Modified impulsive synchronization of fractional order hyperchaotic systems*

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In this paper, a modified impulsive control scheme is proposed to realize the complete synchronization of fractional order hyperchaotic systems. By constructing a suitable response system, an integral order synchronization error system is obtained. Based on the theory of Lyapunov stability and the impulsive differential equations, some effective sufficient conditions are derived to guarantee the asymptotical stability of the synchronization error system. In particular, some simpler and more convenient conditions are derived by taking the fixed impulsive distances and control gains. Compared with the existing results, the main results in this paper are practical and rigorous. Simulation results show the effectiveness and the feasibility of the proposed impulsive control method.

Keywords: hyperchaotic systems, fractional order chaotic systems, synchronization, impulsive control

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1. Introduction

Fractional calculus is a 300-year-old mathematical topic. Although it has a long history, the applications of fractional calculus to physics and engineering are just a recent focus of interest.[1,2] Nowadays, it has been found that some fractional order differential systems can display chaotic behaviour, such as the Chua circuit,[3] jerk model,[4] Lorenz system,[5] Chen system,[6] Lü system,[7] Rössler system,[8] unified system,[9] etc.

Since the pioneering work of Pecora and Carroll,[10] chaos synchronization has become more and more interesting to researchers in different fields. Recently, due to the wide applications in secure communication and process control, many different methods have been used theoretically and experimentally to synchronize the fractional order chaotic systems, such as the active control method,[11,12] sliding mode control method,[13−15] adaptive control method,[16−18] passive control method,[19] impulsive control method,[20,21] etc. Among these methods, impulsive control is an efficient method for dealing with dynamical systems that cannot be controlled by continuous control methods. It has been applied to a large number of chaotic-based communication systems and exhibits good performance for security purposes.[22−27]

In Refs. [20] and [21], the impulsive synchronization schemes for the fractional order hyperchaotic Chen system and fractional order Newton–Leipnik system are studied respectively. However, the proposed methods in Refs. [20] and [21] only apply to integral order chaotic systems and the conditions cannot hold for the fractional order cases theoretically. Motivated by the aforementioned discussion, in this paper we address the modified impulsive synchronization scheme for a wider class of fractional order hyperchaotic systems. The criteria derived in this paper are more reasonable and rigorous than those in Refs. [20] and [21]. Furthermore, the obtained results are helpful for improving existing technologies used

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in chaotic secure communication systems and chaotic spread spectrum communications.

The rest of this paper is organized as follows. In Section 2, the problem formulation for impulsive synchronization is provided. The main results for impulsive synchronization of fractional order hyperchaotic systems are derived in Section 3. In Section 4, an illustrative example of the fractional order hyperchaotic Chen system is presented to demonstrate the effectiveness and the feasibility of the proposed method. Finally, concluding remarks are given in Section 5.

Notations Throughout the paper, $\mathbb{R}$ and $\mathbb{R}^n$ denote the real number and n-dimensional Euclidean space, respectively; $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices; $\mathbb{N} \doteq \{1,2,\ldots\}$. I denotes the identity matrix with appropriate dimension.

2. Problem formulation

Consider the following fractional order hyperchaotic system:

$$D^{\alpha}x(t) = Ax(t) + \phi(x(t)) \tag{1}$$

with the initial condition

$$D^{(\alpha-1)}x(t_0) = x_0,$$

where $A \in \mathbb{R}^{n\times n}$ is a known constant matrix, $\phi : \mathbb{R}^n \to \mathbb{R}^n$ represents the nonlinear part of system (1). $D^{\alpha}x(t)$ denotes the Riemann–Liouville fractional derivative of $\alpha \in (0,1)$ defined as follows:[1]

$$D^{\alpha}x(t) = \frac{1}{\Gamma(1-\alpha)}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t \frac{x(\tau)}{(t-\tau)^{\alpha}}\mathrm{d}\tau,$$

where $\Gamma(\alpha)$ is the Euler gamma function given by

$$\Gamma(\alpha) = \int_0^{\infty} \upsilon^{\alpha-1}\,\mathrm{e}^{-\upsilon}\mathrm{d}\upsilon.$$

To study the impulsive synchronization of fractional order hyperchaotic systems, system (1) is often taken as the drive system, and the response system is characterized by

$$D^{\alpha}y(t) = Ay(t) + \phi(y(t)), \tag{2}$$

where $y(t) \in \mathbb{R}^n$ is the state vector.

