion anisotropic interaction control the changes of $E_{\rm b}$ and $E_{\rm g}$ together.

3.1.1. Case of $D > 0$, $K > 0$

On this condition, both the biquadratic exchange interaction and the single ion anisotropic interaction make $E_{\rm b}, E_{\rm g}$ increase, thus $E_{\rm g}(D > 0, K > 0) > E_{\rm g}(D = K = 0)$, and $E_{\rm b}(D > 0, K > 0) > E_{\rm b}(D = K = 0)$. It means that these interactions are against the generation of the excitation in the easy-axial ferromagnetic chain, but benefit the stability of the excitation that has existed.

3.1.2. Case of $D > 0$, $K < 0$, $D/2 > |6KS^2|$

On this condition, the biquadratic exchange interaction and the single ion anisotropic interaction are competing. But the single ion anisotropy energy is higher than the biquadratic exchange energy, thus $E_{\rm g}(D > 0, K < 0) > E_{\rm g}(D = K = 0)$, and $E_{\rm b}(D > 0, K < 0) > E_{\rm b}(D = K = 0)$. On the one hand, the single ion anisotropic interaction makes $E_{\rm b}$ and $E_{\rm g}$ increase, on the other hand, the biquadratic exchange interaction makes $E_{\rm b}$ and $E_{\rm g}$ decrease. They restrict the changes of $E_{\rm b}$ and $E_{\rm g}$ mutually.

3.1.3. Case of $D > 0$, $K < 0$, $D/2 < |6KS^2|$

In this case, the biquadratic exchange interaction and the single ion anisotropic interaction are also competing. However the single ion anisotropy energy is lower than the biquadratic exchange energy, thus $E_{\rm g}(D > 0, K < 0) > E_{\rm g}(D = K = 0)$, and $E_{\rm b}(D > 0, K < 0) < E_{\rm b}(D = K = 0)$. It means that these interactions are against the generation of the excitation and weaken the stability of the excitation that has existed in the easy-axial ferromagnetic chain.

3.1.4. Case of $D < 0$, $K < 0$

In this case, both the biquadratic exchange interaction and the single ion anisotropic interaction make $E_{\rm b}$ and $E_{\rm g}$ decrease, thus $E_{\rm g}(D < 0, K < 0) < E_{\rm g}(D = K = 0)$, and $E_{\rm b}(D < 0, K < 0) < E_{\rm b}(D = K = 0)$. It shows that these interactions benefit the generation of the excitation, but at the same time weaken the stability of the excitation that has existed in the easy-plane ferromagnetic chain.

For the solution in Eq. (21), there are two limiting cases.

(i) When $q \to 0$, it is the linear limit. The normalization condition for Eq. (21) yields $\phi(\eta) = \sqrt{1/l}$. It represents the spin-like wave excitation.

The excitation energy is

$$E_{\rm sw} = E'_{\rm g} + \frac{1}{2}m_{\rm sw}\nu^2, \tag{29}$$

where $m_{\rm sw}$ is the effective mass of spin-like wave excitation, and $m_{\rm sw} = m$, which is defined by Eq. (26). Now the energy gap is

$$E'_{\rm g} = E_{\rm g} - \frac{4}{l}\left(J' - J + \frac{D}{2} + 6KS^2\right).$$

(ii) When $q \to 1$, then $l \to \infty, K(q) \to \infty$. This is the maximum limit of nonlinearity. Considering the normalization condition, the solution in Eq. (21) becomes a soliton solution

$$\phi(\eta) = \left[\gamma \csc^2 \frac{\sqrt{\mu\gamma}}{2} \cosh^2 \frac{\eta - \eta_0}{L_{\rm s}} - \gamma\right]^{-1/2}, \tag{30}$$

where

$$L_{\rm s} = \frac{1}{\sqrt{\lambda}} = \sqrt{\frac{2J'}{J' - J + D/2 + 6KS^2}}$$

$$\times \cot \frac{\sqrt{2J'(J'-J+D/2+6KS^2)}}{2SJ} \quad (31)$$

is the width of the soliton.

