No-chattering sliding mode control in a class of fractional-order chaotic systems*

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A no-chattering sliding mode control strategy for a class of fractional-order chaotic systems is proposed in this paper. First, the sliding mode control law is derived to stabilize the states of the commensurate fractional-order chaotic system and the non-commensurate fractional-order chaotic system, respectively. The designed control scheme guarantees the asymptotical stability of an uncertain fractional-order chaotic system. Simulation results are given for several fractional-order chaotic examples to illustrate the effectiveness of the proposed scheme.

Keywords: fractional-order system, chaos control, sliding mode, uncertain chaotic systems

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1. Introduction

Although fractional calculus was introduced in the early 17th century,[1,2] its application to physics and engineering has attracted lots of attention only in the recent years.[3−6] Recently, the synchronization of fractional-order chaotic systems has increasingly received attention due to its potential applications in secure communication and control processing.[7−9] For example, Zhou *et al.*[10] designed a controller to synchronize the fractional-order chaotic system with chaotic systems of integer orders, and synchronize the different fractional-order chaotic systems, according to the idea of tracking control and stability theory of fractional-order systems. The chaotic behaviours of a fractional-order generalized Lorenz system and its synchronization were studied by Zhang and Yang.[11] Liu *et al.*[12] studied the anti-synchronization of hyperchaotic systems with slow time-varying parameters. The controller and recognizing rules of the uncertain parameters were designed by Zhao *et al.*,[13] and the tracking control and the synchronization of the fractional hyperchaotic Lorenz system were realized.

It should be noted that most of the research efforts mentioned above concentrated on studying synchronization behaviour of certain systems. There have been few methods of controlling a class of fractional-order chaotic systems, despite their theoretical importance and practical application.[14,15]

In the present paper, a class of fractional-order chaotic systems is introduced, and a no-chattering sliding mode controller is proposed to control the chaos in these systems. This novel control law can make the system states asymptotically stable, which is also robust against uncertainty and external disturbance of the system. Numerical simulation results are presented to demonstrate the validity of the proposed method, which are in agreement with theoretical analysis.

In Section 2, a class of fractional-order chaotic systems is introduced. Based on the sliding mode control theory, a controller is proposed to control the commensurate fractional-order chaotic system and the non-commensurate fractional-order chaotic system in Section 3. The controller design scheme and the stability analysis of a closed loop system are also included in this section. In Section 4, numerical simulation results are presented. Brief comments and conclusions are given in Section 5.

2. General form description

One considers a class of three-dimensional fractional-order chaotic systems as follows:

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$$\begin{cases} \mathrm{d}^{q_1}x/\mathrm{d}t^{q_1} = f(x,y,z) - \alpha x, \\ \mathrm{d}^{q_2}y/\mathrm{d}t^{q_2} = x \cdot g(x,y,z) + z \cdot \phi(x,y,z) - \beta y, \\ \mathrm{d}^{q_3}z/\mathrm{d}t^{q_3} = x \cdot h(x,y,z) - y \cdot \phi(x,y,z) - \gamma z, \end{cases} \tag{1}$$

where x, y, and z are state variables; q_i $(i = 1, 2, 3)$ are fractional-orders, which satisfy $0 < q_i < 1$ correspondingly; each of the three functions $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$ is considered as a smooth function in $\boldsymbol{R}^3 \to \boldsymbol{R}$ space; α, β, and γ are known constants (β and γ are non-negative).

Cited from Refs. [16]–[18], these examples of fractional-order chaotic models can be described by the proposed system (1) and are listed in Table 1.

Table 1. List of cited fractional-order chaotic systems, which can be described by the proposed general model.

Name	Mode	$f(x,y,z)$	$g(x,y,z)$	$h(x,y,z)$	$\varphi(x,y,z)$
Case I	$\begin{cases} \mathrm{d}^{q_1}x/\mathrm{d}t^{q_1} = -yz + ax \\ \mathrm{d}^{q_2}y/\mathrm{d}t^{q_2} = xz - by \\ \mathrm{d}^{q_3}z/\mathrm{d}t^{q_3} = \frac{1}{3}xy - cz \end{cases}$	$-yz$	z	$\frac{1}{3}y$	0
Case II	$\begin{cases} \mathrm{d}^{q_1}x/\mathrm{d}t^{q_1} = a(y-x) \\ \mathrm{d}^{q_2}y/\mathrm{d}t^{q_2} = x(b-z) - y \\ \mathrm{d}^{q_3}z/\mathrm{d}t^{q_3} = xy - cz \end{cases}$	ay	b	0	$-x$
Case III	$\begin{cases} \mathrm{d}^{q_1}x/\mathrm{d}t^{q_1} = a(y-x) + yz \\ \mathrm{d}^{q_2}y/\mathrm{d}t^{q_2} = cx - y - xz \\ \mathrm{d}^{q_3}z/\mathrm{d}t^{q_3} = xy - bz \end{cases}$	$ay + yz$	c	0	$-x$

3. Problem formulation and control design

Adding the control input, the proposed class of chaotic systems in system (1) can be rewritten as follows:

$$\begin{cases} \mathrm{d}^{q_1}x/\mathrm{d}t^{q_1} = f(x,y,z) - \alpha x + u(t), \\ \mathrm{d}^{q_2}y/\mathrm{d}t^{q_2} = x \cdot g(x,y,z) + z \cdot \varphi(x,y,z) - \beta y, \\ \mathrm{d}^{q_3}z/\mathrm{d}t^{q_3} = x \cdot h(x,y,z) - y \cdot \varphi(x,y,z) - \gamma z, \end{cases} \tag{2}$$

where $u(t) \in \boldsymbol{R}$ is the control input. Two steps are needed to design a sliding mode controller. The first step is to construct a sliding surface that represents the desired system dynamics, and the second step is to develop a switching control law such that a sliding mode exists on each point of the sliding surface, and any state outside the surface is driven to the surface in a finite time.[19] As a choice for the sliding surface, we have

$$\begin{aligned} s(t) &= D^{q_1-1}x(t) + D^{-1}\psi(t) \\ &= D^{q_1-1}x(t) + \int_0^t \psi(\tau)\mathrm{d}\tau, \end{aligned} \tag{3}$$

where $\psi(t)$ is a function described by

$$\psi(t) = y \cdot g(x,y,z) + z \cdot h(x,y,z) + \alpha x + \rho x, \tag{4}$$

where ρ is assumed to be an arbitrary positive coefficient satisfying $\rho > |\alpha|$, i.e., $\rho + \alpha > 0$.

