Controlling chaos in power system based on finite-time stability theory*

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(Received 11 January 2011; revised manuscript received 30 June 2011)

Recent investigations show that a power system is a highly nonlinear system and can exhibit chaotic behaviour leading to a voltage collapse, which severely threatens the secure and stable operation of the power system. Based on the finite-time stability theory, two control strategies are presented to achieve finite-time chaos control. In addition, the problem of how to stabilize an unstable nonzero equilibrium point in a finite time is solved by coordinate transformation for the first time. Numerical simulations are presented to demonstrate the effectiveness and the robustness of the proposed scheme. The research in this paper may help to maintain the secure operation of power systems.

Keywords: power system, chaos control, finite-time stability, stabilize unstable nonzero equilibrium point, robust controller

PACS: 05.45.–a **DOI:** 10.1088/1674-1056/20/12/120501

1. Introduction

In the last two decades, extensive investigation has been made of the properties of nonlinear dynamical systems. One of the most important properties of dynamic systems is chaos. Chaotic systems have attracted a great deal of attention because of their powerful application in the areas of engineering science, information processing, biological engineering and secure communication.[1−4] Before 1990, most researchers believed that chaotic systems could not be controlled to a desired target. However, Ott *et al.*[5] first presented the method of chaos control in 1990. Since then enormous studies have been carried out in chaos control by many researchers using different control strategies. In recent years, more and more people have realized the importance of chaos control in power systems because of the demand of the secure and stable operation of the system.[6,7] Recent studies have found that a power system is a highly nonlinear system, which contains many complex dynamic behaviours, such as low-frequency oscillations, bifurcations and even chaotic behaviours. Due to their complexity, the mechanism of chaotic phenomenon in a power system has been very little known up to now, thus chaos is often called the 'strange attractor'. In recent years, some of the major power systems have suffered from stabilization loss and even collapse events in succession.[8,9] At first, scientists thought that the low-frequency oscillations caused by negative damping were the main reason for those destabilization and breakdowns, but these accidents could still happen when people strengthened the systematic damping through the addition of an excitation controller.[8] With the chaos control theory applied to the investigation of electric power system dynamics, researchers began to realize that besides low-frequency oscillations, there are chaotic behaviours in a power system.[10] It is known that a chaotic power system is generally expressed by nonlinear equations and has sensitive dependence on its initial conditions, including system parameters. Chaotic motion can not only destroy the secure and stable operation of a power system, but also immediately damage the whole power system.

*Project supported by the National High Technology Research and Development Program of China (Grant No. 2007AA041401), Tianjin Natural Science Foundation, China (Grant Nos. 08JCZDJC18600 and 09JCZDJC23900), and the University Science and Technology Development Foundation of Tianjin City, China (Grant No. 2006ZD32).

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Therefore, it is significant to study the method of controlling chaos in a power system and to find effective control strategies.

Up to now, many studies have achieved the chaos control by theoretical analysis.[11−15] The main aim of the present paper is to find a proper and applicable method of controlling chaos in a power system. Currently, more and more researchers have noticed the important role of finite-time stability in chaos control and synchronization. In Ref. [16], according to the finite-time stability theory, Wei *et al.* controlled the chaos in a permanent magnet synchronous motor. Finite-time generalized synchronization of chaotic systems with different orders was investigated by Cai *et al.*[17] for the first time. Wang *et al.*[18] proposed a controller to realize the finite-time chaos stabilization of three-dimensional chaotic systems.

Motivated by the above findings, in the present paper, we construct a controller to stabilize unstable states for a power system. Based on the finite-time stability theory, two control strategies are presented to realize the chaos control in a finite time. In addition, the problem about how to stabilize an unstable nonzero equilibrium point in a finite time is solved by coordinate transformation for the first time. Numerical simulation results show that the presented controller is very effective and robust against external noise interference.

2. Power system model

In this paper, we follow Chiang *et al.*[19] and consider a three-bus power system model. It is illustrated in Fig. 1.

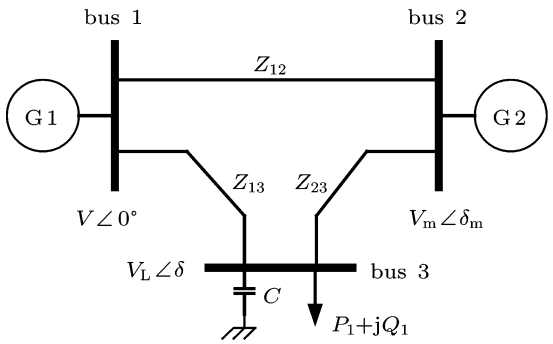


Fig. 1. A three-bus power system.