The elliptic function wave is the intermediary case between spin-like wave and solitons.

The energy and the peak of the soliton, respectively, are

$$\begin{aligned} E_{\rm s} &= E_0 + \frac{1}{2} m_{\rm s} \nu^2, \\ P_{\rm s} &= \sqrt{\frac{\lambda}{\mu}} \\ &= \sqrt{\frac{1}{\gamma}} \tan \frac{\sqrt{\mu\gamma}}{2} \\ &= \sqrt{\frac{SJ}{2J'}} \tan \frac{\sqrt{2J'\,(J'-J+D/2+6KS^2)}}{2SJ}, \end{aligned} \quad (32)$$

where $E_0 = E_{\rm g} - E_{\rm b}$; $E_{\rm g}$ and $m_{\rm s} = m$ are defined previously.

The biding energy is

$$E_{\rm b} = \frac{SJ}{J'} \left(J' - J + \frac{D}{2} + 6KS^2 \right) \times \tan^2 \frac{\sqrt{2J'(J'-J+D/2+6KS^2)}}{2SJ}. \quad (33)$$

In the following, we study the effects of the biquadratic exchange interaction and the single ion anisotropic interaction on the properties of the soliton. In all the calculations, we set $S = 1, J' = 20$, and $J = 2$. The changes of the soliton's shapes with interaction strength can be described by the following figures which can be gained when different parameters are adopted in Eq. (30).

As shown in Fig. 1(a), the changes of the soliton's shape are similar to those in Ref. [34] where the biquadratic exchange interaction is neglected. We find that the peak of the soliton increases gradually with the increase of single ion anisotropic interaction.