At discrete time t_k, the state variables of the drive system are transmitted to the response system as the control input such that the state variables of the response system are suddenly changed at these instants. Therefore, the impulsively controlled response system can be written in the following form:

$$\begin{cases} D^{\alpha}y(t) = Ay(t) + \phi(y(t)), & t \neq t_k,\ k \in \mathbb{N}, \\ \Delta y = y(t_k^+) - y(t_k^-) = B_k e(t_k), & t = t_k, \end{cases} \tag{3}$$

where $e(t) = y(t) - x(t) = [y_1(t) - x_1(t),\ y_2(t) - x_2(t), \ldots,\ y_n(t) - x_n(t)]^{\mathrm{T}}$ is the error vector, $B_k \in \mathbb{R}^{n\times n}$ is the control gain matrix. Suppose that the discrete instants set $\{t_k\}$ satisfies $0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$, $\lim_{k\to\infty} t_k = \infty$. $\lim_{h\to 0^+} x(t_k+h) = y(t_k^+)$, $\lim_{h\to 0^+} y(t_k - h) = y(t_k^-) = y(t_k)$ implies that $y(t)$ is left-continuous at $t = t_k$, Thus, from Eqs. (1) and (3) and $x(t_k) = x(t_k^-)$, the following synchronization error system can be obtained:

$$\begin{cases} D^{\alpha}e(t) = Ae(t) + \psi(x(t),\ y(t)),\ t \neq t_k, \\ \Delta e = e(t_k^+) - e(t_k^-) = B_k e(t_k),\ t = t_k, \end{cases} \tag{4}$$

where $\psi(x(t), y(t)) = \phi(y(t)) - \phi(x(t)) = N(x(t), y(t))e(t)$, and $N(x(t), y(t))$ is a norm-bounded matrix that in determined by vectors $x(t)$ and $y(t)$. Note that most typical fractional order hyperchaotic systems can meet $\psi(x(t), y(t)) = N(x(t), y(t))e(t)$, such as the fractional order hyperchaotic Lorenz system,[28] fractional order hyperchaotic Chen system,[29] fractional order hyperchaotic Rössler system,[30] fractional order hyperchaotic Lü system,[31] etc.

The usual analysis method to achieve the synchronization between the drive system (1) and response system (3) is to study the asymptotical stability of error system (4). However, to the best of our knowledge, there is currently no effective method to deal with the stability analysis for the fractional order impulsive differential system (4). In order to solve this problem, we will convert the fractional order error system (4) into the integral order one by constructing the suitable impulsively controlled response system, which can be constructed in the following form:

$$\begin{cases} \dot{y}(t) = Ay(t) + \phi(y(t)) + \eta(x(t)), & t \neq t_k, \\ \Delta y = y(t_k^+) - y(t_k^-) = B_k e(t_k), & t = t_k, \end{cases} \tag{5}$$

where $\eta(x(t)) = \dot{x}(t) - D^{\alpha}x(t)$, and the terms $\dot{x}(t)$ and $D^{\alpha}x(t)$ can be obtained from drive system (1) (note that the numerical solutions of $\dot{x}(t)$ and $D^{\alpha}x(t)$ can be taken from the simulation of system (1), which is feasible to construct system (5)). Thus, from Eqs. (1) and (5), the new synchronization error system can be

obtained as

$$\begin{cases} \dot{e}(t) = (A + N(x(t), y(t)))e(t), & t \neq t_k, \\ \Delta e = e(t_k^+) - e(t_k^-) = B_k e(t_k), & t = t_k. \end{cases} \tag{6}$$

Obviously, by constructing the response system (5), the synchronization scheme for the fractional order hyperchaotic system is converted into the stability analysis of the integral order error system (6).[9]

3. Impulsive synchronization

In this section, we will give the sufficient conditions to realize the asymptotical stability of error system (6) by designing a suitable impulsive controller (including the control gain B_k and the impulsive distance $\tau_k = t_k - t_{k-1}$ $(k \in \mathbb{N})$).

