Dissipation of a two-mode squeezed vacuum state in the single-mode amplitude damping channel*

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We explore how a two-mode squeezed vacuum state $\mathrm{sech}\theta\, \mathrm{e}^{a^\dagger b^\dagger \tanh\theta}|00\rangle$ evolves when it undergoes a single-mode amplitude dissipative channel with rate of decay κ. We find that in this process not only the squeezing parameter decreases, $\tanh\theta \rightarrow \mathrm{e}^{-\kappa t}\tanh\theta$, but also the second-mode vacuum state evolves into a chaotic state $\exp\{b^\dagger b \ln[(1-\mathrm{e}^{-2\kappa t})\tanh^2\theta]\}$. The outcome state is no more a pure state, but an entangled mixed state.

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1. Introduction

In recent years, quantum operation formalism has been a hot topic in quantum information theory,[1] to be specific, quantum operations describe quantum noise and the behaviour of open quantum systems (both the environment and system under our review). This topic is important and realistic since no quantum systems are ever perfectly closed. The role of quantum operations is transforming an input pure state to output mixed state, denoted by $\rho_0 \rightarrow \sum_i M_i \rho_0 M_i^\dagger = \rho(t)$, such a transformation also defines a quantum channel, where M_i is a kind of Kraus operator. From the viewpoint of mathematical formalism, the density operators $\rho(t)$ at time t are solutions of master equations. For example, for an amplitude-damping channel the master equation is[2–4]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \kappa\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right), \tag{1}$$

where κ is the decaying rate, $\left[a, a^\dagger\right] = 1$. As we have experienced that the evolution of many quantum signals is related to quantum entanglement, for example, a two-mode squeezed state is composed of a signal mode and an idler mode, which is simultaneously an entangled state. In this work we explore how a two-mode squeezed vacuum state, $\mathrm{sech}\,\theta\, \mathrm{e}^{a^\dagger b^\dagger \tanh\theta}|00\rangle$, evolves when it undergoes a single-mode (say, a-mode) amplitude dissipative channel, so that we may see how the b-mode affects a-mode's damping, or *vice versa*.

In the following, first, we shall solve the master equations in the entangled state representation which is denoted by $\langle\eta|$,[3,4] (see Eq. (2) below), whose one mode is a fictitious one. The fictitious mode is a counterpart mode of the system-mode under review, then we can arrange density operator master equations as state-vector evolution equations. In this way the master equation for different physical systems can be concisely solved,[7] and the corresponding Kraus operator can be obtained. Second, we examine when the initial ρ_0 is a two-mode squeezed vacuum state, then how it evolves. As one can see later, we find that in this process not only the squeezing parameter decreases with a factor $\mathrm{e}^{-\kappa t}$, but also the second-mode vacuum state evolves into a chaotic state $\exp\{b^\dagger b \ln[(1-\mathrm{e}^{-2\kappa t})\tanh^2\theta]\}$. The outcome state is no more a pure state, but an entangled mixed state.

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2. $\rho(t)$ obtained by the entangled state representation

To solve the above master equation, similar to Ref. [5] we introduce the two-mode entangled state[6]

$$|\eta\rangle = \exp\left(-\frac{1}{2}|\eta|^2 + \eta a^\dagger - \eta^* \tilde{a}^\dagger + a^\dagger \tilde{a}^\dagger\right)|0\tilde{0}\rangle, \quad (2)$$

where $\tilde{a}^\dagger$ is a fictitious mode independent of the real mode $a^\dagger$, $\tilde{a}|\tilde{0}\rangle = 0$. Historically, Takahashi and Umezawa[8] first introduced the fictitious Fock space to treat the ensemble average of a mixed state as a pure state average. Using the technique of integration within ordered product (IWOP) of operators, we can prove the completeness

$$\int \frac{\mathrm{d}^2\eta}{\pi} |\eta\rangle\langle\eta| = 1.$$

The state $|\eta = 0\rangle$ possesses the following properties:

$$a|\eta = 0\rangle = \tilde{a}^\dagger|\eta = 0\rangle,$$
$$a^\dagger|\eta = 0\rangle = \tilde{a}|\eta = 0\rangle, \quad (3)$$
$$(a^\dagger a)^n|\eta = 0\rangle = (\tilde{a}^\dagger\tilde{a})^n|\eta = 0\rangle. \quad (4)$$

