Evolution of a two-mode squeezed vacuum in the amplitude dissipative channel*

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For the first time we derive the dissipating result of an initial two-mode squeezed pure vacuum state passing through a two-mode amplitude dissipative channel described by the direct product of two independent single-mode master equations. Although these two master equations do not mix the two modes (there is no coupling between them), since the two-mode squeezed state is simultaneously an entangled state, the final state which emerges from passing this channel is a two-mode mixed density operator. The compact expression of the outcoming state is obtained, which manifestly shows that as time evolves, the squeezing effect decreases.

Keywords: squeezed vacuum state, dissipative channel, master equation

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1. Introduction

In nature most systems are immersed in a "thermal reservoir", excitation and de-excitation processes are influenced by the exchange of energy between the system and the reservoir. We inevitably encounter some interactions of the quantum state under consideration with its environment.[1−3] These undesirable couplings give rise to decoherence. In this paper we investigate how a two-mode squeezed vacuum state evolves in a two-mode amplitude dissipative channel. For this channel, the associated loss mechanism in physical processes is governed by the following uncoupled two master equations[4,5]

$$\frac{\mathrm{d}\rho_1(t)}{\mathrm{d}t} = \kappa\left(2a\rho_1 a^\dagger - a^\dagger a\rho_1 - \rho_1 a^\dagger a\right), \quad (1)$$

$$\frac{\mathrm{d}\rho_2(t)}{\mathrm{d}t} = \kappa\left(2b\rho_2 b^\dagger - b^\dagger b\rho_2 - \rho_2 b^\dagger b\right), \quad (2)$$

where ρ is the density operator of the system, and κ is the rate of decay. We shall solve the master equation when the initial state is a two-mode squeezed pure state $S(\lambda)|00\rangle\langle 00|S^\dagger(\lambda)$. The two-mode squeezing operator is $S(\lambda) = \exp[\lambda\left(a^\dagger b^\dagger - ab\right)]$, and

$$S(\lambda)|00\rangle = \mathrm{sech}\lambda \exp\left[a^\dagger b^\dagger \tanh\lambda\right]|00\rangle, \quad (3)$$

where $|00\rangle$ is the vacuum state, and λ is the squeezing parameter. This state can be produced in a parameteric amplifier. An interesting question thus naturally arises: if a two-mode squeezed pure state undergoes the two-mode amplitude dissipative channel, then what is the outcome? It should be noted that although these two master equations do not mix a-mode and b-mode (there is no coupling between them), since the two-mode squeezed state is simultaneously an entangled state (the idler-mode and the signal mode are entangled in the frequency domain), the final state which emerges from passing this channel is a two-mode mixed density operator. We will derive this new time-dependent density operator by virtue of the technique of integration within an ordered product (IWOP) of operators[6−11] and the entangled state representation.

This paper is arranged as follows. In Section 2 by virtue of the entangled state representation we briefly review our way of deriving the infinitive sum represen-

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tation of the density operator as a solution of the master equation. In Section 3 we derive the compact form of $\rho(t)$ when the initial density operator is a two-mode squeezed pure state, which manifestly shows that as time evolves the squeezing effect decreases.

2. Brief review of deducing the infinitive sum representation of the density operator

For solving the above master equation, we first consider the single-mode master equation (Eq. (1)). In a recent paper, Ref. [12], we have introduced a convenient approach in which the two-mode entangled state[13,14]

$$|\eta\rangle = \exp\left(-\frac{1}{2}|\eta|^2 + \eta a^\dagger - \eta^* \tilde{a}^\dagger + a^\dagger \tilde{a}^\dagger\right)|0\tilde{0}\rangle \quad (4)$$

is employed, where $\tilde{a}^\dagger$ is a fictitious mode independent of the real mode $a^\dagger$. The state $|\eta = 0\rangle$ possesses the properties

$$\begin{aligned} &a|\eta=0\rangle = \tilde{a}^\dagger|\eta=0\rangle, \quad a^\dagger|\eta=0\rangle = \tilde{a}|\eta=0\rangle, \\ &(a^\dagger a)^n|\eta=0\rangle = (\tilde{a}^\dagger\tilde{a})^n|\eta=0\rangle. \end{aligned} \quad (5)$$

Acting both sides of Eq. (1) on the state $|\eta = 0\rangle \equiv |I\rangle$, and denoting $|\rho\rangle = \rho\,|I\rangle$, we have

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}|\rho_1\rangle &= \kappa\left(2a\rho_1 a^\dagger - a^\dagger a\rho_1 - \rho_1 a^\dagger a\right)|I\rangle \\ &= \kappa\left(2a\tilde{a} - a^\dagger a - \tilde{a}^\dagger\tilde{a}\right)|\rho_1\rangle, \end{aligned} \quad (6)$$

so its formal solution is

$$|\rho_1(t)\rangle = \exp\{\kappa t\left(2a\tilde{a} - a^\dagger a - \tilde{a}^\dagger\tilde{a}\right)\}|\rho_1(0)\rangle, \quad (7)$$

where $|\rho_1(0)\rangle \equiv \rho_1(0)\,|I\rangle$, $\rho_1(0)$ is the initial density operator.