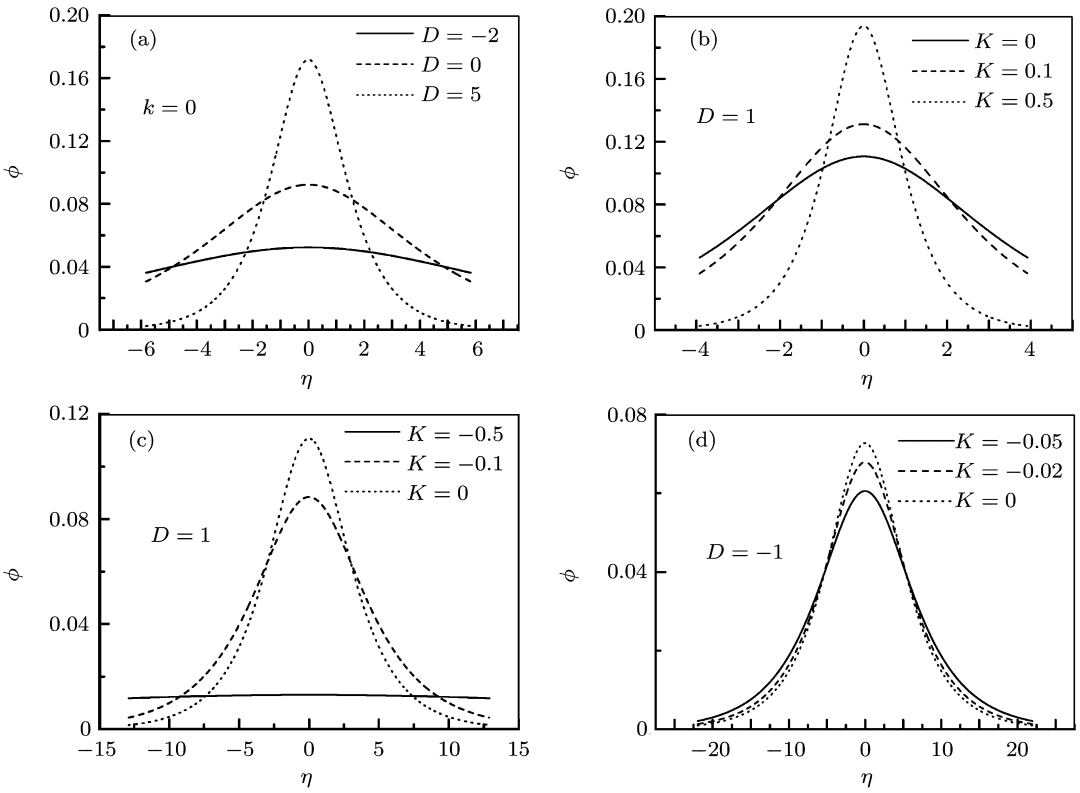


Fig. 1. (a) Effects of the single ion anisotropic interaction on the shape of the soliton, with $K = 0$ and $D = -2,\ 0,\ 5$; (b) effects of the biquadratic exchange interaction and the single ion anisotropic interaction on the shape of the soliton, with $D = 1$ and $K = 0,\ 0.1,\ 0.5$; (c) effects of the biquadratic exchange interaction and the single ion anisotropic interaction on the shape of the soliton, with $D = 1$ and $K = -0.5,\ -0.1,\ 0$; and (d) effects of the biquadratic exchange interaction and the single ion anisotropic interaction on the shape of the soliton, with $D = -1$ and $K = -0.05,\ -0.02,\ 0$.

The width of the soliton becomes narrower. It implies that when the value of the single ion anisotropic interaction is positive, it acts in favour of the stability of the soliton. In contrast, when it is negative, it is against the stability of the soliton. The effects of the biquadratic exchange interaction and the single ion

anisotropic interaction on the shape of the soliton are shown in Fig. 1(b). The values of D and K are both positive. The peak of the soliton increases with the increase of biquadratic exchange interaction. These results imply that the single ion anisotropic interaction and the biquadratic exchange interaction favour the stability of the soliton. That is to say, the biquadratic exchange interaction makes the excitation more stable. In Fig. 1(c), the value of D is positive, but the value of K is negative. It means that the single ion anisotropic interaction favours the stability of the soliton, while the biquadratic exchange interaction acts against the stability of the soliton. So the the effects of the single ion anisotropic interaction on the improvement of the peak is lowered or even offset by the biquadratic exchange interaction. The peak decreases with the increase of biquadratic exchange interaction. When $K = -0.5$, the excitation is not obvious. In Fig. 1(d), the values of D and K both are negative. It means that both the single ion anisotropic interaction and the biquadratic exchange interaction are against the stability of the soliton. The peak is very small and decreases with the increase of biquadratic exchange interaction. The excitation tends to be unstable.

3.2. Case of $\mu > 0, \gamma > 0, C < 0$

The solving process of Eq. (13) is similar to the process when $\mu > 0, \gamma > 0, C > 0$, thus we can obtain the approximate elliptic function wave

$$\begin{aligned}&\phi(\eta)\\ &= \left\{\left(\frac{1}{\phi_1^2}+\gamma\right)\right.\\ &\quad\left.\times\,\mathrm{cn}^{-2}\left[\sqrt{\mu(\phi_1^2-\phi_2^2)}(\eta-\eta_0'),q'\right]\gamma\right\}^{-1/2},\end{aligned} \tag{34}$$

where

$$q'^2 = \frac{\phi_1^2(1+\gamma\phi_2^2)}{\phi_1^2-\phi_2^2}. \tag{35}$$

The meanings of η_0' and q' are the same as those of η_0 and q stated before.

The periodic condition in Eq. (14) leads to

$$\sqrt{\mu(\phi_1^2-\phi_2^2)}\,l = \frac{\sqrt{\mu\phi_1^2(1+\gamma\phi_2^2)}}{q'}\,l = 2K(q'). \tag{36}$$

From Eqs. (35) and (36), ϕ_1^2 and ϕ_2^2 can be expressed as

$$\begin{aligned}\phi_{1,2}^2 &= \frac{1}{2\gamma}\left[-1\pm\gamma B'+\sqrt{(1-\gamma B')^2+4\gamma B'q'^2}\right],\end{aligned} \tag{37}$$

where

$$B' = \frac{4K^2(q')}{\mu l^2}.$$

The excitation energy is

$$E' = E_0' + \frac{1}{2}m\nu^2, \tag{38}$$

where $E_0' = E_\mathrm{g} - E_\mathrm{b}'$ is the rest energy; m and E_g are defined previously.