Acting both sides of Eq. (1) on the state $|\eta = 0\rangle \equiv |I\rangle$, and denoting $|\rho\rangle = \rho|I\rangle$, we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}|\rho\rangle &= \kappa\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)|I\rangle \\ &= \kappa\left(2a\tilde{a} - a^\dagger a - \tilde{a}^\dagger\tilde{a}\right)|\rho\rangle, \end{aligned} \quad (5)$$

so its formal solution is given by

$$|\rho\rangle = \exp\left[\kappa t\left(2a\tilde{a} - a^\dagger a - \tilde{a}^\dagger\tilde{a}\right)\right]|\rho_0\rangle, \quad (6)$$

where $|\rho_0\rangle \equiv \rho_0|I\rangle$, ρ_0 is the initial density operator.

Noticing that the operators in Eq. (5) obey the following commutative relation

$$\left[a\tilde{a}, a^\dagger a\right] = \left[a\tilde{a}, \tilde{a}^\dagger\tilde{a}\right] = \tilde{a}a,$$
$$\left[\frac{a^\dagger a + \tilde{a}^\dagger\tilde{a}}{2}, a\tilde{a}\right] = -\tilde{a}a, \quad (7)$$

and using the operator identity

$$\mathrm{e}^{\lambda(A+\sigma B)} = \mathrm{e}^{\lambda A}\exp\left[\sigma\left(1 - \mathrm{e}^{-\lambda\tau}\right)B/\tau\right] \quad (8)$$

(which is valid for $[A, B] = \tau B$), we have

$$\begin{aligned}&\exp\left[-2\kappa t\left(\frac{a^\dagger a + \tilde{a}^\dagger\tilde{a}}{2} - a\tilde{a}\right)\right] \\ &= \exp\left[-\kappa t\left(a^\dagger a + \tilde{a}^\dagger\tilde{a}\right)\right]\exp\left[T' a\tilde{a}\right], \end{aligned} \quad (9)$$

where

$$T' = 1 - \mathrm{e}^{-2\kappa t}. \quad (10)$$

Then using the operator identity

$$\exp\left(\lambda a^\dagger a\right) = :\exp\left[\left(\mathrm{e}^\lambda - 1\right)a^\dagger a\right]: \quad (11)$$

and the IWOP technique, we can express

$$\begin{aligned}&\exp\left[-\kappa t\left(a^\dagger a + \tilde{a}^\dagger\tilde{a}\right)\right]\exp\left[T' a\tilde{a}\right]|\rho_0\rangle \\ &= :\exp[\left(\mathrm{e}^{-\kappa t} - 1\right)\left(a^\dagger a + \tilde{a}^\dagger\tilde{a}\right) \\ &\quad + \left(1 - \mathrm{e}^{-2\kappa t}\right)a\tilde{a}]:\,|\rho_0\rangle \\ &= \int \frac{\mathrm{d}^2\eta}{\pi}:\exp[-|\eta|^2 + \eta\left(a^\dagger - \mathrm{e}^{-\kappa t}\tilde{a}\right) \\ &\quad + \eta^*\left(\mathrm{e}^{-\kappa t}a - \tilde{a}^\dagger\right) + a^\dagger\tilde{a}^\dagger + a\tilde{a} - a^\dagger a - \tilde{a}^\dagger\tilde{a}]:\,|\rho_0\rangle \\ &= \int \frac{\mathrm{d}^2\eta}{\pi}\,\mathrm{e}^{-\frac{1}{2}T'|\eta|^2}|\eta\rangle\left\langle\eta\,\mathrm{e}^{-\kappa t}\right|\rho_0\rangle. \end{aligned} \quad (12)$$