Theorem 1 If there exists a positive number $\gamma > 1$ such that the following condition holds:

$$\gamma\beta_k \exp((\lambda_A + \lambda_N)\tau_k) \leq 1, \quad k \in \mathbb{N}, \tag{7}$$

then error system (6) is globally asymptotically stable, which implies that the impulsively controlled response system (5) asymptotically synchronizes with the drive system (1), where β_k, λ_A, and λ_N are the largest eigenvalues of $(I + B_k)^{\mathrm{T}}(I + B_k)$, $A + A^{\mathrm{T}}$, and $N(x, y) + N^{\mathrm{T}}(x, y)$ respectively.

Proof Construct the Lyapunov function in the form of $V(e) = e^{\mathrm{T}}e$, and for $t \in (t_{k-1}, t_k]$, $k \in \mathbb{N}$, its derivative along the trajectory of error system (6) is

$$\begin{aligned} \dot{V}(e) &= \dot{e}^{\mathrm{T}}e + e^{\mathrm{T}}\dot{e} \\ &= (Ae + N(x, y)e)^{\mathrm{T}}e + e^{\mathrm{T}}(Ae + N(x, y)e) \\ &= e^{\mathrm{T}}(A + A^{\mathrm{T}})e + e^{\mathrm{T}}(N(x, y) + N^{\mathrm{T}}(x, y))e \\ &\leq \lambda_A e^{\mathrm{T}}e + \lambda_N e^{\mathrm{T}}e \\ &= (\lambda_A + \lambda_N)V(e), \end{aligned} \tag{8}$$

which implies that

$$\begin{aligned} V(e(t)) \leq\ & V(e(t_{k-1}^+))\exp((\lambda_A + \lambda_N)(t - t_{k-1})), \\ & t \in (t_{k-1}, t_k]. \end{aligned} \tag{9}$$

On the other hand, for $t = t_k$, we have

$$\begin{aligned} V(e(t_k^+)) &\leq ((I + B_k)e(t_k))^{\mathrm{T}}(I + B_k)e(t_k) \\ &\leq e^{\mathrm{T}}(t_k)(I + B_k)^{\mathrm{T}}(I + B_k)e(t_k) \\ &\leq \beta_k V(e(t_k)). \end{aligned} \tag{10}$$

For $t \in (t_0, t_1]$, it follows from inequality (9) that

$$V(e(t)) \leq V(e(t_0^+))\exp((\lambda_A + \lambda_N)(t - t_0)),$$

which leads to

$$V(e(t_1)) \leq V(e(t_0^+))\exp((\lambda_A + \lambda_N)(t_1 - t_0)).$$

From inequality (10), it follows that

$$\begin{aligned} V(e(t_1^+)) &\leq \beta_1 V(e(t_1)) \\ &\leq \beta_1 V(e(t_0^+))\exp((\lambda_A + \lambda_N)(t_1 - t_0)). \end{aligned}$$

Similarly, for $t \in (t_1, t_2]$,

$$\begin{aligned} V(e(t)) &\leq V(e(t_1^+))\exp((\lambda_A + \lambda_N)(t - t_1)) \\ &\leq \beta_1 V(e(t_0^+))\exp((\lambda_A + \lambda_N)(t - t_0)). \end{aligned}$$

In general, for $t \in (t_k, t_{k+1}]$,

$$\begin{aligned} V(e(t)) \leq\ & V(e(t_0^+))\beta_1\beta_2\cdots\beta_k \\ & \times \exp((\lambda_A + \lambda_N)(t - t_0)). \end{aligned} \tag{11}$$