This model consists of two generators feeding a load, which is represented by an induction motor in parallel with a capacitor and a PQ load. One generator is an infinite bus and the other generator has a constant voltage magnitude $E_{\rm m}$. The load model is described by

$$\begin{aligned} P_d &= P_0 + P_1 + K_{pw}\dot{\delta} + K_{pv}\left(V + T\dot{V}\right), \\ Q_d &= Q_0 + Q_1 + K_{qw}\dot{\delta} + K_{qv}V + K_{qv2}V^2, \end{aligned} \quad (1)$$

where P_0 and Q_0 represent the constant real and the reactive powers of the motor, respectively, and P_1 and Q_1 are the P–Q load. Then the equations that governs the power system can be written as[9]

$$\begin{aligned} \dot{\delta}_{\rm m} &= \omega, \\ \dot{\omega} &= 16.6667\sin\left(\delta - \delta_{\rm m} + 0.0873\right)V_{\rm L} \\ &\quad - 0.1667\omega + 1.8807, \\ \dot{\delta} &= 496.8718V_{\rm L}^2 - 166.6667\cos\left(\delta - \delta_{\rm m} - 0.0873\right)V_{\rm L} \\ &\quad -666.6667\cos\left(\delta - 0.2094\right)V_{\rm L} \\ &\quad - 93.3333V_{\rm L} + 33.3333Q_1 + 43.333, \\ \dot{V}_{\rm L} &= -78.7638V_{\rm L}^2 + 26.2172\cos\left(\delta - \delta_{\rm m} - 0.0124\right)V_{\rm L} \\ &\quad + 104.8689\cos\left(\delta - 0.1346\right)V_{\rm L} \\ &\quad + 14.5229V_{\rm L} - 5.2288Q_1 - 7.0327, \end{aligned} \quad (2)$$

where $\delta_{\rm m}$, ω, δ, and $V_{\rm L}$ are the state variables; $\delta_{\rm m}$ and δ are the power angles of the machine and the load, respectively; ω is the radian frequency of the machine; $V_{\rm L}$ is the load voltage; Q_1 plays an important role in the dynamical behaviour of the entire power system and is usually taken as the bifurcation parameter of the power system. Chiang *et al.*[19] found that when the Q_1 set as different values, power systems experience complicated dynamic bifurcations, as shown in Fig. 2.

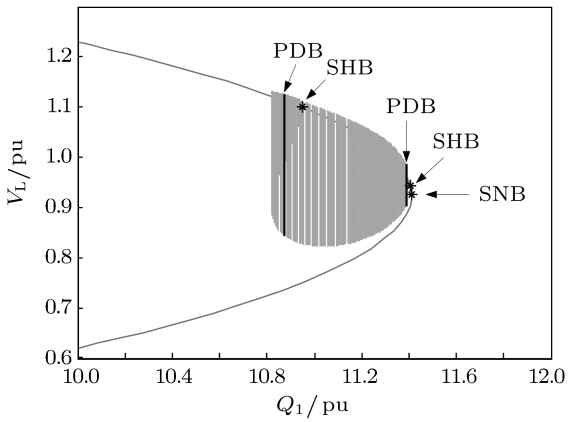


Fig. 2. Bifurcation diagram.

From Fig. 2, we can see that the value of parameter Q_1 depends on the type of bifurcation, which is summarized as follows:[20]

(i) periodic doubling bifurcation at $Q_1 = 10.8859$;

(ii) subcritical Hopf bifurcation at $Q_1 = 10.9461$;

(iii) periodic doubling bifurcation at $Q_1 = 11.3776$;

(iv) subcritical Hopf bifurcation at $Q_1 = 11.4066$;

(v) saddle-node bifurcation at $Q_1 = 11.4106$.

We calculate the Lyapunov exponents by using the method of singular-value decomposition with setting $Q_1 = 11.3776$ and choosing initial condition $(\delta_{\rm m},\ \omega,\ \delta,\ V_{\rm L})_{t=0} = (0.3,\ 0.2,\ 0.97)$. The four Lyapunov exponents of power system (2) are obtained to be $\lambda_1 = 0.3136$, $\lambda_2 = 0$, $\lambda_3 = -3.7496$, and $\lambda_4 = -93.0108$, which satisfy the necessary and sufficient conditions for the existence of a chaotic attractor. Therefore, chaos exists in power system (2). Taking the above mentioned analyses into consideration, the power system undergoes the chaotic state and the typical chaotic attractor is shown in Fig. 3.

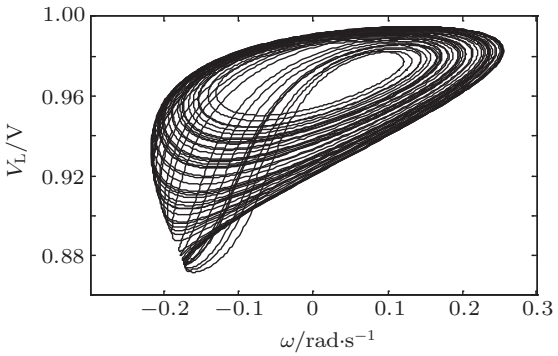


Fig. 3. Typical chaotic attractor.

From Fig. 3, we can see that when the power system falls into the chaotic behaviour, the stable operation of power system is severely hampered. Therefore, it is necessary to control the chaos in the power system. In the present paper, we focus mainly on how to control the chaos in the power system.

3. Preliminary definitions and lemmas

Finite-time stability means that the state of the dynamic system converges to a desired target in a finite time. First, some necessary preliminary lemmas are introduced. Consider the nonlinear dynamical system modeled by[21]

$$\dot{x} = f(x), \tag{3}$$

where the state variable $x \in R^n$. If there exists a constant $T > 0$ ($T > 0$ may depend on the initial state $x(0)$), such that

$$\lim_{t \to T} \|x(t)\| = 0, \tag{4}$$

and $\|x(t)\| \equiv 0$, if $t \geq T$, then system (3) is finite-time stable.