When the system operates in the sliding mode, it satisfies the following equation:

$$s(t) = D^{q_1-1}x(t) + D^{-1}\psi(t) = 0. \tag{5}$$

Differentiating Eq. (5), one obtains

$$\begin{aligned} \dot{s}(t) &= \frac{\mathrm{d}}{\mathrm{d}t}s(t) = \frac{\mathrm{d}}{\mathrm{d}t}\left[D^{q_1-1}x(t) + \int_0^t \psi(\tau)\mathrm{d}\tau\right] \\ &= \frac{\mathrm{d}}{\mathrm{d}t}D^{q_1-1}x(t) + \frac{\mathrm{d}}{\mathrm{d}t}\int_0^t \psi(\tau)\mathrm{d}\tau \\ &= D^{q_1}x(t) + \psi(t) = 0, \\ &\Rightarrow D^{q_1}x(t) = -\psi(t) \\ &= -y \cdot g(x,y,z) - z \cdot h(x,y,z) - \alpha x - \rho x, \end{aligned} \tag{6}$$

Using Eq. (6), it is straightforward to verify that the sliding mode dynamics can be obtained as

$$\begin{cases} \dfrac{\mathrm{d}^{q_1}x}{\mathrm{d}t^{q_1}} = -y \cdot g(x,y,z) - z \cdot h(x,y,z) - \alpha x - \rho x, \\ \dfrac{\mathrm{d}^{q_2}y}{\mathrm{d}t^{q_2}} = x \cdot g(x,y,z) - \beta y, \\ \dfrac{\mathrm{d}^{q_3}z}{\mathrm{d}t^{q_3}} = x \cdot h(x,y,z) - \gamma z. \end{cases} \tag{7}$$

According to the sliding mode control theory, Eqs. (2) and (7), the equivalent control law is calculated as

$$\begin{aligned} u_{\mathrm{eq}}(t) &= \frac{\mathrm{d}^{q_1}x}{\mathrm{d}t^{q_1}} - f(x,y,z) + \alpha x \\ &= -y \cdot g(x,y,z) - z \cdot h(x,y,z) - \alpha x \\ &\quad - \rho x - f(x,y,z) + \alpha x \\ &= -f(x,y,z) - y \cdot g(x,y,z) \\ &\quad - z \cdot h(x,y,z) - \rho x. \end{aligned} \tag{8}$$

To satisfy the sliding condition, the discontinuous reaching law is chosen as

$$u_r(t) = -k \cdot \mathrm{sat}(s/\varepsilon), \tag{9}$$

where

$$\mathrm{sat}(y) = \begin{cases} \mathrm{sgn}(y), & |y| > 1, \\ y, & |y| \le 1, \end{cases} \tag{10}$$

and k is the gain of the controller.

Now, the total control law can be defined as follows:

$$\begin{aligned} u(t) &= u_{\mathrm{eq}}(t) + u_r(t) \\ &= -f(x,y,z) - y \cdot g(x,y,z) - z \cdot h(x,y,z) \\ &\quad - \rho x - k \cdot \mathrm{sat}(s/\varepsilon). \end{aligned} \tag{11}$$

Remark 1 Consider the fractional-order chaotic system (2) and the control law (11). If the controller gain $k > 0$, the closed loop system is globally asymptotically stable.

Proof Selecting the Lyapunov candidate as

$$V = s^2, \tag{12}$$

we have

$$\begin{aligned} \dot{V} &= 2s\dot{s} = 2s\left[D^{q_1}x(t) + \psi(t)\right] \\ &= 2s[f(x,y,z) - \alpha x + u(t) + y \cdot g(x,y,z) \\ &\quad + z \cdot h(x,y,z) + \alpha x + \rho x] \\ &= 2s[f(x,y,z) + u(t) + g \cdot f(x,y,z) \\ &\quad + z \cdot h(x,y,z) + \rho x] \\ &= 2s[f(x,y,z) - f(x,y,z) - y \cdot g(x,y,z) \\ &\quad - z \cdot h(x,y,z) - \rho x - k \cdot \mathrm{sat}(s/\varepsilon) \\ &\quad + y \cdot g(x,y,z) + z \cdot h(x,y,z) + \rho x] \\ &= -2k \cdot s \cdot \mathrm{sat}(s/\varepsilon). \end{aligned} \tag{13}$$

Thus, outside the boundary layer, i.e. $\{|s| > \varepsilon\}$, we have

$$\dot{V} = -2k \cdot s \cdot \mathrm{sgn}(s) = -2k \cdot |s| < 0. \tag{14}$$

If $k > 0$ and $|s(0)| > \varepsilon$, $|s|$ will strictly be decreasing until it reaches the set $|S| \le \varepsilon$ in a finite time, and remains inside thereafter.

On the other hand, inside the boundary layer, i.e. $\{|s| \le \varepsilon\}$, we have

$$\dot{V} = -2k \cdot s \cdot \frac{s}{\varepsilon} = -\frac{2k}{\varepsilon}s^2 < 0. \tag{15}$$

Obviously, inside the boundary layer we have $V > 0$ and $\dot{V} < 0$. Therefore the closed loop system in the presence of the controller (11) is globally asymptotically stable.