Noticing that the operators in Eq. (7) obey the following commutative relation,

$$\begin{aligned} &\left[a\tilde{a}, a^\dagger a\right] = \left[a\tilde{a}, \tilde{a}^\dagger\tilde{a}\right] = \tilde{a}a, \\ &\left[\frac{a^\dagger a + \tilde{a}^\dagger\tilde{a}}{2}, a\tilde{a}\right] = -\tilde{a}a, \end{aligned} \quad (8)$$

and using the operator identity

$$\mathrm{e}^{\lambda(A+\sigma B)} = \mathrm{e}^{\lambda A}\exp\left[\sigma\left(1 - \mathrm{e}^{-\lambda\tau}\right)B/\tau\right], \quad (9)$$

(which is valid for $[A, B] = \tau B$), we have

$$\begin{aligned} &\exp\left[-2\kappa t\left(\frac{a^\dagger a + \tilde{a}^\dagger\tilde{a}}{2} - a\tilde{a}\right)\right] \\ &= \exp\left[-\kappa t\left(a^\dagger a + \tilde{a}^\dagger\tilde{a}\right)\right]\exp\left[T' a\tilde{a}\right], \end{aligned} \quad (10)$$

where

$$T' = 1 - \mathrm{e}^{-2\kappa t}. \quad (11)$$

Then substituting Eq. (10) into Eq. (7) yields

$$\begin{aligned} |\rho_1(t)\rangle &= \exp\left[-\kappa t\left(a^\dagger a + \tilde{a}^\dagger\tilde{a}\right)\right]\sum_{n=0}^{\infty}\frac{T'^n}{n!}a^n\tilde{a}^n|\rho(0)\rangle \\ &= \mathrm{e}^{-\kappa t a^\dagger a}\sum_{n=0}^{\infty}\frac{T'^n}{n!}a^n\rho(0)\,a^{\dagger n}\,\mathrm{e}^{-\kappa t\tilde{a}^\dagger\tilde{a}}|I\rangle \\ &= \sum_{n=0}^{\infty}\frac{T'^n}{n!}\,\mathrm{e}^{-\kappa t a^\dagger a}a^n\rho_1(0)\,a^{\dagger n}\,\mathrm{e}^{-\kappa t a^\dagger a}|I\rangle, \end{aligned} \quad (12)$$

which leads to the infinitive operator-sum representation of ρ,

$$\rho_1(t) = \sum_{n=0}^{\infty}\frac{T'^n}{n!}\,\mathrm{e}^{-\kappa t a^\dagger a}a^n\rho_1(0)\,a^{\dagger n}\,\mathrm{e}^{-\kappa t a^\dagger a}. \quad (13)$$

Introducing

$$M_n \equiv \sqrt{\frac{T'^n}{n!}}\,\mathrm{e}^{-\kappa t a^\dagger a}a^n, \quad (14)$$

such that

$$\rho_1(t) = \sum_{n=0}^{\infty}M_n\rho_1(0)\,M_n^\dagger, \quad (15)$$

then using the normally ordered expansion of

$$\mathrm{e}^{\lambda a^\dagger a} = :\exp\{\left(\mathrm{e}^\lambda - 1\right)a^\dagger a\}: \quad (16)$$

and Eq. (14) we can prove

$$\begin{aligned} \sum_n M_n^\dagger M_n &= \sum_n\frac{T'^n}{n!}a^{\dagger n}\,\mathrm{e}^{-2\kappa t a^\dagger a}a^n \\ &= \sum_n\frac{T'^n}{n!}\,\mathrm{e}^{2n\kappa t}:a^{\dagger n}a^n:\,\mathrm{e}^{-2\kappa t a^\dagger a} \\ &= :\,\mathrm{e}^{T'\,\mathrm{e}^{2\kappa t}a^\dagger a}:\,\mathrm{e}^{-2\kappa t a^\dagger a} \\ &= :\,\mathrm{e}^{\left(\mathrm{e}^{2\kappa t}-1\right)a^\dagger a}:\,\mathrm{e}^{-2\kappa t a^\dagger a} = 1. \end{aligned} \quad (17)$$