Comparing Eq. (12) with

$$|\rho(t)\rangle = \int \frac{\mathrm{d}^2\eta}{\pi}|\eta\rangle\langle\eta|\,\rho(t)\rangle, \quad (13)$$

we can see

$$\langle\eta|\,\rho(t)\rangle = \mathrm{e}^{-\frac{1}{2}T'|\eta|^2}\left\langle\eta\,\mathrm{e}^{-\kappa t}\right|\rho_0\rangle, \quad (14)$$

which manifestly shows that the wave function of the mixed state $\rho(t)$ in $\langle\eta|$ representation is proportional to that of the initial state ρ_0 in the decayed entangled state $\langle\eta\,\mathrm{e}^{-\kappa t}|$, accompanied by a Gaussian damping factor $\mathrm{e}^{-\frac{T'}{2}|\eta|^2}$. Thus we see clearly how the dissipative channel plays its role in time evolution. This is the advantage of employing $\langle\eta|$ representation. Further, substituting Eq. (9) into Eq. (6) yields

$$\begin{aligned}|\rho\rangle &= \exp\left[-\kappa t\left(a^\dagger a + \tilde{a}^\dagger\tilde{a}\right)\right]\sum_{n=0}^{\infty}\frac{T'^n}{n!}a^n\tilde{a}^n|\rho_0\rangle \\ &= \mathrm{e}^{-\kappa t a^\dagger a}\sum_{n=0}^{\infty}\frac{T'^n}{n!}a^n\rho_0 a^{\dagger n}\,\mathrm{e}^{-\kappa t\tilde{a}^\dagger\tilde{a}}|I\rangle \\ &= \sum_{n=0}^{\infty}\frac{T'^n}{n!}\,\mathrm{e}^{-\kappa t a^\dagger a}a^n\rho_0 a^{\dagger n}\,\mathrm{e}^{-\kappa t a^\dagger a}|I\rangle, \end{aligned} \quad (15)$$

which leads to the infinitive operator-sum representation of $\rho(t)$,

$$\rho(t) = \sum_{n=0}^{\infty} M_n\rho_0 M_n^\dagger, \quad (16)$$

where

$$M_n \equiv \sqrt{\frac{T'^n}{n!}}\,\mathrm{e}^{-\kappa t a^\dagger a}a^n. \quad (17)$$

Using Eq. (11) we can prove

$$\sum_n M_n^\dagger M_n = \sum_n \frac{T'^n}{n!}a^{\dagger n}\,\mathrm{e}^{-2\kappa t a^\dagger a}a^n$$

$$= \sum_n \frac{T'^n}{n!} \mathrm{e}^{2n\kappa t} {:}\, a^{\dagger n} a^n {:}\, \mathrm{e}^{-2\kappa t a^\dagger a}$$
$$= {:}\, \mathrm{e}^{T' \mathrm{e}^{2\kappa t} a^\dagger a} {:}\, \mathrm{e}^{-2\kappa t a^\dagger a} = 1. \quad (18)$$

Thus M_n is a kind of Kraus operator, and $\rho(t)$ in Eq. (16) is qualified to be a density operator, i.e.,

$$\mathrm{Tr}\,\rho(t) = \mathrm{Tr}\left[\sum_{n=0}^{\infty} M_n \rho_0 M_n^\dagger\right] = \mathrm{Tr}\,\rho_0. \quad (19)$$

Therefore, for any given initial state ρ_0, the density operator $\rho(t)$ can be directly calculated from Eq. (16). The entangled state representation provides us with an elegant way of deriving the infinitive sum representation of the density operator as a solution of the master equation.

3. Evolution of a two-mode squeezed vacuum state in the single-mode amplitude damping channel

The realistic two-mode squeezed vacuum state is expressed by

$$S(\theta)|00\rangle = \mathrm{sech}\theta \exp\left[a^\dagger b^\dagger \tanh\theta\right]|00\rangle, \quad (20)$$

where the vacuum state $|00\rangle$ is annihilated by either a or b, $\left[a, a^\dagger\right] = 1 = \left[b, b^\dagger\right]$, and $S(\theta)$ is the squeezing operator with the squeezing parameter θ, $S(\theta) = \exp\left[\theta\left(a^\dagger b^\dagger - ab\right)\right]$. When the initial state is the two-mode squeezed vacuum

$$\rho(0) = S(\theta)|00\rangle\langle 00|S^\dagger(\theta), \quad (21)$$

then

$$\rho(t) = \sum_{n=0}^{\infty} \frac{T'^n}{n!} \mathrm{e}^{-\kappa t a^\dagger a} a^n S(\theta)|00\rangle \times \langle 00|S^\dagger(\theta) a^{\dagger n} \mathrm{e}^{-\kappa t a^\dagger a}. \quad (22)$$