Lemma 1[17] Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leq -\alpha V^{\lambda}(t), \quad \forall t \geq t_0, \quad V(t_0) \geq 0, \tag{5}$$

where $\alpha > 0$ and $0 < \lambda < 1$ are constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\lambda}(t) \leq V^{1-\lambda}(t_0) - \alpha(1-\lambda)(t-t_0), \quad t_0 \leq t \leq t_1, \tag{6}$$

and

$$V(t) \equiv 0, \quad \forall t \geq t_1 \tag{7}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\lambda}(t_0)}{\alpha(1-\lambda)}. \tag{8}$$

Proof Consider the following differential equation:

$$\dot{X}(t) = -\alpha X^{\lambda}(t), \quad X(t_0) = V(t_0). \tag{9}$$

Although this differential equation does not satisfy the global Lipschitz condition, the unique solution to this equation can be found to be

$$X^{1-\lambda}(t) = X^{1-\lambda}(t_0) - \alpha(1-\lambda)(t-t_0), \tag{10}$$

and

$$x(t) \equiv 0, \quad \forall t \geq t_1. \tag{11}$$

It is direct to prove that $x(t)$ is differential for $t > t_0$. From the comparison lemma,[22] one obtains

$$V^{1-\lambda}(t) \leq V^{1-\lambda}(t_0) - \alpha(1-\lambda)(t-t_0), \quad t_0 \leq t \leq t_1, \tag{12}$$

and

$$V(t) \equiv 0, \quad \forall t \geq t_1, \tag{13}$$

where t_1 is given by expression (8).

Lemma 2 When a, b, and $\eta < 1$ are all positive numbers, the following inequality holds:

$$(a+b)^{\eta} \leq a^{\eta} + b^{\eta}. \tag{14}$$

This result is quite straightforward and the proof is omitted here.

4. Chaos control in a power system based on the finite-time stability theory

As is well known, a power system is a practical engineering plant, the feasibility of control must be taken into account, namely, the operational variable must be accessible and the control objective must be realizable. In order to control chaotic behaviour in a power system, we add the control law u_i ($i = 1, 2, 3, 4$) to the right-hand side of system (2) and then the controlled system can be expressed by

$$\begin{aligned}
\dot{\delta}_{\rm m} &= \omega + u_1,\\
\dot{\omega} &= 16.6667\sin(\delta - \delta_{\rm m} + 0.0873)\,V_{\rm L}\\
&\quad - 0.1667\omega + 1.8807 + u_2,\\
\dot{\delta} &= 496.8718V_{\rm L}^2 - 166.6667\cos(\delta - \delta_{\rm m} - 0.0873)\,V_{\rm L}\\
&\quad - 666.6667\cos(\delta - 0.2094)\,V_{\rm L}\\
&\quad - 93.3333V_{\rm L} + 33.3333Q_1 + 43.333 + u_3,\\
\dot{V}_{\rm L} &= -78.7638V_{\rm L}^2 + 26.2172\cos(\delta - \delta_{\rm m} - 0.0124)\,V_{\rm L}\\
&\quad + 104.8689\cos(\delta - 0.1346)\,V_{\rm L}\\
&\quad + 14.5229V_{\rm L} - 5.2288Q_1 - 7.0327 + u_4. \qquad (15)
\end{aligned}$$

So far, there have been a few control methods for chaos control using the finite-time stability theory. The existing control strategies usually design a controller to stabilize the chaotic system at the equilibrium point $X_{\rm eq} = \{0\}$ within a finite time.[13,16−18] However, to the best of our knowledge, in most cases the equilibrium point is nonzero, that is, $X_{\rm eq} \neq \{0\}$ (e.g. system (2)). To overcome this limitation, coordinate transformation $y = x - X_{\rm eq}$ is introduced for the first time. In this paper, based on the finite-time stability theory, two control strategies are presented to realize the chaos control in a finite time. Assume that the equilibrium point is $X_{\rm eq} = (\delta_{\rm m}^*, \omega^*, \delta^*, V_{\rm L}^*)$ ($X_{\rm eq} \neq \{0\}$), then the controlled new system (15) by coordinate transformation will be given as

$$\begin{aligned}
\dot{\delta}_{\rm m} &= (\omega - \omega^*) + u_1,\\
\dot{\omega} &= 16.6667\sin((\delta - \delta^*) - (\delta_{\rm m} - \delta_{\rm m}^*) + 0.0873)\\
&\quad \times (V_{\rm L} - V_{\rm L}^*) - 0.1667(\omega - \omega^*) + 1.8807 + u_2,\\
\dot{\delta} &= 496.8718(V_{\rm L} - V_{\rm L}^*)^2 - 166.6667\cos((\delta - \delta^*)\\
&\quad - (\delta_{\rm m} - \delta_{\rm m}^*) - 0.0873)(V_{\rm L} - V_{\rm L}^*)\\
&\quad - 666.6667\cos((\delta - \delta^*) - 0.2094)(V_{\rm L} - V_{\rm L}^*)\\
&\quad - 93.3333(V_{\rm L} - V_{\rm L}^*) + 33.3333Q_1 + 43.333 + u_3,\\
\dot{V}_{\rm L} &= -78.7638(V_{\rm L} - V_{\rm L}^*)^2 + 26.2172\cos((\delta - \delta^*)\\
&\quad - (\delta_{\rm m} - \delta_{\rm m}^*) - 0.0124)(V_{\rm L} - V_{\rm L}^*)\\
&\quad + 104.8689\cos((\delta - \delta^*) - 0.1346)\\
&\quad \times (V_{\rm L} - V_{\rm L}^*) + 14.5229(V_{\rm L} - V_{\rm L}^*)\\
&\quad - 5.2288Q_1 - 7.0327 + u_4. \qquad (16)
\end{aligned}$$

Now, our aim is to use the finite-time stability theory to design two controllers and make the power system (2) asymptotically stable at $X_{\rm eq}$. With the above knowledge, our main results are proposed in the following.