It follows from inequalities (7) and (11) that

$$\begin{aligned} V(e(t)) &\leq V(e(t_0^+))\beta_1\beta_2\cdots\beta_k\exp((\lambda_A + \lambda_N)(t - t_0)) \\ &\leq V(e(t_0^+))\beta_1\beta_2\cdots\beta_k \\ &\quad \times \exp((\lambda_A + \lambda_N)(t_{k+1} - t_0)) \\ &= V(e(t_0^+))\beta_1\exp((\lambda_A + \lambda_N)(t_1 - t_0)) \\ &\quad \times \beta_2\exp((\lambda_A + \lambda_N)(t_2 - t_1))\ldots \\ &\quad \times \beta_k\exp((\lambda_A + \lambda_N)(t_k - t_{k-1})) \\ &\quad \times \exp((\lambda_A + \lambda_N)(t_{k+1} - t_k)) \\ &\leq \frac{V(e(t_0^+))}{\gamma^k}\exp((\lambda_A + \lambda_N)(t_{k+1} - t_k)). \end{aligned}$$

Obviously, $V(e(t)) \to 0$ and $e(t) \to 0$ as $t \to 0$ (or $k \to 0$). Therefore, error system (6) is globally asymptotically stable, which implies that the impulsively controlled response system (5) asymptotically synchronizes with drive system (1). This completes the proof.

Theorem 2 If there exists a positive number $\gamma > 1$ such that the following conditions hold:

$$\gamma\beta_{2k-1}\beta_{2k}\exp((\lambda_A + \lambda_N)(t_{2k+1} - t_{2k-1})) \leq 1, \quad k \in \mathbb{N}, \tag{12}$$

$$\sup_k\{\beta_k \exp((\lambda_A + \lambda_N)(t_{k+1} - t_k))\} = \varepsilon < \infty, \tag{13}$$

then error system (6) is globally asymptotically stable, which implies that the impulsively controlled response system (5) asymptotically synchronizes with drive system (1), where β_k, λ_A, and λ_N are the largest eigenvalues of $(I + B_k)^{\mathrm{T}}(I + B_k)$, $A + A^{\mathrm{T}}$, and $N(x, y) + N^{\mathrm{T}}(x, y)$ respectively.

Proof In a way similar to that in the first half of the proof of Theorem 1, let the Lyapunov function be $V(e) = e^{\mathrm{T}}e$, and for $t \in (t_k, t_{k+1}]$, $k \in \mathbb{N}$, the following inequality can be obtained (i.e., inequality (11)):

$$V(e(t)) \leq V(e(t_0^+))\beta_1\beta_2\ldots\beta_k \times \exp((\lambda_A + \lambda_N)(t - t_0)). \quad (14)$$

From inequalities (12), (13), and (14), we can further obtain

(i) for $t \in (t_{2k-1}, t_{2k}]$,

$$\begin{aligned} V(e(t)) &\leq V(e(t_0^+)) \prod_{i=1}^{2k-1} \beta_i \exp((\lambda_A + \lambda_N)(t - t_0)) \\ &\leq V(e(t_0^+)) \prod_{i=1}^{2k-1} \beta_i \exp((\lambda_A + \lambda_N)(t_{2k} - t_0)) \\ &= V(e(t_0^+))\beta_1\beta_2 \exp((\lambda_A + \lambda_N)(t_3 - t_1)) \cdots \\ &\quad \times \beta_{2k-3}\beta_{2k-2} \exp((\lambda_A+\lambda_N)(t_{2k-1} - t_{2k-3})) \\ &\quad \times \beta_{2k-1} \exp((\lambda_A + \lambda_N)(t_{2k} - t_{2k-1})) \\ &\quad \times \exp((\lambda_A + \lambda_N)(t_1 - t_0)) \\ &\leq \varepsilon \frac{V(e(t_0^+))}{\eta^{k-1}} \exp((\lambda_A + \lambda_N)(t_1 - t_0)); \end{aligned} \quad (15)$$

(ii) For $t \in (t_{2k}, t_{2k+1}]$,

$$\begin{aligned} V(e(t)) &\leq V(e(t_0^+)) \prod_{i=1}^{2k} \beta_i \exp((\lambda_A + \lambda_N)(t - t_0)) \\ &\leq V(e(t_0^+)) \prod_{i=1}^{2k} \beta_i \exp((\lambda_A + \lambda_N)(t_{2k+1} - t_0)) \\ &= V(e(t_0^+))\beta_1\beta_2 \exp((\lambda_A + \lambda_N)(t_3 - t_1)) \cdots \\ &\quad \times \beta_{2k-1}\beta_{2k} \exp((\lambda_A + \lambda_N)(t_{2k+1} - t_{2k-1})) \\ &\quad \times \exp((\lambda_A + \lambda_N)(t_1 - t_0)) \\ &\leq \frac{V(e(t_0^+))}{\eta^k} \exp((\lambda_A + \lambda_N)(t_1 - t_0)). \end{aligned} \quad (16)$$