Substituting Eq. (20) into Eq. (16), and using

$$a^n S(\theta)|00\rangle = \left(b^\dagger \tanh\theta\right) a^{n-1} S(\theta)|00\rangle = \left(b^\dagger \tanh\theta\right)^n S(\theta)|00\rangle, \quad (23)$$

we obtain

$$\rho(t) = \sum_{n=0}^{\infty} \frac{T'^n \tanh^{2n}\theta}{n!} \mathrm{e}^{-\kappa t a^\dagger a} b^{\dagger n} S(\theta)|00\rangle \times \langle 00|S^\dagger(\theta) b^n \mathrm{e}^{-\kappa t a^\dagger a} = \mathrm{sech}^2\theta \sum_{n=0}^{\infty} \frac{T'^n \tanh^{2n}\theta}{n!} \mathrm{e}^{-\kappa t a^\dagger a} b^{\dagger n} \times \mathrm{e}^{a^\dagger b^\dagger \tanh\theta}|00\rangle\langle 00| \mathrm{e}^{ab\tanh\theta} b^n \mathrm{e}^{-\kappa t a^\dagger a}. \quad (24)$$

Then using the relation

$$\mathrm{e}^{-\kappa t a^\dagger a} a^\dagger \mathrm{e}^{\kappa t a^\dagger a} = \mathrm{e}^{-\kappa t} a^\dagger, \quad (25)$$

we obtain

$$\rho(t) = \mathrm{sech}^2\theta \sum_{n=0}^{\infty} \frac{T'^n \tanh^{2n}\theta}{n!} b^{\dagger n} \times \mathrm{e}^{\mathrm{e}^{-\kappa t} a^\dagger b^\dagger \tanh\theta}|00\rangle\langle 00| \mathrm{e}^{\mathrm{e}^{-\kappa t} ab\tanh\theta} b^n. \quad (26)$$

By introducing another damping squeezing parameter,

$$\tanh\theta' = \mathrm{e}^{-\kappa t}\tanh\theta, \quad (27)$$

we can introduce a new time-dependent squeezed vacuum state

$$\mathrm{sech}\theta' \mathrm{e}^{\mathrm{e}^{-\kappa t} a^\dagger b^\dagger \tanh\theta}\left|0\tilde{0}\right\rangle = S(\theta')|00\rangle, \quad (28)$$

where $S(\theta') = \exp\left[\theta'\left(a^\dagger b^\dagger - ab\right)\right]$. The state $S(\theta')|00\rangle$ includes less real photons, since

$$\langle 00|S^\dagger(\theta') a^\dagger a S(\theta')|00\rangle = \sinh^2\theta' = \frac{\tanh^2\theta}{\mathrm{e}^{2\kappa t} - \tanh^2\theta} < \sinh^2\theta. \quad (29)$$

With Eq. (28), $\rho(t)$ becomes

$$\rho(t) = \frac{\mathrm{sech}^2\theta}{\mathrm{sech}^2\theta'} \sum_{n=0}^{\infty} \frac{T'^n \tanh^{2n}\theta}{n!} \times b^{\dagger n} S(\theta')|00\rangle\langle 00|S^\dagger(\theta') b^n, \quad (30)$$

which indicates that in the dissipation process, the a-mode photon decreases, while the b-mode photon increases, since $b^{\dagger n} S(\theta')|00\rangle$ is an excitation of $S(\theta')|00\rangle$. Further, using the normal product form

$$|00\rangle\langle 00| = {:}\, \mathrm{e}^{-a^\dagger a - b^\dagger b} {:}, \quad |0\rangle_{aa}\langle 0| = {:}\, \mathrm{e}^{-a^\dagger a} {:}, \quad |0\rangle_{bb}\langle 0| = {:}\, \mathrm{e}^{-b^\dagger b} {:}, \quad (31)$$