Remark 2 Consider that system (2) is perturbed by uncertainties and an external disturbance, which can be modeled as follows:

$$\begin{cases} \dfrac{\mathrm{d}^{q_1}x}{\mathrm{d}t^{q_1}} = f(x,y,z) - \alpha x + \Delta f(x,y,z) + \xi(t) + u(t), \\ \dfrac{\mathrm{d}^{q_2}y}{\mathrm{d}t^{q_2}} = x \cdot g(x,y,z) + z \cdot \phi(x,y,z) - \beta y, \\ \dfrac{\mathrm{d}^{q_3}z}{\mathrm{d}t^{q_3}} = x \cdot h(x,y,z) - y \cdot \phi(x,y,z) - \gamma z, \end{cases} \tag{16}$$

where $\Delta f(x,y,z)$ and $\xi(t)$ are assumed to be bounded, i.e., $|\Delta f(x,y,z)| \le d_1$ and $|\xi(t)| \le d_2$.

The closed loop system with sliding mode control (11) is globally asymptotically stable, when $k > d_1 + d_2$.

Proof Selecting Lyapunov candidate (12), we have

$$\begin{aligned} \dot{V} &= 2s\dot{s} = 2s[D^{q_1}x(t) + \psi(t)] \\ &= 2s[f(x,y,z) - \alpha x + \Delta f(x,y,z) + \xi(t) + u(t) \\ &\quad + y \cdot g(x,y,z) + z \cdot h(x,y,z) + \alpha x + \rho x] \\ &= 2s[f(x,y,z) + \Delta f(x,y,z) + \xi(t) + u(t) \\ &\quad + g \cdot f(x,y,z) + z \cdot h(x,y,z) + \rho x] \\ &= 2s[f(x,y,z) + \Delta f(x,y,z) + \xi(t) - f(x,y,z) \\ &\quad - y \cdot g(x,y,z) - z \cdot h(x,y,z) - \rho x - k \cdot \mathrm{sat}(s/\varepsilon) \\ &\quad + y \cdot g(x,y,z) + z \cdot h(x,y,z) + \rho x] \\ &= 2s \cdot (\Delta f(x,y,z) + \xi(t) - k \cdot \mathrm{sat}(s/\varepsilon)). \end{aligned} \tag{17}$$

Outside the boundary layer, i.e. $\{|s| > \varepsilon\}$, we have

$$\begin{aligned} \dot{V} &= 2s \cdot (\Delta f(x,y,z) + \xi(t) - k \cdot \mathrm{sgn}(s)) \\ &< 2(d_1 + d_2 - k) \cdot |s| < 0. \end{aligned} \tag{18}$$

Inside the boundary layer, i.e. $\{|s| \le \varepsilon\}$, we have

$$\begin{aligned} \dot{V} &= 2s \cdot \left(\Delta f(x,y,z) + \xi(t) - k \cdot \frac{s}{\varepsilon}\right) \\ &< 2(d_1 + d_2) \cdot |s| - \frac{2k}{\varepsilon} \cdot s^2 \\ &= 2(d_1 + d_2) \cdot V^{1/2} - \frac{2k}{\varepsilon} \cdot V, \end{aligned} \tag{19}$$

which implies that

$$V^{1/2} \le V^{1/2}(0) \cdot \mathrm{e}^{-\varepsilon t/k} + \frac{\varepsilon(d_1 + d_2)}{k}. \tag{20}$$

This means that

$$|S| \le |S(0)| \cdot \mathrm{e}^{-\varepsilon t/k} + \varepsilon(d_1 + d_2)/k,$$

i.e. $\lim\limits_{t\to\infty} |S| \le \varepsilon(d_1 + d_2)/k$.

Obviously, we can have a constant N such that system (2) is controlled within an ultimate error range of $N(d_1+d_2)$, i.e., $\lim\limits_{t\to\infty} |\mathrm{e}_i(t)| \le N(d_1+d_2)$, $i = 1, 2, 3$.

4. Simulation results

In this section, we evaluate the performance of our control scheme by applying the method to three different chaotic systems. The simulation is carried out using the MATLAB software, and the control actions are implemented at time t=0. We set $\varepsilon = 0.1$ in this paper.

Case I Here, we consider a new fractional-order chaotic system, which is proposed by Sheu *et al.*[16]

$$\begin{cases} \dfrac{\mathrm{d}^{q_1}x}{\mathrm{d}t^{q_1}} = -yz + ax, \\ \dfrac{\mathrm{d}^{q_2}y}{\mathrm{d}t^{q_2}} = xz - by, \\ \dfrac{\mathrm{d}^{q_3}z}{\mathrm{d}t^{q_3}} = \dfrac{1}{3}xy - cz. \end{cases} \tag{21}$$

The fractional-order system is called commensurate fractional-order system, when $q_1 = q_2 = q_3 = q$. Here $q = 0.90$. The simulation is performed with the initial value $[x, y, z]^{\mathrm{T}} = [5, 3, 1]^{\mathrm{T}}$. In the case where $a = 5$, $b = 10$, and $c = 3.8$, the system exhibits chaotic behaviour as shown in Figs. 1(a)–1(d) and the state variables are shown in Figs. 2(a)–2(c).

To guarantee reaching the sliding surface and finally stabilizing the system, an adaptive surface and a proper $u(t)$ are chosen as

$$\begin{aligned} s(t) &= D^{q-1}x(t) + D^{-1}\psi(t) \\ &= D^{q-1}x(t) + \int_0^t \psi(\tau)\mathrm{d}\tau \end{aligned} \tag{22}$$

with

$$\psi(t) = \frac{4}{3}yz + \alpha x + \rho x, \tag{23}$$

and

$$u(t) = -\frac{1}{3}yz - \rho x - k \cdot \mathrm{sat}(s/\varepsilon). \tag{24}$$

Figures 3(a)–3(d) show the state trajectories of the states x, y, z and the sliding surface s in the presence of the control with respect to $k = 3$ and $\rho = 8$, which shows that the obtained theoretical results are feasible and efficient for the controlled fractional-order chaotic system.

