One-step generation of qutrit entanglement via adiabatic passage in cavity quantum electrodynamics*

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A scheme is proposed for generating a three-dimensional entangled state for two atoms trapped in a cavity by one step via adiabatic passage. In the scheme, the two atoms are always in ground states and the field mode of the cavity excited is negligible under a certain condition. Therefore, the scheme is very robust against decoherence. Furthermore, it needs neither the exact control of all parameters nor the accurate control of the interaction time. It is shown that qutrit entanglement can be generated with a high fidelity.

Keywords: qutrit entanglement, adiabatic passage, cavity quantum electrodynamics

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1. Introduction

Entanglement is one of the most striking features of quantum mechanics. Entanglement is not only fundamental for demonstrating quantum nonlocality, but also useful in quantum information theory, such as quantum cryptographic conference[1] and quantum secrete sharing.[2] In recent years, with the development of quantum information, much attention has been paid to the entanglement of multiple particles. Greenberger *et al.*[3] have studied maximally entangled states of three or more particles, i.e., the Greenberger–Horne–Zeilinger (GHZ) states, which can provide possibilities to test quantum mechanics against local hidden theory without inequality.[4] On the other hand, Dür *et al.*[5] have shown that there are two inequivalent classes of tripartite entanglement states, the GHZ class and the W class, under stochastic local operation and classical communication. The W state, whose general form is $W_n = (1/\sqrt{n})\,|n-1\,,\,1\rangle$ with $|n-1\,,\,1\rangle$ being all the totally symmetric states involving $n-1$ zeros and 1 one, is famous for its robustness to local measurement, even under qubit loss. For example, it retains bipartite entanglement when any one of the three qubits is traced out, which is argued as an important property in those situations where any one of the three particles decides not to cooperate with the others. W state can be used to test quantum nonlocality without inequality.[6] It is also useful in quantum information processing.[7]

However, both the GHZ state and the W state are the entangled states that are defined in Hilbert spaces with two dimensions, i.e., entangled qubits. Recently, entangled states of high-dimensional systems, i.e., entangled qudits (with dimension $d \geq 3$), have attracted great attention. It has been shown that the violations of local realism by entangled qudits are stronger than those by entangled qubits.[8] In optical systems, entangled qudits have been demonstrated and used to verify quantum nonlocal realism.[9−11] It has also been demonstrated that quantum key distribution based on qudit systems is more secure than those based on qubit systems, by use of spatial degrees of freedom of photons based on the standard BB84 protocol.[12]

Cavity quantum electrodynamics (CQED) system is one of the most effective systems for generating entangled states. People have demonstrated the Bell state, the Einstein–Podolsky–Rosen (EPR) state and the GHZ state in the system.[13−16] There have also been many schemes proposed for generating entangled qutrits (with dimension $d = 3$). For example, Zou *et al.*[17] proposed a scheme for generating a maximally entangled state of two three-level atoms with a nonresonant cavity by cavity-assisted collisions. Zheng[18] presented a scheme for generating entangled states for two or more multilevel atoms in a thermal cavity. Re-

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cently, Lü *et al.*[19] proposed a scheme for deterministically generating three-dimensional entanglement between two distant five-level atoms based on the dispersive atom-field interaction. All these mentioned schemes were based on the exact knowledge of all parameters and require the exact control of the interaction time. Furthermore, they needed several steps to generate the three-dimensional entanglement. More recently, Li and Huang[20] proposed a scheme for the generation of three-dimensional entangled state for two atoms trapped in a cavity only by one step via quantum Zeno dynamics. However, the scheme was also based on the exact knowledge of all parameters and requires exact control of the interaction time.

In recent years, another kind of scheme was proposed to generate entangled states or implement quantum processing based on a technique known as stimulated Raman scattering involving adiabatic passage (STIRAR). The adiabatic passage is more robust against moderate fluctuations of experimental parameters than normal dynamical evolution. Furthermore, during the adiabatic passage, the atoms have no probability of being excited and thus the atomic spontaneous emission is suppressed. Ye *et al.*[21] have proposed a scheme for the deterministic generation of three-dimensional entanglement of two distant atoms separately trapped in two optical cavities connected by an optical fibre via adiabatic passage along dark state. Recently, Yang *et al.*[22] proposed an alternative scheme for the deterministic generation of qutrit entanglement for two atoms trapped in an optical cavity via adiabatic passage. The drawback of the two schemes is that they could not generate three-dimensional entanglement by one step, which would make it difficult to implement the schemes in experiment.

Motivated by these studies,[20−22] we propose an alternative scheme to generate qutrit entanglement for two atoms trapped in a cavity via adiabatic passage. Our scheme has the following advantages: (i) taking advantage of adiabatic passage, entanglement generation is realized by using only the lower states of the atoms, and the field mode of the cavity being excited is negligible under a certain condition, so the scheme is very robust against decoherent; (ii) the scheme needs neither the exact control of all parameters, nor the accurate control of the interaction time; (iii) the qutrit entanglement can be generated only by one step with a high fidelity.