Control strategy 1

If the controller is designed as

$$\begin{aligned}
u_1 &= -(\omega - \omega^*) - (\delta_{\rm m} - \delta_{\rm m}^*) - (\delta_{\rm m} - \delta_{\rm m}^*)^{\beta},\\
u_2 &= -16.6667\sin((\delta - \delta^*) - (\delta_{\rm m} - \delta_{\rm m}^*) + 0.0873)\\
&\quad \times (V_{\rm L} - V_{\rm L}^*)\\
&\quad - 1.8807 - 0.8333(\omega - \omega^*) - (\omega - \omega^*)^{\beta},\\
u_3 &= -496.8718(V_{\rm L} - V_{\rm L}^*)^2 + 166.6667\cos((\delta - \delta^*)\\
&\quad - (\delta_{\rm m} - \delta_{\rm m}^*) - 0.0873)(V_{\rm L} - V_{\rm L}^*)\\
&\quad + 666.6667\cos((\delta - \delta^*) - 0.2094)(V_{\rm L} - V_{\rm L}^*)\\
&\quad + 93.3333(V_{\rm L} - V_{\rm L}^*) - 33.3333Q_1 - 43.333\\
&\quad - (\delta - \delta^*) - (\delta - \delta^*)^{\beta},\\
u_4 &= 78.7638(V_{\rm L} - V_{\rm L}^*)^2 - 26.2172\cos((\delta - \delta^*)\\
&\quad - (\delta_{\rm m} - \delta_{\rm m}^*) - 0.0124)(V_{\rm L} - V_{\rm L}^*)\\
&\quad - 104.8689\cos((\delta - \delta^*) - 0.1346)(V_{\rm L} - V_{\rm L}^*)\\
&\quad + 5.2288Q_1 + 7.0327 - 15.5229(V_{\rm L} - V_{\rm L}^*)\\
&\quad - (V_{\rm L} - V_{\rm L}^*)^{\beta}, \qquad (17)
\end{aligned}$$

where $\beta = q/p$ is a proper rational number; p and q are positive odd integers and $p > q$. The chaos in power system will be controlled, namely, the power system will be asymptotically stabilized at the equilibrium point $X_{\rm eq}$.

Proof For the equation consisting of Eqs. (16) and (17), which we call the augment equation, a positive-definite Lyapunov function is introduced as follows:

$$\begin{aligned}
V &= \frac{1}{2}(\delta_{\rm m} - \delta_{\rm m}^*)^2 + \frac{1}{2}(\omega - \omega^*)^2\\
&\quad + \frac{1}{2}(\delta - \delta^*)^2 + \frac{1}{2}(V_{\rm L} - V_{\rm L}^*)^2. \qquad (18)
\end{aligned}$$

By differentiating the function V along the trajectories of the augment equation, we have

$$\begin{aligned}
\dot{V} &= (\delta_{\rm m} - \delta_{\rm m}^*)\dot{\delta}_{\rm m} + (\omega - \omega^*)\dot{\omega} + (\delta - \delta^*)\dot{\delta}\\
&\quad + (V_{\rm L} - V_{\rm L}^*)\dot{V}_{\rm L}\\
&= (\delta_{\rm m} - \delta_{\rm m}^*)(-(\delta_{\rm m} - \delta_{\rm m}^*) - (\delta_{\rm m} - \delta_{\rm m}^*)^{\beta})\\
&\quad + (\omega - \omega^*)(-(\omega - \omega^*) - (\omega - \omega^*)^{\beta})\\
&\quad + (\delta - \delta^*)(-(\delta - \delta^*) - (\delta - \delta^*)^{\beta})\\
&\quad + (V_{\rm L} - V_{\rm L}^*)(-(V_{\rm L} - V_{\rm L}^*) - (V_{\rm L} - V_{\rm L}^*)^{\beta})\\
&= -(\delta_{\rm m} - \delta_{\rm m}^*)^2 - (\delta_{\rm m} - \delta_{\rm m}^*)^{1+\beta}\\
&\quad - (\omega - \omega^*)^2 - (\omega - \omega^*)^{1+\beta}
\end{aligned}$$

$$
\begin{aligned}
&-(\delta-\delta^*)^2-(\delta-\delta^*)^{1+\beta}\\
&-(V_{\rm L}-V_{\rm L}^*)^2-(V_{\rm L}-V_{\rm L}^*)^{1+\beta}\\
\le\ &-(\delta_{\rm m}-\delta_{\rm m}^*)^{1+\beta}-(\omega-\omega^*)^{1+\beta}-(\delta-\delta^*)^{1+\beta}\\
&-(V_{\rm L}-V_{\rm L}^*)^{1+\beta}\\
\le\ &-\left(\frac{1}{2}\right)^{-(1+\beta)/2}\left(\frac{1}{2}(\delta_{\rm m}-\delta_{\rm m}^*)^2+\frac{1}{2}(\omega-\omega^*)^2\right.\\
&\left.+\frac{1}{2}(\delta-\delta^*)^2+\frac{1}{2}(V_{\rm L}-V_{\rm L}^*)^2\right)^{(1+\beta)/2}\\
=\ &-\left(\frac{1}{2}\right)^{-(1+\beta)/2}V^{(1+\beta)/2}. \qquad (19)
\end{aligned}
$$