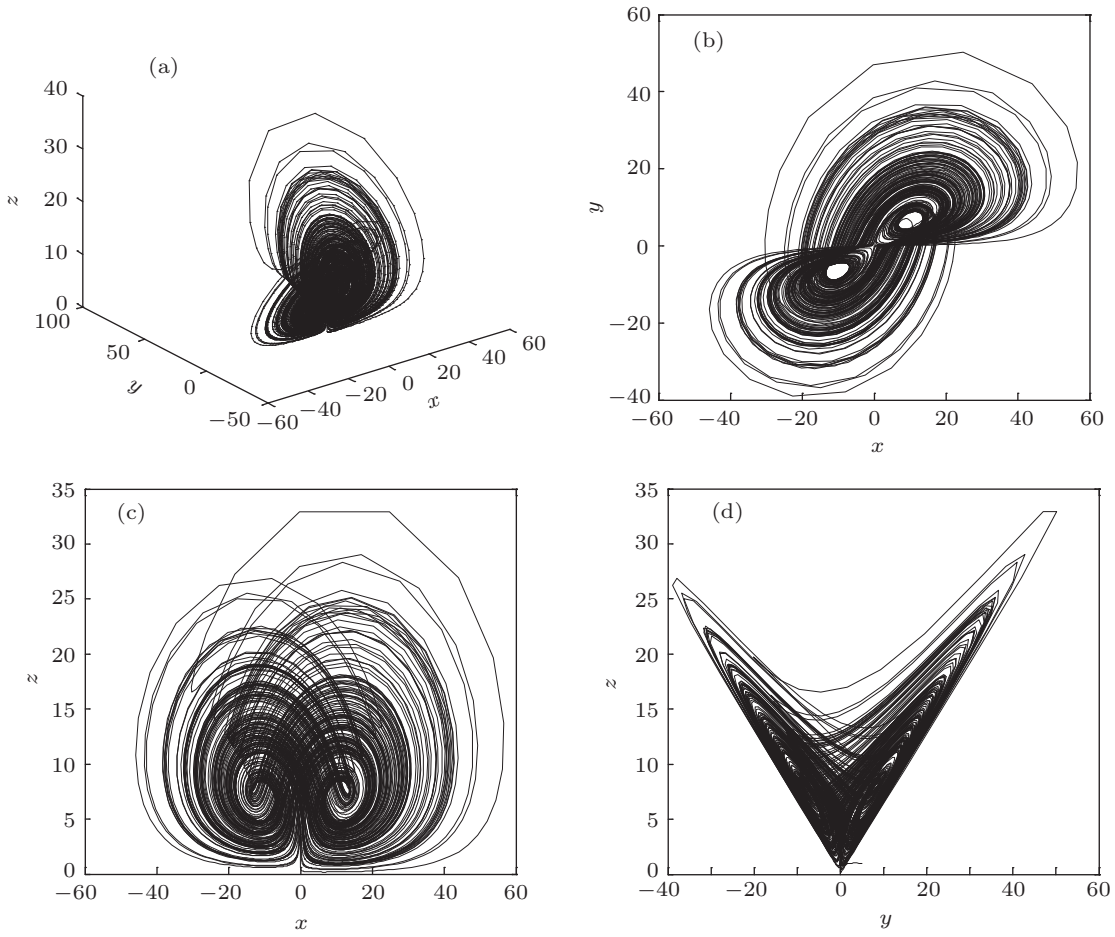


Fig. 1. State trajectories of the new fractional-order chaotic system: (a) state trajectory of x, y, z; (b) state trajectory of x, y; (c) state trajectory of x, z; (d) state trajectory of y, z.

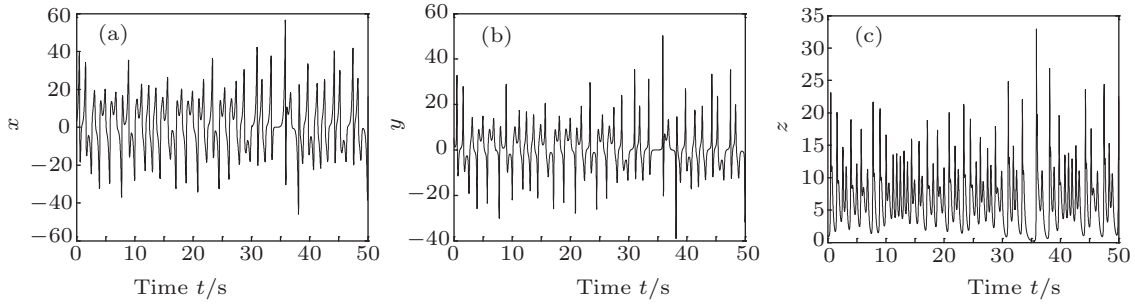


Fig. 2. State variables of the new fractional-order chaotic system without control signals: (a) state variable of x; (b) state variable of y; (c) state variable of z.