2. One-step generation of qutrit entanglement

Consider two atoms (atom A and atom B) are trapped in a double-mode cavity. Atom A is a tripod-type atom with two degenerate ground states $|g_{\rm L}\rangle$ and $|g_{\rm R}\rangle$, atom B is an M-type atom with two degenerate excited states $|e_{\rm L}\rangle$ and $|e_{\rm R}\rangle$ and two degenerate ground states $|g_{\rm L}\rangle$ and $|g_{\rm R}\rangle$. The energy levels of the two atoms are shown in Fig. 1. The atomic transitions $|e_0\rangle_A \leftrightarrow |g_{\rm L}\rangle_A$ ($|g_{\rm R}\rangle_A$) and $|g_0\rangle_B \leftrightarrow |e_{\rm L}\rangle_B$ ($|e_{\rm R}\rangle_B$) are coupled resonantly to the left-circularly (right-circularly) polarized mode of the cavity with coupling constants $g_{A,{\rm L}}$ ($g_{A,{\rm R}}$) and $g_{B,{\rm L}}$ ($g_{B,{\rm R}}$), respectively. The transitions $|g_0\rangle_A \leftrightarrow |e_0\rangle_A$ and $\left|e_{\rm L\,(R)}\right\rangle_B \leftrightarrow \left|g_{\rm L\,(R)}\right\rangle_B$ are driven resonantly through classical laser fields with the coupling coefficients Ω_A and Ω_B, respectively. In the interaction picture, the Hamiltonian of the system can be written as

$$\begin{aligned} H \;=\; & \sum_{i={\rm L,\,R}} (g_{A,\,i} a_i \left|e_0\right\rangle_A \left\langle g_i\right| + g_{B,\,i} a_i \left|e_i\right\rangle_B \left\langle g_0\right| \\ & + \Omega_B \left|e_i\right\rangle_B \left\langle g_i\right| + \Omega_A \left|e_0\right\rangle_A \left\langle g_0\right|) + {\rm H.c.}, \quad (1) \end{aligned}$$

where $a_i^\dagger$ and a_i are the creation and the annihilation operators for photons with polarization i ($i = {\rm L}$, R corresponding to left-circular and right-circular polarizations, respectively) associated with the corresponding quantized cavity modes. We assume that $g_{A,\,i} = g_{B,\,j} = g$ ($i, j = {\rm L}, {\rm R}$) for convenience. If the initial state is $\left|g_0\right\rangle_A \left|g_0\right\rangle_B \left|0\right\rangle_{\rm L} \left|0\right\rangle_{\rm R}$, the whole system evolves in the subspace spanned by the basis state vectors

$$\begin{aligned} |\phi_1\rangle &= \left|g_0\right\rangle_A \left|g_0\right\rangle_B \left|0\right\rangle_{\rm L} \left|0\right\rangle_{\rm R}, \\ |\phi_2\rangle &= \left|e_0\right\rangle_A \left|g_0\right\rangle_B \left|0\right\rangle_{\rm L} \left|0\right\rangle_{\rm R}, \\ |\phi_3\rangle &= \left|g_{\rm L}\right\rangle_A \left|g_0\right\rangle_B \left|1\right\rangle_{\rm L} \left|0\right\rangle_{\rm R}, \\ |\phi_4\rangle &= \left|g_{\rm R}\right\rangle_A \left|g_0\right\rangle_B \left|0\right\rangle_{\rm L} \left|1\right\rangle_{\rm R}, \\ |\phi_5\rangle &= \left|g_{\rm L}\right\rangle_A \left|e_{\rm L}\right\rangle_B \left|0\right\rangle_{\rm L} \left|0\right\rangle_{\rm R}, \\ |\phi_6\rangle &= \left|g_{\rm R}\right\rangle_A \left|e_{\rm R}\right\rangle_B \left|0\right\rangle_{\rm L} \left|0\right\rangle_{\rm R}, \\ |\phi_7\rangle &= \left|g_{\rm L}\right\rangle_A \left|g_{\rm L}\right\rangle_B \left|0\right\rangle_{\rm L} \left|0\right\rangle_{\rm R}, \\ |\phi_8\rangle &= \left|g_{\rm R}\right\rangle_A \left|g_{\rm R}\right\rangle_B \left|0\right\rangle_{\rm L} \left|0\right\rangle_{\rm R}. \end{aligned}$$