Thus M_n is a kind of Kraus operator. Therefore, for any given initial state $\rho_1(0)$, the density operator $\rho_1(t)$ can be directly calculated from Eq. (13). Generalizing it to the two-mode case, we know that the solution to master equations (1) and (2) is

$$\begin{aligned} \rho(t) = \mathrm{sech}^2\lambda\sum_{m,n=0}^{\infty}&\frac{T'^{n+m}}{n!m!}\,\mathrm{e}^{-\kappa t\left(a^\dagger a + b^\dagger b\right)}a^n b^m\rho(0) \\ &\times a^{\dagger n}b^{\dagger m}\,\mathrm{e}^{-\kappa t\left(a^\dagger a + b^\dagger b\right)}. \end{aligned} \quad (18)$$

3. Evolution of a two-mode squeezed vacuum in the amplitude dissipative channel

When the initial state is a pure two-mode squeezed state

$$\rho(0) = \operatorname{sech}^2\lambda\, \mathrm{e}^{a^\dagger b^\dagger \tanh\lambda} |00\rangle\langle 00|\, \mathrm{e}^{ab\tanh\lambda}, \quad (19)$$

then substituting Eq. (19) into Eq. (18) yields

$$\begin{aligned}\rho(t) = \operatorname{sech}^2\lambda \sum_{m,n=0}^{\infty} \frac{T'^{n+m}}{n!m!}\, \mathrm{e}^{-\kappa t\left(a^\dagger a + b^\dagger b\right)} a^n b^m \\ \times \mathrm{e}^{a^\dagger b^\dagger \tanh\lambda} |00\rangle\langle 00|\, \mathrm{e}^{ab\tanh\lambda} a^{\dagger n} b^{\dagger m} \\ \times \mathrm{e}^{-\kappa t\left(a^\dagger a + b^\dagger b\right)}. \end{aligned} \quad (20)$$

To simplify it we first analyse

$$\begin{aligned} a^n b^m\, \mathrm{e}^{a^\dagger b^\dagger \tanh\lambda} |00\rangle &= a^n \left(a^\dagger \tanh\lambda\right)^m \mathrm{e}^{a^\dagger b^\dagger \tanh\lambda} |00\rangle, \quad (21) \end{aligned}$$

then using the operator identity

$$a^n a^{\dagger m} = (-\mathrm{i})^{m+n} : \mathrm{H}_{m,n}\left(\mathrm{i}\, a^\dagger, \mathrm{i}\, a\right) :, \quad (22)$$

(which can be deduced by virtue of the technique of integration within an ordered product of operators,[15] where

$$\begin{aligned} &\mathrm{H}_{m,n}(\lambda, \lambda^*) \\ &= \sum_{l=0}^{\min(m,n)} \frac{m!n!}{l!(m-l)!(n-l)!} (-1)^l \lambda^{m-l} \lambda^{*n-l} \end{aligned} \quad (23)$$

are the two variable Hermite polynomials, we can reform

$$\begin{aligned} a^n b^m\, \mathrm{e}^{a^\dagger b^\dagger \tanh\lambda} |00\rangle &= (-\mathrm{i})^{m+n} \tanh^m\lambda : \mathrm{H}_{m,n}\left(\mathrm{i}\, a^\dagger, \mathrm{i}\, a\right) :\, \mathrm{e}^{a^\dagger b^\dagger \tanh\lambda} |00\rangle \\ &= (-\mathrm{i})^{m+n} \tanh^m\lambda : \sum_{l=0}^{\min(m,n)} \frac{m!n!(-1)^l}{l!(m-l)!(n-l)!} \left(\mathrm{i}\, a^\dagger\right)^{m-l} (\mathrm{i}\, a)^{n-l}\, \mathrm{e}^{a^\dagger b^\dagger \tanh\lambda} |00\rangle \\ &= \tanh^m\lambda \sum_{l=0}^{\min(m,n)} \frac{m!n!\left(a^\dagger\right)^{m-l}}{l!(m-l)!(n-l)!} a^{n-l}\, \mathrm{e}^{a^\dagger b^\dagger \tanh\lambda} |00\rangle \\ &= \tanh^m\lambda \sum_{l=0}^{\min(m,n)} \frac{m!n!\left(a^\dagger\right)^{m-l}\left(b^\dagger \tanh\lambda\right)^{n-l}}{l!(m-l)!(n-l)!}\, \mathrm{e}^{a^\dagger b^\dagger \tanh\lambda} |00\rangle. \end{aligned} \quad (24)$$