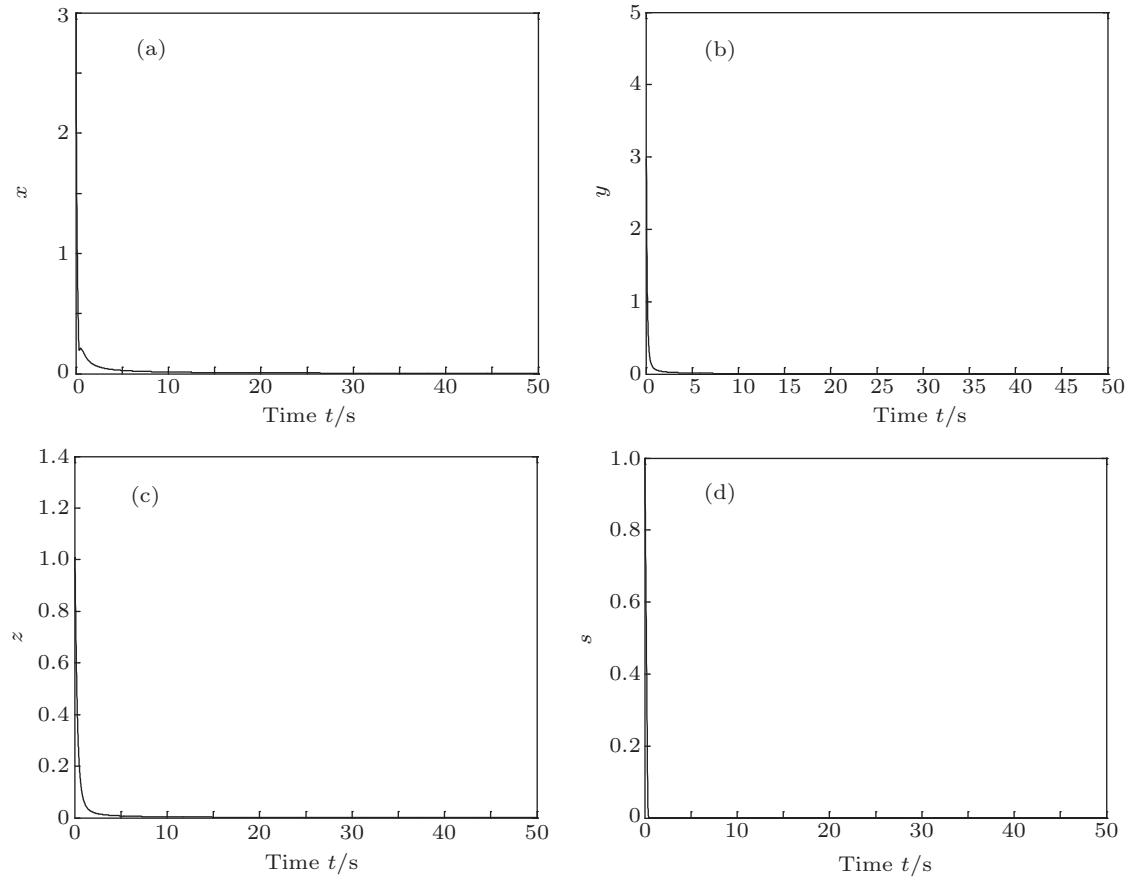


Fig. 3. State trajectories of new fractional-order chaotic system when the controller is activated: (a) state trajectory of x; (b) state trajectory of y; (c) state trajectory of z; (d) state trajectory of s.

Case II Consider the fractional-order Lorenz system which is written as[17]

$$\begin{cases} \dfrac{\mathrm{d}^{q_1}x}{\mathrm{d}t^{q_1}} = a(y-x), \\ \dfrac{\mathrm{d}^{q_2}y}{\mathrm{d}t^{q_2}} = x(b-z)-y, \\ \dfrac{\mathrm{d}^{q_3}z}{\mathrm{d}t^{q_3}} = xy-cz, \end{cases} \tag{25}$$

where the fractional-order $q_1 = q_3 = 1.0$, and $q_2 = 0.98$, i.e., the system is a non-commensurate fractional-order system. When $(a, b, c) = (10, 28, 8/3)$, system (25) demonstrates a chaotic attractor. The chaotic figures for system (25) are shown in Figs. 4(a)–4(d), and the state variables are shown in Figs. 5(a)–5(c).

Regarding Eqs. (3) and (11), the sliding surface and the control law are chosen as

$$\begin{aligned} s(t) &= D^{q_1-1}x(t) + D^{-1}\psi(t) \\ &= D^{q_1-1}x(t) + \int_0^t \psi(\tau)\mathrm{d}\tau, \end{aligned} \tag{26}$$

with

$$\psi(t) = yb + \alpha x + \rho x, \tag{27}$$

and

$$u(t) = -ay - yb - \rho x - k \cdot \mathrm{sat}(s/\varepsilon). \tag{28}$$

The simulation results are shown in Figs. 6(a)–6(d), when the reach gain $k = 2$, $\rho = 6$, and the initial value $[x, y, z]^{\mathrm{T}} = [-2\sqrt{17} + 2.5, -2\sqrt{17}, 14]$. The states of system (25) under controller (28) and sliding surface (26) are illustrated in Fig. 6, showing that the sliding control law guarantees reaching the sliding surface and finally stabilizing.