It is easy to verify that there is a dark state with null eigenvalue

$$\begin{aligned} |\psi_0\rangle \;=\; & \cos\alpha \cos\beta \left|\phi_1\right\rangle - \frac{1}{\sqrt{2}} \sin\alpha (|\phi_3\rangle + |\phi_4\rangle) \\ & + \frac{1}{\sqrt{2}} \cos\alpha \sin\beta (|\phi_7\rangle + |\phi_8\rangle), \quad (2) \end{aligned}$$

where

$$\sin\alpha = \frac{\Omega_A \Omega_B}{\sqrt{2g^2\Omega_B^2 + g^2\Omega_A^2 + \Omega_A^2\Omega_B^2}},$$

$$\cos\alpha = \frac{\sqrt{\Omega_A^2 g^2 + 2\Omega_B^2 g^2}}{\sqrt{2g^2\Omega_B^2 + g^2\Omega_A^2 + \Omega_A^2\Omega_B^2}},$$
$$\sin\beta = \frac{\Omega_A g}{\sqrt{\Omega_A^2 g^2 + 2\Omega_B^2 g^2}},$$
$$\cos\beta = \frac{\sqrt{2}\Omega_B g}{\sqrt{\Omega_A^2 g^2 + 2\Omega_B^2 g^2}}.$$

Suppose that $\Omega_B = g/10$ and $\Omega_A = 0$ initially, then the initial state of the whole system will be $|\phi_1\rangle$. Under the adiabatic condition, the state undergoes the evolution determined by Eq. (2). If we keep Ω_B unchanged and change Ω_A slowly to $\Omega_{A\max} = 2\Omega_B \ll g$, then the state $|\phi_1\rangle$ evolves to

$$\begin{aligned}|\psi\rangle &= \frac{1}{\sqrt{3}}(|\phi_1\rangle + |\phi_7\rangle + |\phi_8\rangle)\\ &= \frac{1}{\sqrt{3}}(|g_0\rangle_A|g_0\rangle_B + |g_{\rm L}\rangle_A|g_{\rm L}\rangle_B\\ &\quad + |g_{\rm R}\rangle_A|g_{\rm R}\rangle_B)|0\rangle_{\rm L}|0\rangle_{\rm R}. \end{aligned} \tag{3}$$

We obtain the qutrit entanglement of the normal form

$$\begin{aligned}|\psi'\rangle &= \frac{1}{\sqrt{3}}(|g_0\rangle_A|g_0\rangle_B + |g_{\rm L}\rangle_A|g_{\rm L}\rangle_B\\ &\quad + |g_{\rm R}\rangle_A|g_{\rm R}\rangle_B), \end{aligned} \tag{4}$$

leaving the cavity field in the vacuum state.

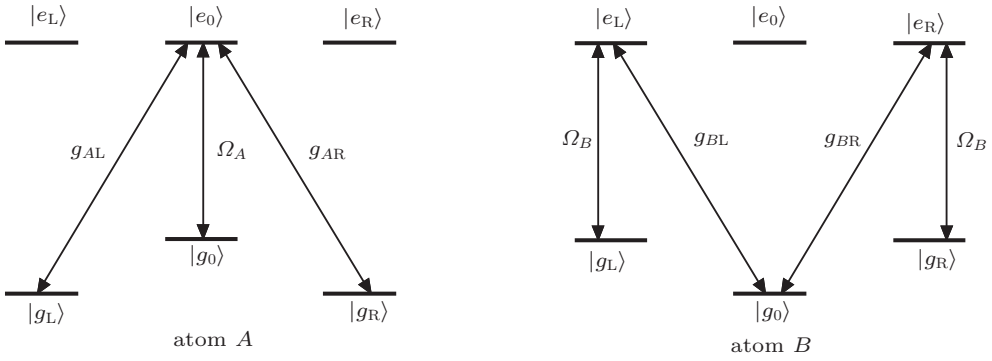


Fig. 1. Energy levels of tripod-type atom A and M-type atom B.

We can see from Eq. (2) that in the adiabatically process $|\phi_1\rangle \to |\phi_1\rangle + |\phi_7\rangle + |\phi_8\rangle$, the atoms in the cavity are always in ground states and the field mode of the cavity being excited is negligible on condition that all the Rabi frequencies of the classical fields are smaller than the cavity–atom coupling strength $g \gg \Omega_A$, Ω_B. However, if the condition $g \gg \Omega_A$, Ω_B is not ideally satisfied, there is a probability of the cavity mode being excited. The probability of the cavity modes being excited can be estimated as[23] $P = \frac{1}{\alpha_{\max}}\int_0^{\alpha_{\max}} \sin^2\alpha \,{\rm d}\alpha$. If we set $\Omega_{A\max} = 2\Omega_B = g/5$ during the adiabatic passage, we can obtain $\alpha_{\max} = \arcsin(1/\sqrt{151})$ and $P \approx 0.006$, which shows that the probability of the cavity modes being excited is very small.