From inequalties (15) and (16) it follows that $V(e(t)) \to 0$ and $e(t) \to 0$ as $t \to 0$ (or $k \to 0$). Therefore, error system (6) is globally asymptotically stable, which implies that the impulsively controlled response system (5) asymptotically synchronizes with drive system (1). This completes the proof.

Remark 1 In Theorem 2, we can choose the odd impulsive sequence $\{t_{2k-1}\}$ to realize the synchronization scheme. Compared with most existing results as derived in Theorem 1 (the full impulsive sequence $\{t_k\}$ is used), the condition in Theorem 2 is less conservative.

For simplicity, the impulsive distance and the control gain are often chosen to be constant, then we can obtain the following corollaries on the base of Theorems 1 and 2 easily.

Corollary 1 Assume that $\tau_k = \tau$ and $B_k = B$, if there exists a positive number $\gamma > 1$ such that the following condition holds:

$$\gamma\beta\exp((\lambda_A + \lambda_N)\tau) \leq 1, \quad (17)$$

then error system (6) is globally asymptotically stable, which implies that the impulsivly controlled response system (5) asymptotically synchronizes with drive system (1), where β, λ_A, and λ_N are the largest eigenvalues of $(I+B)^{\mathrm{T}}(I+B)$, $A+A^{\mathrm{T}}$, and $N(x,y)+N^{\mathrm{T}}(x,y)$ respectively.

Corollary 2 Assume that $t_{2k+1} - t_{2k-1} = \tau^*$ and $B_k = B$, if there exists a positive number $\gamma > 1$ such that the following condition holds:

$$\gamma\beta^2\exp((\lambda_A + \lambda_N)\tau^*) \leq 1, \quad (18)$$

then error system (6) is globally asymptotically stable, which implies that the impulsively controlled response system (5) asymptotically synchronizes with drive system (1), where β, λ_A, and λ_N are the largest eigenvalues of $(I+B)^{\mathrm{T}}(I+B)$, $A+A^{\mathrm{T}}$, and $N(x,y)+N^{\mathrm{T}}(x,y)$ respectively.

In Theorems 1 and 2, and Corollaries 1 and 2, since the matrix $N(x,y)$ is norm-bounded, the estimation of λ_N can be further obtained by the boundary of chaotic states. In the following, we will take fractional order hyperchaotic Chen system for example to obtain some more practical results.

The fractional order hyperchaotic Chen system can also be presented as system (1) with

$$A = \begin{bmatrix} -a & a & 0 & 0 \\ b & c & 0 & -1 \\ 0 & 0 & -d & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \phi(x) = \begin{bmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \\ g \end{bmatrix}.$$

When $a = 36$, $b = -16$, $c = 28$, $d = 3$, and $g = 0.3$, the system possesses a hyperchaotic behaviour as shown in Fig. 1.

Furthermore, we have

$$\psi(x,y) = \phi(y) - \phi(x) = \begin{bmatrix} 0 \\ x_1x_3 - y_1y_3 \\ y_1y_2 - x_1x_2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ -x_3 & 0 & -y_1 & 0 \\ x_2 & y_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = N(x,y)e,$$

which leads to

$$N(x,y) + N^{\mathrm{T}}(x,y) = \begin{bmatrix} 0 & -x_3 & x_2 & 0 \\ -x_3 & 0 & 0 & 0 \\ x_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

thus we have $0 \le \lambda_N = \sqrt{x_2^2 + x_3^2} \le \sqrt{\mathscr{M}^2 + \mathscr{M}^2} = \sqrt{2}\mathscr{M}$ (Due to the boundedness of chaotic signals, there exists a positive constant $\mathscr{M}$ such that $|x_2| \le \mathscr{M}$ and $|x_3| \le \mathscr{M}$). Therefore, for the fractional order hyperchaotic Chen system, inequality (8) can be converted into

$$\dot{V}(e) \le (\lambda_A + \sqrt{2}\mathscr{M})V(e). \tag{19}$$

From inequality (19) and similar analytic process to those for Theorems 1 and 2, we can obtain the following results easily.