In the proof of inequality (19), we have used Lemma 2 to deduce the above result. Since $0<\beta<1$ and $0<(1+\beta)/2<0$. Then, according to Lemma 1, controlled system (16) is finite-time stable. It implies that there exists a $T>0$ such that $\delta_{\rm m}=\delta_{\rm m}^*$, $\omega=\omega^*$, $\delta=\delta^*$, and $V_{\rm L}=V_{\rm L}^*$, if $t>T$.

Control strategy 2

The design procedure is divided into two steps.

Step 1 Let u_3 be designed as follows:

$$
\begin{aligned}
u_3 =\ &-496.8718(V_{\rm L}-V_{\rm L}^*)^2+166.6667\cos((\delta-\delta^*)\\
&-(\delta_{\rm m}-\delta_{\rm m}^*)-0.0873)(V_{\rm L}-V_{\rm L}^*)\\
&+666.6667\cos((\delta-\delta^*)-0.2094)\\
&\times(V_{\rm L}-V_{\rm L}^*)+93.3333(V_{\rm L}-V_{\rm L}^*)-33.3333Q_1\\
&-43.333-(\delta-\delta^*)-(\delta-\delta^*)^{\beta}, \qquad (20)
\end{aligned}
$$

where $\beta=q/p$ and $p>q$, which are positive odd integers. With the controller u_3, the third equation of expression (16) is

$$\dot{\delta}=-(\delta-\delta^*)-(\delta-\delta^*)^{\beta}. \qquad (21)$$

Consider the candidate Lyapunov function

$$V_1=\frac{1}{2}(\delta-\delta^*)^2, \qquad (22)$$

then the time derivative of V_1 along the trajectory of expression (21) will be

$$
\begin{aligned}
\dot{V}_1 &= (\delta-\delta^*)\dot{\delta}\\
&= (\delta-\delta^*)(-(\delta-\delta^*)-(\delta-\delta^*)^{\beta})\\
&\le -(\delta-\delta^*)^{1+\beta}\\
&= -\left(\frac{1}{2}\right)^{-(1+\beta)/2}\left(\frac{1}{2}(\delta-\delta^*)^2\right)^{(1+\beta)/2}\\
&= -\left(\frac{1}{2}\right)^{-(1+\beta)/2}V_1^{(1+\beta)/2}. \qquad (23)
\end{aligned}
$$

Since $0<\beta<1$, then $0<(1+\beta)/2<1$. From Lemma 1, the state δ will reach $\delta=\delta^*$ at a finite time $T_1=\delta(\delta^*)/(1-\beta)$.

Step 2 If $t>T_1$, then $\delta=\delta^*$. Substituting $\delta=\delta^*$ into system (16) yields

$$
\begin{aligned}
\dot{\delta}_{\rm m} &= (\omega-\omega^*)+u_1,\\
\dot{\omega} &= 16.6667\sin(-(\delta_{\rm m}-\delta_{\rm m}^*)+0.0873)(V_{\rm L}-V_{\rm L}^*)\\
&\quad -0.1667(\omega-\omega^*)+1.8807+u_2,\\
\dot{V}_{\rm L} &= -78.7638(V_{\rm L}-V_{\rm L}^*)^2\\
&\quad +26.2172\cos(-(\delta_{\rm m}-\delta_{\rm m}^*)-0.0124)(V_{\rm L}-V_{\rm L}^*)\\
&\quad +104.8689\cos(-0.1346(V_{\rm L}-V_{\rm L}^*))+14.5229\\
&\quad \times(V_{\rm L}-V_{\rm L}^*)-5.2288Q_1-7.0327+u_4. \qquad (24)
\end{aligned}
$$

Select u_1, u_2, and u_4 as follows:

$$
\begin{aligned}
u_1 &= -(\omega-\omega^*)-(\delta_{\rm m}-\delta_{\rm m}^*)-(\delta_{\rm m}-\delta_{\rm m}^*)^{\beta},\\
u_2 &= -16.6667\sin(-(\delta_{\rm m}-\delta_{\rm m}^*)+0.0873)(V_{\rm L}-V_{\rm L}^*)\\
&\quad -1.8807+H_1(\omega-\omega^*)-(\omega-\omega^*)^{\beta},\\
u_4 &= 78.7638(V_{\rm L}-V_{\rm L}^*)^2\\
&\quad -26.2172\cos(-(\delta_{\rm m}-\delta_{\rm m}^*)-0.0124)(V_{\rm L}-V_{\rm L}^*)\\
&\quad -104.8689\cos(-0.1346(V_{\rm L}-V_{\rm L}^*))+5.2288Q_1\\
&\quad +7.0327-H_2(V_{\rm L}-V_{\rm L}^*)-(V_{\rm L}-V_{\rm L}^*)^{\beta}, \qquad (25)
\end{aligned}
$$

where $H_1\le 0.1667$, and $H_2\ge 14.5229$. Let Eqs. (24) and (25) be the argument equations. Choosing a non-negative Lyapunov function