Because of the cavity decay, the probability for the state $|\phi_1\rangle$ evolving to $(|g_{\rm L}\rangle_A + |g_{\rm R}\rangle_A)|g_0\rangle_B|0\rangle_{\rm L}|0\rangle_{\rm R}/\sqrt{2}$ through the intermediate state $(|\phi_3\rangle + |\phi_4\rangle)/\sqrt{2} = (|g_{\rm L}\rangle_A|1\rangle_{\rm L}|0\rangle_{\rm R} + |g_{\rm R}\rangle_A|0\rangle_{\rm L}|1\rangle_{\rm R}(|g_0\rangle_B/\sqrt{2}$ is approximately $\varepsilon = \kappa t P$,[23] where κ is the cavity decay rate and t is the interaction time. Then the final state is approximately given by

$$\begin{aligned}\rho &= (1-\varepsilon)|\psi'\rangle\langle\psi'| + \frac{\varepsilon}{2}(|g_{\rm L}\rangle_A|g_0\rangle_B\langle g_{\rm L}|_A\langle g_0|_B\\ &\quad + |g_{\rm R}\rangle_A|g_0\rangle_B\langle g_{\rm R}|_A\langle g_0|_B). \end{aligned} \tag{5}$$

In recent experiments,[24,25] for an optical cavity with the wavelength in a range of 630 nm–850 nm, the condition ($g/2\pi = 750$ MHz, $\kappa/2\pi = 2.62$ MHz) is realizable. If we set the interaction time $t = 20/g$, we can obtain the fidelity of the qutrit entanglement $F = \langle\psi'|\rho|\psi'\rangle \approx 0.99958$. Our scheme can reach a high fidelity.

Notice that we obtain the above fidelity under the condition $\Omega_{A\max} = 2\Omega_B = g/5$. If the condition cannot be satisfied ideally, the fidelity would be decreased. Assume that we cannot control strictly the two Rabi frequencies as $\Omega_{A\max} = 2\Omega_B$, if set $2\Omega_B = g/5$ and $\Omega_{A\max} = 0.99 \times 2\Omega_B$, according to Eq. (2), the final state is given approximately as

$$\begin{aligned}\rho' &= (1-\varepsilon)|\psi''\rangle\langle\psi''| + \frac{\varepsilon}{2}(|g_{\rm L}\rangle_A|g_0\rangle_B\langle g_{\rm L}|_A\langle g_0|_B\\ &\quad + |g_{\rm R}\rangle_A|g_0\rangle_B\langle g_{\rm R}|_A\langle g_0|_B), \end{aligned} \tag{6}$$

with

$$|\psi''\rangle = \frac{1}{\sqrt{2.9798}}(|g_0\rangle_A |g_0\rangle_B + 0.99 |g_{\rm L}\rangle_A |g_{\rm L}\rangle_B + 0.99 |g_{\rm R}\rangle_A |g_{\rm R}\rangle_B). \quad (7)$$

The fidelity is approximately $F' = \langle\psi'| \rho' |\psi'\rangle \approx 0.99305$. On the other hand, assume that we can only satisfy $\Omega_{A\max} = 2\Omega_B = g/4$ during implementing, we can also obtain a fidelity $F'' \approx 0.98927$. How does the interaction time affect the fidelity? We obtain the above fidelities F, F', and F'' under the condition $t = 20/g$. If set $t = 50/g$, we can have $F \approx 0.99895$, $F' \approx 0.99243$ and $F'' \approx 0.98865$, respectively. It is obvious that our scheme is insensitive to the fluctuation of parameters and does not need to control the interaction time accurately.

For the atomic level structure, the atom ^{87}Rb which was reported in the experiment[26] can be used to implement our scheme. The level structure is shown in Fig. 2. Each atom is assumed to be coupled resonantly to a classical field and both left-circularly and right-circularly polarized mode of the cavity.

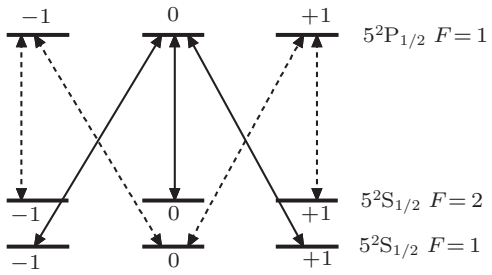


Fig. 2. Energy levels of ^{87}Rb.

3. Conclusion

We proposed a scheme to generate qutrit entanglement for two atoms trapped in a cavity via adiabatic passage. Taking advantage of adiabatic passage, entanglement generation is realized by using only the lower states of the atoms and the field mode of the cavity being excited is negligible under a certain condition, so the scheme is very robust against decoherent. The scheme is insensitive to the fluctuation of parameters and does not require to accurate control of the interaction time. Furthermore, the qutrit entanglement can be generated only by one step with a high fidelity.

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