Again using Eq. (23) we see

$$a^n b^m\, \mathrm{e}^{a^\dagger b^\dagger \tanh\lambda} |00\rangle = (-\mathrm{i})^{m+n} \tanh^m\lambda \mathrm{H}_{m,n}\left(\mathrm{i}\, a^\dagger, \mathrm{i}\, b^\dagger \tanh\lambda\right) \mathrm{e}^{a^\dagger b^\dagger \tanh\lambda} |00\rangle. \quad (25)$$

According to the generating function formula of the product of two Hermite polynomials[15]

$$\sum_{m,n=0}^{\infty} \mathrm{H}_{m,n}(\xi,\eta)\, \mathrm{H}_{m,n}(\sigma,\kappa) \frac{t^n s^m}{m!n!} = \frac{1}{1-ts} \exp\left\{\frac{1}{1-ts}\left(s\sigma\xi + t\eta\kappa - st\sigma\kappa - st\xi\eta\right)\right\} \quad (26)$$

and the normally ordered form of the vacuum projector

$$|00\rangle\langle 00| =: \mathrm{e}^{-a^\dagger a - b^\dagger b} :, \quad (27)$$

as well as $\mathrm{e}^{-\kappa t a^\dagger a} a^\dagger\, \mathrm{e}^{\kappa t a^\dagger a} = a^\dagger\, \mathrm{e}^{-\kappa t}$, we have the compact expression of density operator

$$\begin{aligned} \rho(t) &= \operatorname{sech}^2\lambda \sum_{m,n=0}^{\infty} \frac{T'^{n+m}}{n!m!}\, \mathrm{e}^{-\kappa t\left(a^\dagger a + b^\dagger b\right)} \tanh^{2m}\lambda \mathrm{H}_{m,n}\left(\mathrm{i}\, a^\dagger, \mathrm{i}\, b^\dagger \tanh\lambda\right) \mathrm{e}^{a^\dagger b^\dagger \tanh\lambda} \\ &\quad \times |00\rangle\langle 00|\, \mathrm{e}^{ab\tanh\lambda} \mathrm{H}_{m,n}\left(-\mathrm{i}\, a, -\mathrm{i}\, b\tanh\lambda\right) \mathrm{e}^{-\kappa t\left(a^\dagger a + b^\dagger b\right)} \\ &= \operatorname{sech}^2\lambda \sum_{m,n=0}^{\infty} \frac{T'^n \left(T' \tanh^2\lambda\right)^m}{n!m!} : \mathrm{H}_{m,n}\left(\mathrm{i}\, a^\dagger\, \mathrm{e}^{-\kappa t}, \mathrm{i}\, b^\dagger\, \mathrm{e}^{-\kappa t}\tanh\lambda\right) \\ &\quad \times \mathrm{H}_{m,n}\left(-\mathrm{i}\, a\, \mathrm{e}^{-\kappa t}, -\mathrm{i}\, b\, \mathrm{e}^{-\kappa t}\tanh\lambda\right) \mathrm{e}^{\left(a^\dagger b^\dagger + ab\right)\mathrm{e}^{-2\kappa t}\tanh\lambda - a^\dagger a - b^\dagger b} : \\ &= \frac{\operatorname{sech}^2\lambda}{1 - T'^2\tanh^2\lambda} : \exp\left\{\frac{T'\tanh^2\lambda\, \mathrm{e}^{-2\kappa t}}{1 - T'^2\tanh^2\lambda}\left[\left(aa^\dagger + b^\dagger b\right) + T'\tanh\lambda\left(ab + a^\dagger b^\dagger\right)\right]\right. \end{aligned}$$

$$+\left(a^{\dagger}b^{\dagger}+ab\right)\mathrm{e}^{-2\kappa t}\tanh\lambda-a^{\dagger}a-b^{\dagger}b\Big\}:$$
$$=\frac{\mathrm{sech}^2\lambda}{1-T'^2\tanh^2\lambda}:\exp\{\frac{T'\tanh^2\lambda-1}{1-T'^2\tanh^2\lambda}\left(aa^{\dagger}+b^{\dagger}b\right)+\frac{\mathrm{e}^{-2\kappa t}\tanh\lambda}{1-T'^2\tanh^2\lambda}\left(ab+a^{\dagger}b^{\dagger}\right)\}:, \tag{28}$$

which is a mixed state. Where we have used $T'=1-\mathrm{e}^{-2\kappa t}$.