The binding energy is

$$\begin{aligned}E_\mathrm{b}' &= \frac{SJ}{J'}\left(J'-J+\frac{D}{2}+6KS^2\right)\\ &\quad\times\left[\sqrt{(1-\gamma B')^2+4\gamma B'q'^2}-1\right].\end{aligned} \tag{39}$$

The solution in Eq. (34) also has two limiting cases.

(I) When $q \to 0$, $\phi(\eta) = 0$, it is a trivial solution and is meaningless. In fact, it can be proved that when $q < 1/2$, the binding energy E_b' is negative, and the excitation of elliptic function wave is unstable.

(II) When $q \to 1$, the elliptic function $\mathrm{cn}(x, q') \to \mathrm{sech}\ x$. Obviously, the discussion about this case is similar to that in Subsection 3.1 and the same results can be obtained.

3.3. Case of $\mu < 0, \gamma > 0, C < 0$

By multiplying both sides of Eq. (13) with $2\mathrm{d}\phi/\mathrm{d}\eta$ and integrating once, we obtain

$$(1+\gamma\phi^2)\left(\frac{\mathrm{d}\phi}{\mathrm{d}\eta}\right)^2 = -\mu(\phi_1^2-\phi^2)(\phi_2^2-\phi^2), \tag{40}$$

where

$$\phi_1^2 > \phi_2^2 > \phi^2, \quad \phi_1^2+\phi_2^2 = \frac{\lambda}{\mu}, \quad \phi_1^2\phi_2^2 = \frac{C}{\mu}, \tag{41}$$

and C is a negative constant.

By integrating Eq. (40) once again, we can gain the approximative elliptic function wave

$$\begin{aligned}&\phi(\eta)\\ &= \left\{\left(\frac{1}{\phi_2^2}+\gamma\right)\right.\\ &\quad\left.\times\,\mathrm{sn}^{-2}\left[\sqrt{-\mu\phi_1^2(1+\gamma\phi_2^2)}(\eta-\eta_0''),q''\right]-\gamma\right\}^{-1/2},\end{aligned} \tag{42}$$

where

$$q''^2 = \frac{\phi_2^2(1+\gamma\phi_1^2)}{\phi_1^2(1+\gamma\phi_2^2)}. \tag{43}$$

The meanings of η_0'' and q'' are the same as those of η_0 and q, respectively as stated before.

By the periodic condition Eq. (14), we have

$$\frac{1}{q''}\sqrt{-\mu\phi_2^2(1+\gamma\phi_1^2)}l = 2K(q''). \tag{44}$$

From Eqs. (12), (41), and (44), we gain the binding energy

$$\begin{aligned} & E_{\rm b}'' \\ = & \frac{SJ}{J'}\left(J'-J+\frac{D}{2}+6KS^2\right) \\ & \times\{\sqrt{[1+\gamma B''(1-q''^2)]^2+4\gamma B''q''^2}-1\}, \end{aligned} \tag{45}$$

where $B''=-4K^2(q'')/\mu l^2$.

It can be proved that the binding energy is negative in this case and the excitation is unstable.

4. Conclusion

In this paper, solutions of the elliptic function wave and the solitary wave are obtained in a ferromagnetic chain with a biquadratic exchange interaction, a single ion anisotropic interaction, and an anisotropic nearest neighbour interaction by the traveling wave method. The results are improved on the basis of Ref. [34]. It is found that the dynamical equation of the Bose operator can be controlled by different kinds of exchange interactions and the strengths of interactions. The stability of the soliton changes with the strength of interaction. However, we should point out that relative problems are solved without considering the inhomogeneity of the media[39] which may influence the properties of the soliton. Therefore, models which are closer to the real materials should be studied in the future. It is firmly believed that the soliton can highlight its importance in application, such as data storage or communication. Henceforth, further investigation is worthwhile.

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