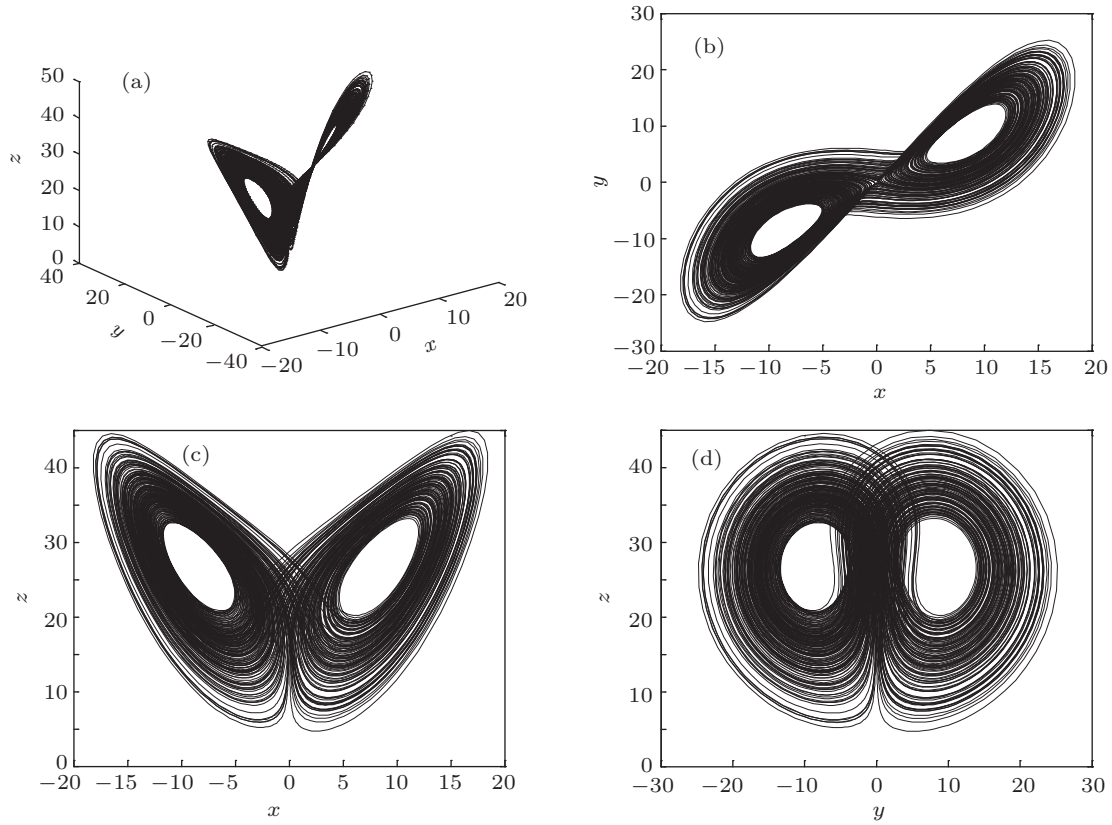


Fig. 4. State trajectories of the non-commensurate fractional-order Lorenz system: (a) state trajectory of x, y, z; (b) state trajectory of x, y; (c) state trajectory of x, z; (d) state trajectory of y, z.

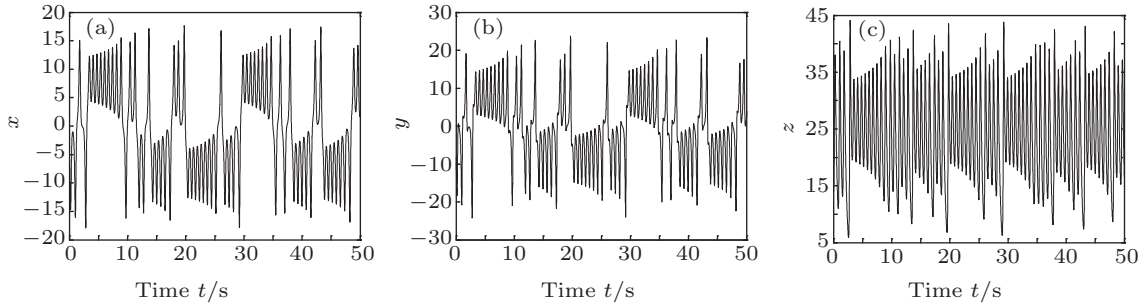
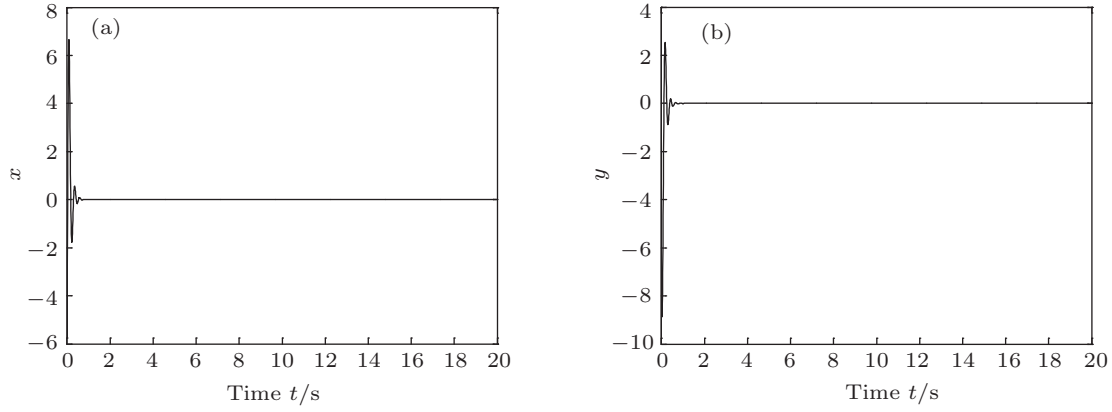


Fig. 5. State variables of the non-commensurate fractional-order Lorenz system without control signals: (a) state variable of x; (b) state variable of y; (c) state variable of z.



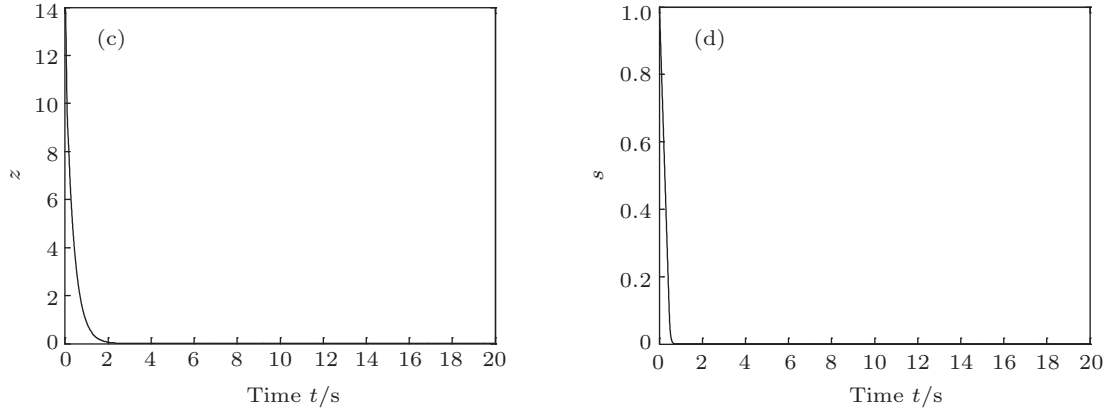


Fig. 6. State trajectories of the non-commensurate fractional-order Lorenz system when the controller is activated: (a) state trajectory of x; (b) state trajectory of y; (c) state trajectory of z; (d) state trajectories of s.