$$
\begin{aligned}
V_2 = &\frac{1}{2}(\delta_{\rm m}-\delta_{\rm m}^*)^2+\frac{1}{2}(\omega-\omega^*)^2\\
&+\frac{1}{2}(V_{\rm L}-V_{\rm L}^*)^2, \qquad (26)
\end{aligned}
$$

and differentiating the function V_2 along the trajectories of the augment equation, we obtain

$$
\begin{aligned}
\dot{V}_2 &= (\delta_{\rm m}-\delta_{\rm m}^*)\dot{\delta}_{\rm m}+(\omega-\omega^*)\dot{\omega}+(V_{\rm L}-V_{\rm L}^*)\dot{V}_{\rm L}\\
&= (\delta_{\rm m}-\delta_{\rm m}^*)(-(\delta_{\rm m}-\delta_{\rm m}^*)-(\delta_{\rm m}-\delta_{\rm m}^*)^{\beta})\\
&\quad +(\omega-\omega^*)(-(\omega-\omega^*)(0.1667-H_1)\\
&\quad -(\omega-\omega^*)^{\beta})+(V_{\rm L}-V_{\rm L}^*)(-(V_{\rm L}-V_{\rm L}^*)\\
&\quad \times(H_2-14.5229)-(V_{\rm L}-V_{\rm L}^*)^{\beta})\\
&= -(\delta_{\rm m}-\delta_{\rm m}^*)^2-(\delta_{\rm m}-\delta_{\rm m}^*)^{1+\beta}-(\omega-\omega^*)^2\\
&\quad \times(0.1667-H_1)-(\omega-\omega^*)^{1+\beta}-(V_{\rm L}-V_{\rm L}^*)^2\\
&\quad \times(H_2-14.5229)-(V_{\rm L}-V_{\rm L}^*)^{1+\beta}\\
&\le -(\delta_{\rm m}-\delta_{\rm m}^*)^{1+\beta}-(\omega-\omega^*)^{1+\beta}-(V_{\rm L}-V_{\rm L}^*)^{1+\beta}\\
&\le -\left(\frac{1}{2}\right)^{-(1+\beta)/2}\left(\frac{1}{2}(\delta_{\rm m}-\delta_{\rm m}^*)^2+\frac{1}{2}(\omega-\omega^*)^2\right.\\
&\quad \left.+\frac{1}{2}(V_{\rm L}-V_{\rm L}^*)^2\right)^{(1+\beta)2}\\
&= -\left(\frac{1}{2}\right)^{-(1+\beta)/2}V_2^{(1+\beta)/2}. \qquad (27)
\end{aligned}
$$

Since $0<\beta<1$, then $0<(1+\beta)/2<1$. From Lemma 1 it follows that the quantities $\delta_{\rm m}$, ω, and $V_{\rm L}$

will converge to $\delta_{\rm m} = \delta_{\rm m}^*$, $\omega = \omega^*$, and $V_{\rm L} = V_{\rm L}^*$ in a finite time T_2. This means that controlled power system (16) is stabilized at $X_{\rm eq} = (\delta_{\rm m}^*,\ \omega^*,\ \delta^*,\ V_{\rm L}^*)$ ($X_{\rm eq} \neq \{0\}$) in a finite time. To validate the control strategy, effectiveness, numerical simulation results will be shown later.

5. Numerical simulation results

In this section, we give numerical simulations to show the effectiveness of the proposed two control methods. The controlled system (16) of the ordinary differential equations is integrated by using the fourth-order Runge–Kutta in time steps of 0.001 s. In the numerical simulations, we set $Q_1 = 11.3376$, with which the unstable equilibrium point of the system is $X_{\rm eq} = (0.3366,\ 0,\ 0.1330,\ 0.9727)$. The initial conditions $(\delta_{\rm m},\ \omega,\ \delta,\ V_{\rm L})_{t=0} = (0.3,\ 0.2,\ 0.97)$ are chosen in all simulations, and the control parameters of the second controller are taken to be $H_1 = 0.0667$ and $H_2 = 15$. Power system has experienced chaotic behaviours before the two proposed controllers are carried out. To demonstrate why the presented controllers are so effective in controlling the chaos in a power system, we turn on the controller u at $t = 100$ s.