For checking the validity of the above derivation, we calculate $\mathrm{Tr}\,\rho(t)$ to see if it is equal to 1. In fact using the coherent state completeness relation $\int\frac{\mathrm{d}^2z_1\mathrm{d}^2z_2}{\pi^2}\left|z_1z_2\right\rangle\left\langle z_1z_2\right|=1$, we have

$$\begin{aligned}\mathrm{Tr}\,\rho\left(t\right)&=\frac{\mathrm{sech}^2\lambda}{1-T'^2\tanh^2\lambda}\int\frac{\mathrm{d}^2z_1\mathrm{d}^2z_2}{\pi}\left\langle z_1z_2\right|:\exp\left\{\frac{T'\tanh^2\lambda-1}{1-T'^2\tanh^2\lambda}\left(aa^{\dagger}+b^{\dagger}b\right)\right.\\&\left.+\frac{\mathrm{e}^{-2\kappa t}\tanh\lambda}{1-T'^2\tanh^2\lambda}\left(ab+a^{\dagger}b^{\dagger}\right)\right\}:\left|z_1z_2\right\rangle\\&=\frac{\mathrm{sech}^2\lambda}{1-T'^2\tanh^2\lambda}\int\frac{\mathrm{d}^2z_1\mathrm{d}^2z_2}{\pi^2}\exp\left\{\frac{T'\tanh^2\lambda-1}{1-T'^2\tanh^2\lambda}\left(|z_1|^2+|z_2|^2\right)\right.\\&\left.+\left(z_1^*z_2^*+z_1z_2\right)\frac{\mathrm{e}^{-2\kappa t}\tanh\lambda}{1-T'^2\tanh^2\lambda}\right\},\end{aligned} \tag{29}$$

where

$$\begin{aligned}&\int\frac{\mathrm{d}^2z_1}{\pi}\exp\left\{\frac{T'\tanh^2\lambda-1}{1-T'^2\tanh^2\lambda}|z_1|^2+\left(z_1^*z_2^*+z_1z_2\right)\frac{\mathrm{e}^{-2\kappa t}\tanh\lambda}{1-T'^2\tanh^2\lambda}\right\}\\&=\frac{1-T'^2\tanh^2\lambda}{1-T'\tanh^2\lambda}\exp\left\{\frac{1}{1-T'\tanh^2\lambda}\frac{\left(1-T'\right)^2\tanh^2\lambda}{1-T'^2\tanh^2\lambda}|z_2|^2\right\}.\end{aligned} \tag{30}$$

Substituting Eq. (30) into Eq. (29) we obtain

$$\begin{aligned}\mathrm{Tr}\,\rho\left(t\right)&=\frac{\mathrm{sech}^2\lambda}{1-T'\tanh^2\lambda}\int\frac{\mathrm{d}^2z_2}{\pi}\\&\times\exp\left(\frac{-\mathrm{sech}^2\lambda}{1-T'\tanh^2\lambda}|z_2|^2\right)=1,\end{aligned} \tag{31}$$

as expected. Further, using

$$\mathrm{e}^{\lambda a^{\dagger}a}=:\exp\left[\left(\mathrm{e}^{\lambda}-1\right)a^{\dagger}a\right]:,$$

we can rewrite Eq. (28) as

$$\begin{aligned}\rho\left(t\right)&=\frac{\mathrm{sech}^2\lambda}{1-T'^2\tanh^2\lambda}\exp\left(\frac{\mathrm{e}^{-2\kappa t}\tanh\lambda}{1-T'^2\tanh^2\lambda}a^{\dagger}b^{\dagger}\right)\\&\times\exp\left\{\left(a^{\dagger}a+b^{\dagger}b\right)\ln\frac{T'\tanh^2\lambda\,\mathrm{e}^{-2\kappa t}}{1-T'^2\tanh^2\lambda}\right\}\\&\times\exp\left(\frac{\mathrm{e}^{-2\kappa t}\tanh\lambda}{1-T'^2\tanh^2\lambda}ab\right).\end{aligned} \tag{32}$$

Comparing Eq. (32) with the initial state expression Eq. (19) we see the squeezing amount

$$\tanh\lambda\rightarrow\tanh\lambda\frac{\mathrm{e}^{-2\kappa t}}{1-T'^2\tanh^2\lambda}.$$

Due to $T'=1-\mathrm{e}^{-2\kappa t}$, and $T'>T'^2\tanh^2\lambda$, so

$$\frac{\mathrm{e}^{-2\kappa t}}{1-T'^2\tanh^2\lambda}<1,$$

this implies that the squeezing amount decreases in the process of dissipation. Thus we know that as time evolves the squeezing effect decreases whereas decoherence increases. Using Eq. (32) one can calculate the corresponding Wigner function evolution, etc.

In summary, for the first time we have investigated how a two-mode squeezed pure state evolves into a mixed state upon undergoing a two-mode amplitude dissipative channel.

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