Case III Fractional-order Qi oscillator

The fractional-order Qi oscillator is described as follows:[18]

$$\frac{\mathrm{d}^{q_1}x}{\mathrm{d}t^{q_1}} = a(y-x) + yz, \tag{29a}$$

$$\frac{\mathrm{d}^{q_2}y}{\mathrm{d}t^{q_2}} = cx - y - xz, \tag{29b}$$

$$\frac{\mathrm{d}^{q_3}z}{\mathrm{d}t^{q_3}} = xy - bz, \tag{29c}$$

where x, y, and z are state variables and $q_1 = q_2 = q_3 = q = 0.96$. The simulation is conducted with the initial value $[x, y, z]^{\mathrm{T}} = [-40, -30, 60]^{\mathrm{T}}$. When $a = 35$, $b = 8/3$, and $c = 80$, the system exhibits chaotic behaviour as shown in Figs. 7(a)–7(d).

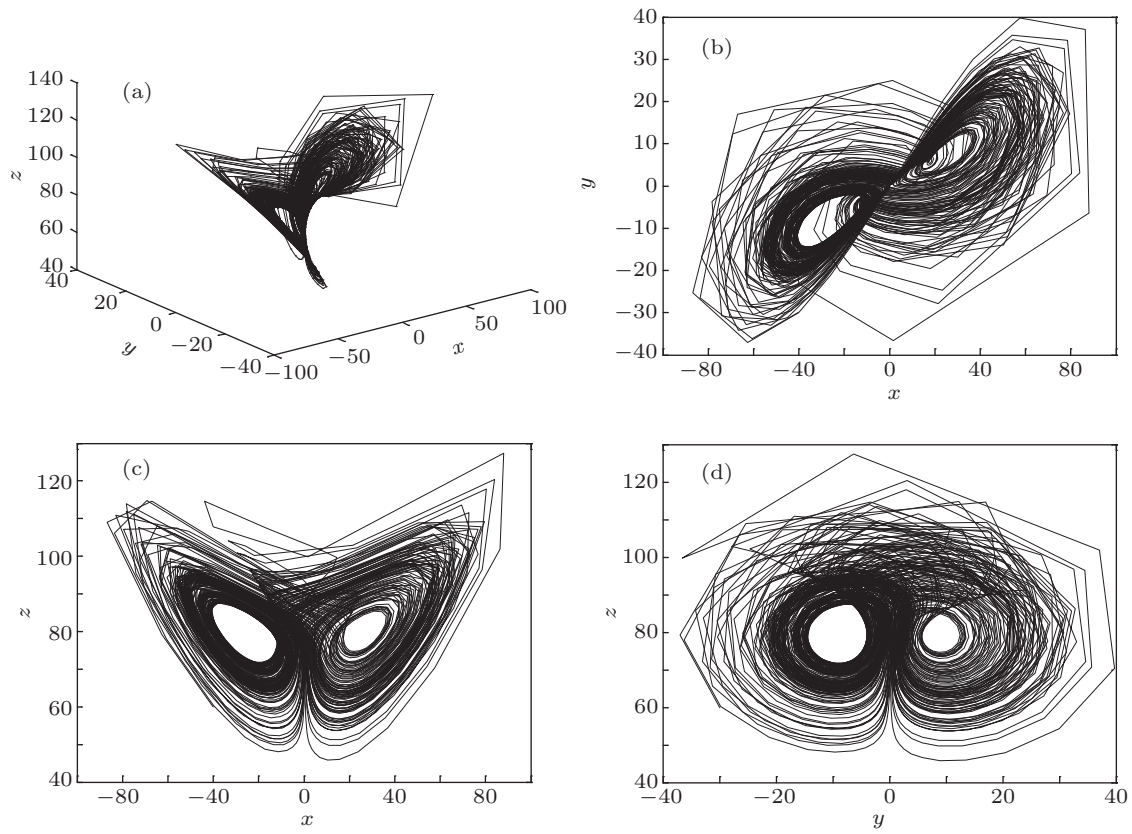


Fig. 7. State trajectories of the fractional-order Qi oscillator: (a) state trajectory of x, y, z; (b) state trajectory of x, y; (c) state trajectory of x, z; (d) state trajectory of y, z.

By adding the controller and considering $\Delta f(x, y, z)$ as an uncertainty term, $\xi(t)$ as an external disturbance, equation (29) can be rewritten as

$$\begin{cases} \dfrac{\mathrm{d}^{q_1}x}{\mathrm{d}t^{q_1}} = a(y-x) + yz + \Delta f(x,y,z) + \xi(t) + u(t), \\ \dfrac{\mathrm{d}^{q_2}y}{\mathrm{d}t^{q_2}} = cx - y - xz, \\ \dfrac{\mathrm{d}^{q_3}z}{\mathrm{d}t^{q_3}} = xy - bz. \end{cases} \tag{30}$$