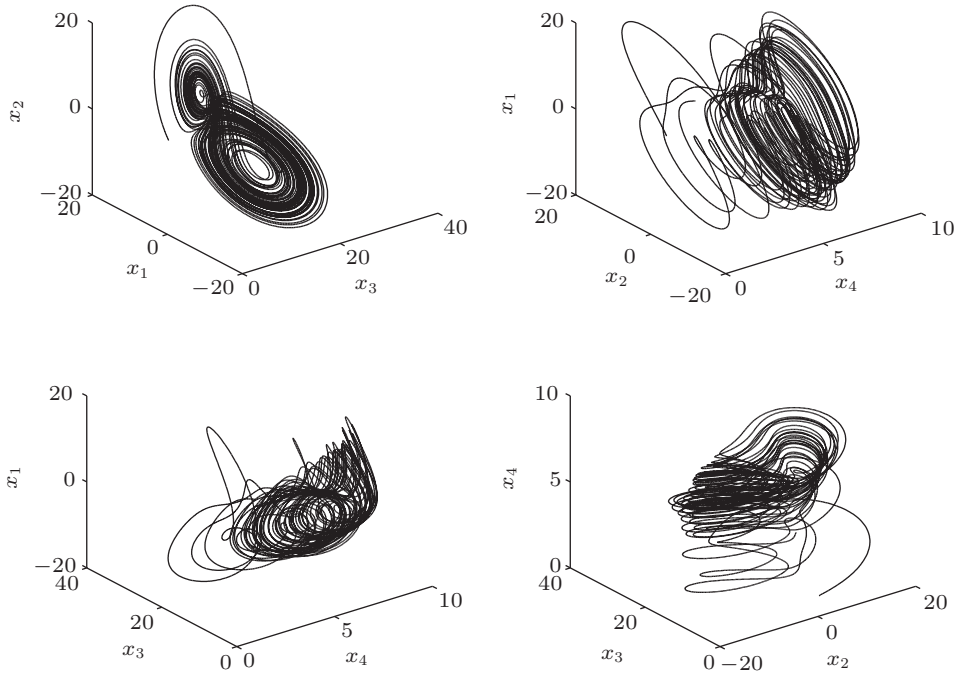


Fig. 1. Chaotic attractors of fractional order hyperchaotic Chen system.

Theorem 3 If there exists a positive number $\gamma > 1$ such that the following condition holds:

$$\gamma\beta_k \exp((\lambda_A + \sqrt{2}\mathscr{M})\tau_k) \le 1, \quad k \in \mathbb{N}, \tag{20}$$

then error system (6) is globally asymptotically stable, which implies that the impulsively controlled response system (5) asymptotically synchronizes with drive system (1), where β_k and λ_A are the largest eigenvalues of $(I + B_k)^{\mathrm{T}}(I + B_k)$ and $A + A^{\mathrm{T}}$ respectively.

Theorem 4 If there exists a positive number $\gamma > 1$ such that the following conditions hold

$$\gamma\beta_{2k-1}\beta_{2k} \exp((\lambda_A + \sqrt{2}\mathscr{M})(t_{2k+1} - t_{2k-1})) \le 1,$$
$$k \in \mathbb{N}, \tag{21}$$
$$\sup_k \{\beta_k \exp((\lambda_A + \sqrt{2}\mathscr{M})(t_{k+1} - t_k))\} = \varepsilon < \infty, \tag{22}$$

then error system (6) is globally asymptotically stable, which implies that the impulsively controlled response system (5) asymptotically synchronizes with drive system (1), where β_k and λ_A are the largest eigenvalues of $(I + B_k)^{\mathrm{T}}(I + B_k)$ and $A + A^{\mathrm{T}}$, respectively.

Similarly, for simplicity, the impulsive distance and the control gain are often chosen to be constant, then we can obtain the following corollaries on the basis of Theorems 3 and 4 easily.