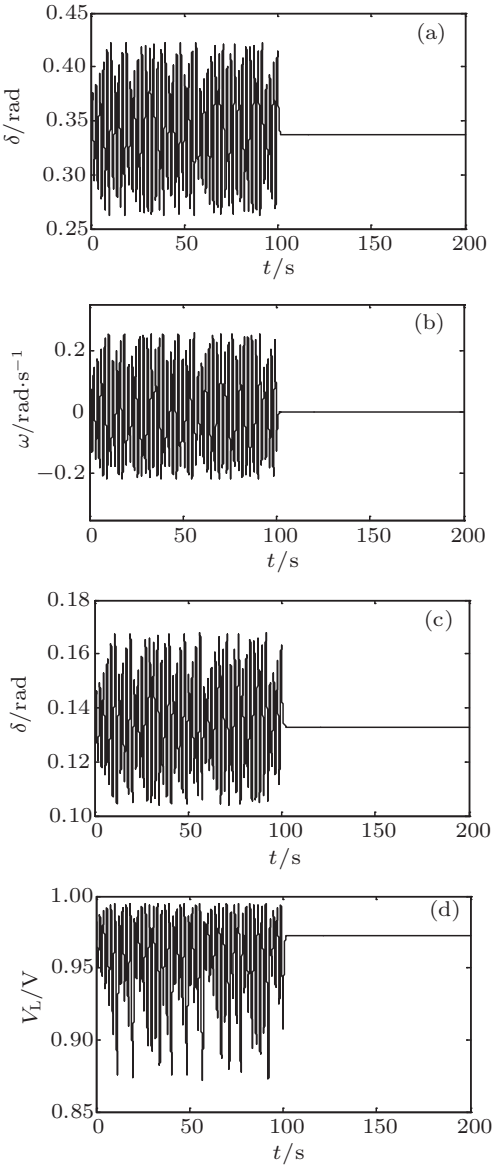


Fig. 4. Control strategy 1: Control chaotic power system to the equilibrium point $X_{\rm eq} = (0.3366,\ 0,\ 0.1330,\ 0.9727)$, with uncontrolled chaotic evolution $t \in [0, 100)$ and controlled dynamics $t \in [100,\ 200)$. Panel (a) shows the dynamic responding curve of $\delta_{\rm m}$, panel (b) the dynamic responding curve of ω, panel (c) the dynamic responding curve of δ, and panel (d) the dynamic responding curve of $V_{\rm L}$.

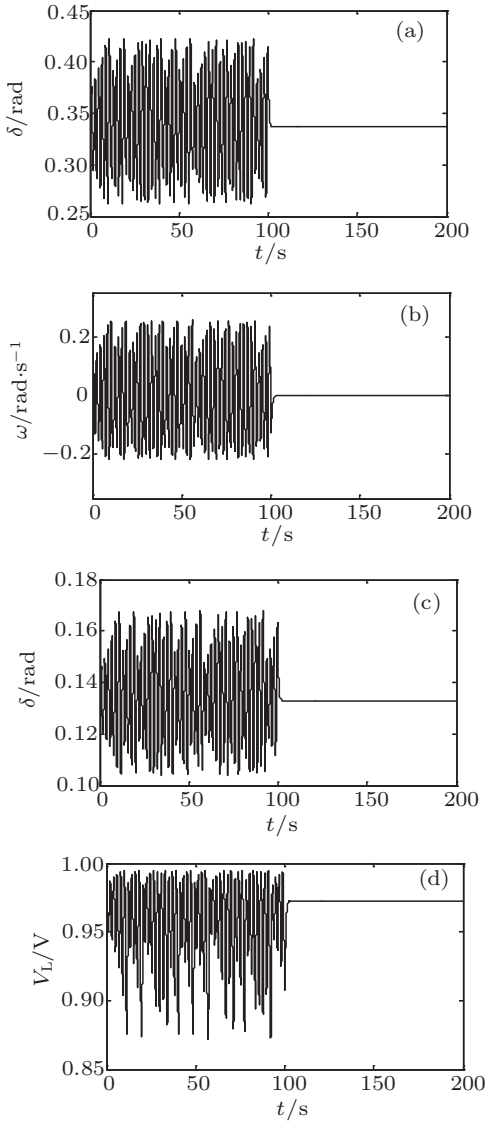


Fig. 5. Control strategy 2: control the chaotic power system to the equilibrium point $X_{\rm eq} = (0.3366,\ 0,\ 0.1330,\ 0.9727)$ when $H_1 = 0.0667$ and $H_2 = 15$, with uncontrolled chaotic evolution $t \in [0,\ 100)$ and controlled dynamics $t \in [100,\ 200)$. Panel (a) shows the dynamic responding curve of $\delta_{\rm m}$, panel (b) the dynamic responding curve of ω, panel (c) the dynamic responding curve of δ, and panel (d) the dynamic responding curve of $V_{\rm L}$.

The dynamic responding curves of the two proposed controllers are shown in Figs. 4 and 5, respectively.

From Figs. 4 and 5, we can see that it takes only a very short time to suppress the chaotic power system and stabilize the controlled system at the equilibrium point $X_{\rm eq}$.

6. Robustness analysis of the adaptive controller

The robustness of a control technique is also an important issue in control theory. Thus we verify the robustness of the presented two control strategies. The parametric values take the same form as those in Section 5. We apply an external noise interference $n(t) = 0.001\cos(0.2\pi t)$ to the first equation of the controlled system (16). Suppose that the control target is still the equilibrium point $X_{\rm eq}$. The dynamic responding curves of the two proposed controllers are shown in Fig. 6 and Fig. 7, respectively.