To guarantee reaching the sliding surface and finally stabilizing the system, an adaptive surface and a proper $u(t)$ are chosen, respectively, as

$$\begin{aligned} s(t) &= D^{q_1-1}x(t) + D^{-1}\psi(t) \\ &= D^{q_1-1}x(t) + \int_0^t \psi(\tau)\,\mathrm{d}\tau, \end{aligned} \tag{31}$$

with

$$\psi(t) = yc + \alpha x + \rho x, \tag{32}$$

and

$$u(t) = -ay - yz - yc - \rho x - k \cdot \mathrm{sat}(s/\varepsilon). \tag{33}$$

The state variables are shown in Figs. 8(a)–8(c) in the presence of an uncertainty term and an external disturbance. When the reach gain $k = 3$, $\rho = 8$, $\Delta f(x, y, z) = \sin(\pi x)\sin(2\pi y)\cos(\pi z)$, and $\xi(t) = 0.1\sin(t)$, the states of system (30) under controller (33) and sliding surface (31) are shown in Figs. 9(a)–9(d). The simulation results show that the obtained theoretical results are feasible and efficient for controlling uncertain Qi oscillator (Figs. 8 and 9).

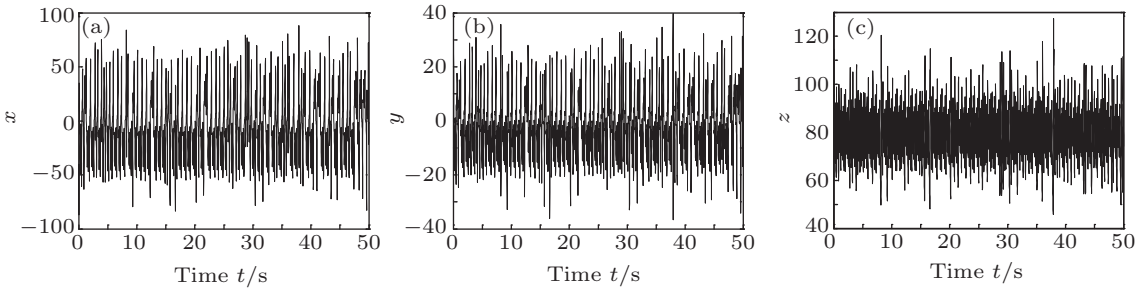


Fig. 8. State variables of the uncertain fractional-order Qi oscillator without control signals: (a) state variable of x; (b) state variable of y; (c) state variable of z.

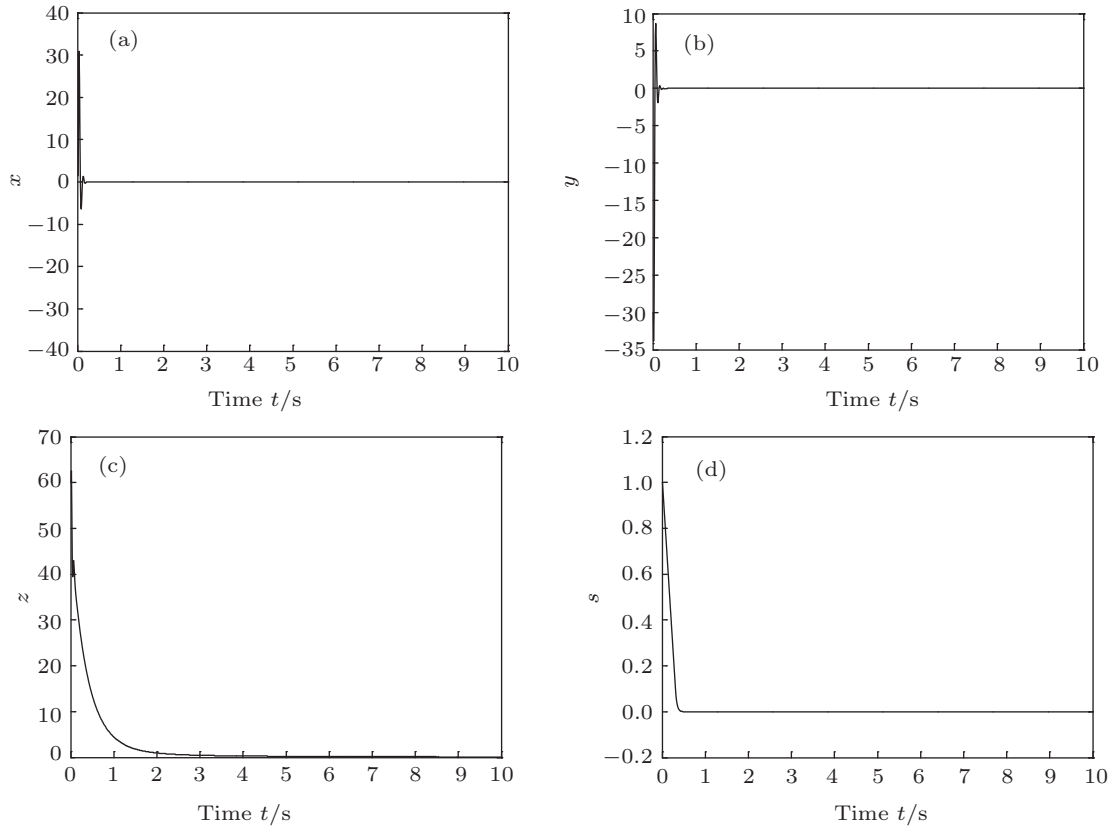


Fig. 9. State trajectories of the uncertain fractional-order Qi oscillator when the controller is activated: (a) state trajectory of x; (b) state trajectory of y; (c) state trajectory of z; (d) state trajectories of s.

5. Conclusion

In this paper, a no-chattering sliding mode control scheme is proposed for the stabilizing of a class of fractional-order chaotic systems. Based on the sliding mode control method, the states of the commensurate fractional-order system and the non-commensurate fractional-order system are stabilized. The proposed technique is also applied to the systems in the presence of uncertainty and external disturbance. Finally, numerical simulations are given to verify the effectiveness of the proposed control scheme.

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