Corollary 3 Assume that $\tau_k = \tau$ and $B_k = B$, if there exists a positive number $\gamma > 1$ such that the following condition holds:

$$\gamma\beta \exp((\lambda_A + \sqrt{2}\mathscr{M})\tau) \le 1, \tag{23}$$

then error system (6) is globally asymptotically stable,

which implies that the impulsively controlled response system (5) asymptotically synchronizes with drive system (1), where β and λ_A are the largest eigenvalues of $(I+B)^{\mathrm{T}}(I+B)$ and $A+A^{\mathrm{T}}$ respectively.

Corollary 4 Assume that $t_{2k+1}-t_{2k-1}=\tau^*$ and $B_k=B$, if there exists a positive number $\gamma>1$ such that the following condition holds:

$$\gamma\beta^2\exp((\lambda_A+\sqrt{2}\mathscr{M})\tau^*)\leq 1, \tag{24}$$

then error system (6) is globally asymptotically stable, which implies that the impulsively controlled response system (5) asymptotically synchronizes the drive system (1), where β and λ_A are the largest eigenvalues of $(I+B)^{\mathrm{T}}(I+B)$ and $A+A^{\mathrm{T}}$, respectively.

Remark 2 In Theorems 3 and 4, and Corollaries 3 and 4, the impulsive synchronization scheme for fractional order hyperchaotic Chen systems is presented. For the most typical fractional order hyperchaotic systems, although the nonlinear function $\phi(x)$ has different forms, they all satisfy $\psi(x,y)=\phi(y)-\phi(x)=N(x,y)e$. By the boundedness of chaotic states, λ_N can be estimated availably. Therefore, the proposed method in this paper can be applied to a wider class of fractional order hyperchaotic systems.

4. Simulation results

In the simulation, the fractional order hyperchaotic Chen system will be taken for example to confirm the proposed method.

From the chaotic attractor of fractional order hyperchaotic Chen system as shown in Fig. 1, we can obtain the bound $\mathscr{M}=30$. From

$$A+A^{\mathrm{T}}=\begin{bmatrix}-72 & 20 & 0 & 1\\ 20 & 56 & 0 & -1\\ 0 & 0 & -6 & 0\\ 1 & -1 & 0 & 0\end{bmatrix},$$

we have $\lambda=59.0641$. Let $\gamma=1.05$ and $B_k=B=-0.7I$, then we will obtain $\beta=0.09$. By Corollary 3, the bound of the stable region can be determined to be

$$0<\tau\leq -\ln(\gamma\beta)/(\lambda+\sqrt{2}\mathscr{M})=0.0232.$$

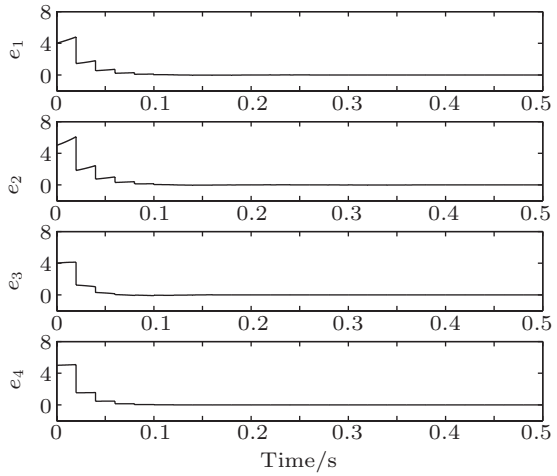


Fig. 2. Synchronization errors with $\tau=0.02$ and $B=-0.7I$.

Figure 2 shows the time response curves of error system (6) with $B=-0.7I$ and $\tau=0.02$. The initial conditions of drive and response systems are taken as $[2,-1,1,-2]^{\mathrm{T}}$ and $[6,4,5,3]^{\mathrm{T}}$, respectively.

5. Conclusions

In this paper, we present a modified impulsive control method to synchronize fractional order hyperchaotic systems. By constructing a new response system, the fractional order impulsive control scheme is converted into the integral case, and some effective and practical sufficient conditions are presented to guarantee that the synchronization error dynamics can be converged to the origin. Finally, a numerical example for fractional order hyperchaotic Chen system is given to demonstrate the effectiveness of the method.

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