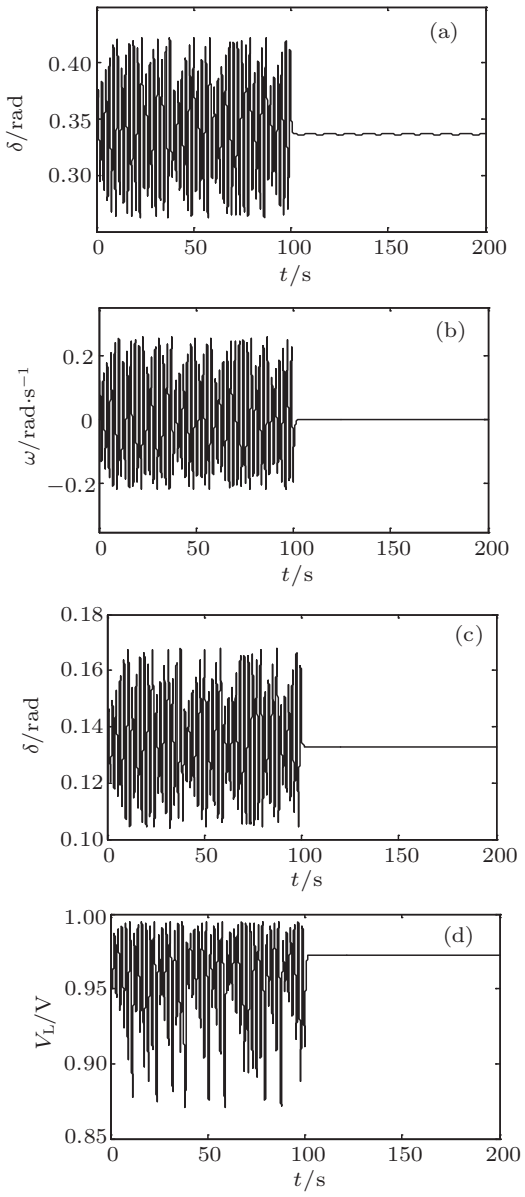


Fig. 6. Control strategy 1: control the chaotic power system to the equilibrium point $X_{\rm eq}$ = (0.3366, 0, 0.1330, 0.9727), with uncontrolled chaotic evolution $t \in [0, 100)$ and controlled dynamics $t \in [100, 200)$. Panel (a) shows the dynamic responding curve of $\delta_{\rm m}$, panel (b) the dynamic responding curve of ω, panel (c) the dynamic responding curve of δ, and panel (d) the dynamic responding curve of $V_{\rm L}$.

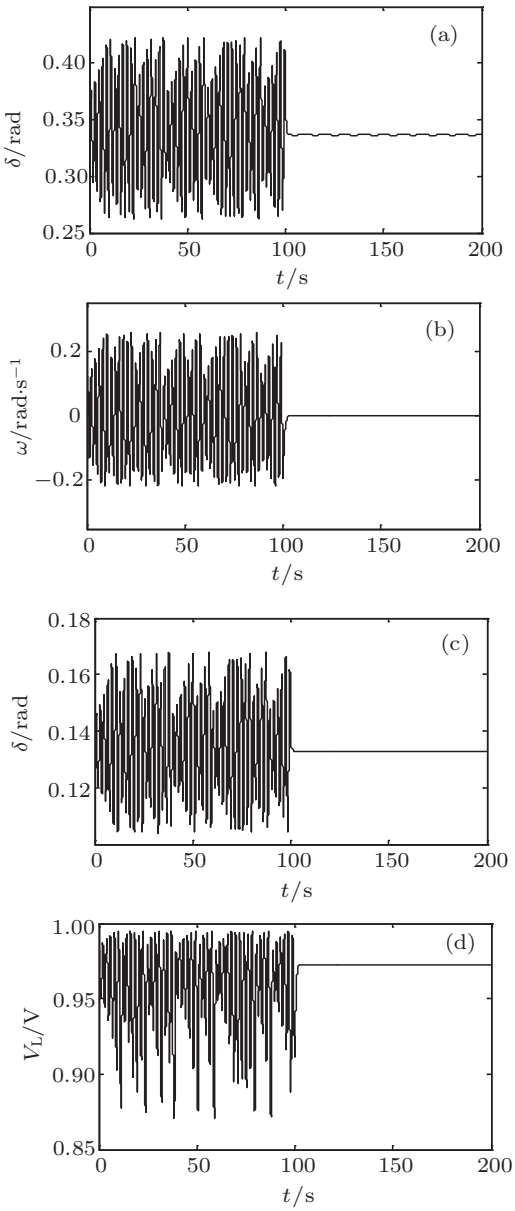


Fig. 7. Control strategy 2: control the chaotic power system to the equilibrium point $X_{\rm eq}$ = (0.3366, 0, 0.1330, 0.9727) when $H_1 = 0.0667$ and $H_2 = 15$, with uncontrolled chaotic evolution $t \in [0, 100)$ and controlled dynamics $t \in [100, 200)$. Panel (a) shows the dynamic responding curve of $\delta_{\rm m}$, panel (b) the dynamic responding curve of ω, panel (c) the dynamic responding curve of δ, and panel (d) the dynamic responding curves of $V_{\rm L}$.

From Figs. 6 and 7, one can see that when the power system has an external noise interference, the proposed two control strategies can still control the chaotic power system to the equilibrium points $X_{\rm eq1}$ = (0.3369, 0, 0.1332, 0.9729) and $X_{\rm eq2}$ = (0.3366, 0, 0.1330, 0.9726), but the control accuracy will be reduced to some extent and the adjustment of time will be increased slightly. Thus, the adaptive controller is certainly robust against external noise interference.

7. Conclusion

In this paper, finite-time chaos control of a chaotic power system is considered. Based on the finite-time stability theory, two control strategies are presented to realize the chaos control in a finite time. In addition, the problem of how to stabilize an unstable nonzero equilibrium point in a finite time is solved by coordinate transformation for the first time. Simulation results show that the presented controllers are very effective and robust against external noise interference. The study of this paper is significant because it is helpful for maintaining the secure operation of a power system. How to control the chaos in a power system with uncertainties in a finite-time is our future work.

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