we can rewrite $\rho(t)$ in Eq. (26) as

$$\begin{aligned}\rho(t) &= \operatorname{sech}^2\theta{:}\ \sum_{n=0}^{\infty}\frac{T'^n\tanh^{2n}\theta}{n!}b^{\dagger n}b^n\,\mathrm{e}^{\,\mathrm{e}^{-\kappa t}a^\dagger b^\dagger\tanh\theta}\,\mathrm{e}^{\,\mathrm{e}^{-\kappa t}ab\tanh\theta-a^\dagger a-b^\dagger b}{:}\\ &= \operatorname{sech}^2\theta{:}\ \mathrm{e}^{b^\dagger b\left(1-\mathrm{e}^{-2\kappa t}\right)\tanh^2\theta+\mathrm{e}^{-\kappa t}\tanh\theta\left(a^\dagger b^\dagger+ab\right)-a^\dagger a-b^\dagger b}{:}\\ &= \operatorname{sech}^2\theta\,\mathrm{e}^{\,\mathrm{e}^{-\kappa t}a^\dagger b^\dagger\tanh\theta}\,|0\rangle_{aa}\,\langle 0|\,\mathrm{e}^{b^\dagger b\ln\left[\left(1-\mathrm{e}^{-2\kappa t}\right)\tanh^2\theta\right]}\,\mathrm{e}^{\,\mathrm{e}^{-\kappa t}ab\tanh\theta},\end{aligned}\tag{32}$$

where in the last step we have used the operator identity $:\mathrm{e}^{b^\dagger b\left(\mathrm{e}^{\lambda}-1\right)}:\,=\mathrm{e}^{\lambda b^\dagger b}$. To confirm the validity of the above conclusion, using the completeness relation of the two-mode coherent state[9,10]

$$\int\frac{\mathrm{d}^2z_1\mathrm{d}^2z_2}{\pi^2}\,|z_1,z_2\rangle\,\langle z_1,z_2|=1,\tag{33}$$

where

$$\begin{aligned}|z_1,z_2\rangle\,=\,&\exp\Big[-\frac{1}{2}\left(|z_1|^2+|z_2|^2\right)\\ &+z_1a^\dagger+z_2b^\dagger\Big]\,|00\rangle\,,\end{aligned}\tag{34}$$

we check the trace-preserving by evaluating

$$\begin{aligned}\operatorname{Tr}\rho(t) &= \operatorname{sech}^2\theta\int\frac{\mathrm{d}^2z_1\mathrm{d}^2z_2}{\pi^2}\,\langle z_1,z_2|\,0\rangle_{aa}\\ &\times\,\langle 0|:\,\mathrm{e}^{b^\dagger b\left[\left(1-\mathrm{e}^{-2\kappa t}\right)\tanh^2\theta-1\right]}:\\ &\times\,|z_1,z_2\rangle\,\mathrm{e}^{\,\mathrm{e}^{-\kappa t}\left(z_1z_2+z_1^*z_2^*\right)\tanh\theta}\\ &= \operatorname{sech}^2\theta\int\frac{\mathrm{d}^2z_2}{\pi}\exp\left[\left(\tanh^2\theta-1\right)|z_2|^2\right]=1,\end{aligned}\tag{35}$$

where we have used the following integration formula

$$\int\frac{\mathrm{d}^2z}{\pi}\,\mathrm{e}^{\zeta|z|^2+\xi z+\eta z^*}=-\frac{1}{\zeta}\exp\left(-\frac{\xi\eta}{\zeta}\right),\quad \operatorname{Re}(\zeta)<0.\tag{36}$$

4. Evolution of the Wigner function

We now see how the Wigner function evolves in this process. Using the Wigner operator coherent state representation[11]

$$\begin{aligned}\Delta(\alpha,\beta)\,=\,&\mathrm{e}^{2|\alpha|^2+2|\beta|^2}\int\frac{\mathrm{d}^2z_1\mathrm{d}^2z_2}{\pi^4}\,|z_1,z_2\rangle\,\langle -z_1,-z_2|\\ &\times\,\mathrm{e}^{2(\alpha z_1^*-\alpha^*z_1)}\,\mathrm{e}^{2(\beta z_2^*-\beta^*z_2)},\end{aligned}\tag{37}$$

and noticing that $\langle -z_2|\,\mathrm{e}^{b^\dagger b\ln[(1-\mathrm{e}^{-2\kappa t})\tanh^2\theta]}\,|z_2\rangle = \exp[(-\left(1-\mathrm{e}^{-2\kappa t}\right)\tanh^2\theta-1)z_2^*z_2]$, as well as using Eq. (32), we can calculate the Wigner function at time t as:

$$\begin{aligned}W(\alpha,\beta)\,=\,\operatorname{Tr}\left[\rho(t)\,\Delta(\alpha,\beta)\right]=\frac{1}{\pi^2}\frac{\operatorname{sech}^2\theta}{1+R}\exp\Big\{&-2\frac{(1-R)\,|\beta|^2+|\alpha|^2\left(1+\tanh^2\theta\right)}{1+R}\\ &+\frac{4\,\mathrm{e}^{-\kappa t}\tanh\theta}{1+R}\left(\alpha^*\beta^*+\alpha\beta\right)\Big\},\end{aligned}\tag{38}$$

where $R=\left(1-2\,\mathrm{e}^{-2\kappa t}\right)\tanh^2\theta$.

In particular, when $\kappa t=0$, i.e., the case of two-mode squeezed state, $R=-\tanh^2\theta$, equation (41) reduces to the initial state Wigner function[12,13]

$$\begin{aligned}W(\alpha,\beta)\,=\,&\frac{1}{\pi^2}\exp[2\left(\alpha^*\beta^*+\alpha\beta\right)\sinh 2\theta\\ &-2(|\alpha|^2+|\beta|^2)\cosh 2\theta].\end{aligned}\tag{39}$$

On the other hand, when $\kappa t\to\infty$, then $R\to\tanh^2\theta$, equation (38) becomes

$$W(\alpha,\beta)=\frac{1}{\pi}\,\mathrm{e}^{-2|\alpha|^2}\frac{\operatorname{sech}2\theta}{\pi}\exp\left[-2\,|\beta|^2\operatorname{sech}2\theta\right].\tag{40}$$

It is interesting to note that the Wigner function of the thermal vacuum state is

$$\begin{aligned}W_{\mathrm{th}}\,=\,&\frac{1-\mathrm{e}^{-\omega/kT}}{\pi\left(1+\mathrm{e}^{-\omega/kT}\right)}\\ &\times\exp\left[-2\frac{\left(1-\mathrm{e}^{-\omega/kT}\right)}{\left(1+\mathrm{e}^{-\omega/kT}\right)}\,|\beta|^2\right],\end{aligned}\tag{41}$$

thus equation (40) is just the product of the Wigner function $W_{|0\rangle\langle 0|}$ of the vacuum state and the W_{th} of the thermal vacuum state, i.e.,

$$W(\alpha,\beta)=W_{|0\rangle\langle 0|}(\alpha)\cdot W_{\mathrm{th}}(\beta),\tag{42}$$

where

$$W_{|0\rangle\langle 0|}(\alpha)=\frac{1}{\pi}\,\mathrm{e}^{-2|\alpha|^2},$$

$$W_{\rm th}(\beta) = \frac{\operatorname{sech} 2\theta}{\pi} \mathrm{e}^{-2|\beta|^2 \operatorname{sech} 2\theta}, \qquad (43)$$

with $\mathrm{e}^{-\omega/kT} = \tanh^2\theta$, $\langle a^\dagger a\rangle_{\rm th} = \sinh^2\theta$. From Eq. (42) we can see that the output state is a product of a vacuum state and a "thermal" state after a long interaction time. In fact, this point can also be understood from Eq. (29).

5. Conclusion and discussion

As shown in Eq. (29), where $\mathrm{e}^{b^\dagger b \ln[(1-\mathrm{e}^{-2\kappa t})\tanh^2\theta]}$ represents a time-dependent chaotic field of b-mode which is in sharp contrast to the initial pure state $|0\rangle_{bb}\langle 0|$ in $\rho(0)$. The formula $\rho(0) \to \rho(t)$ exhibits how a pure two-mode squeezed vacuum state evolves into the mixed state, i.e., not only its squeezing parameter $\tanh\theta \to \mathrm{e}^{-\kappa t}\tanh\theta$, but also $|00\rangle\langle 00|$ evolves into $|0\rangle_{aa}\langle 0|\,\mathrm{e}^{b^\dagger b \ln[(1-\mathrm{e}^{-2\kappa t})\tanh^2\theta]}$. The outcome state is no more a pure state, but an entangled mixed state. This is because a two-mode squeezed state is simultaneously an entangled state, while the a-mode undergoes a damping, the entangled b-mode also varies correspondingly. For this channel, the associated loss mechanism for the two-mode squeezed state is in an entangled way.

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