Dynamics of organizational rumor communication on connecting multi-small-world networks

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We study the dynamics of an epidemic-like model for the spread of a rumor on a connecting multi-small-world-network (CM-SWN) model, which represents organizational communication in the real world. It has been shown that this model exhibits a transition between regimes of localization and propagation at a finite value of network randomness. Here, by numerical means, we perform a quantitative characterization of the evolution in the three groups under two evolution rules, namely the conformity and obeying principles. The variant of a dynamic CM-SWN, where the quenched disorder of small-world networks is replaced by randomly changing connections between individuals in a single network and stable connection by star nodes between networks, is also analysed in detail and compared with a mean-field approximation.

Keywords: organizational communication, small-world network, rumor propagation

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1. Introduction

When the finance crisis broke out in 2007, news spread more widely than in 1998 with the help of the internet. People hurriedly sell their stock and hold the money, thus accelerate the arrival of a bear market and economic depression. This rumor spreading is a kind of social spreading phenomena, so investigation is necessary and essential, not only because of the would-be worst scenario, but also because we are part of this process of propagation, when we are uploading a piece of information or talking with our friends whether in person or through MSN. Research on social spreading phenomena, such as the diffusion of fads, the adoption of technological innovations, and the success of consumer products mediated by word of mouth, has a long tradition in sociology and economics. With the development of the internet, many socialists and computer scientists are absorbed by the propagation of rumor, which was traditionally the field of psychology.[1] In reality, many technological networks[2−4] often revealed scale-free behaviour. Some modular engineering networks[5,6] exhibited both the power-law and exponential behaviours.

As Barabasi once said, "Our world is small because our society is a dense network."[7] One of the classes of a complex network is a small-world network, which has a shorter path length and is highly clustered. It is neither wholly regular nor wholly random, but is counterbalanced between these two extremes. In 1998, Watts and Strogatz first promoted a small-world network,[8] which randomly rewired links and is well-known as a WS-SWN. Later, Newman and Watts proposed a modified WS-SWN named NW-SWN,[9] which randomly adds edges instead of reconnecting in order to minimize the number of orphan clusters. The use of these two networks to simulate social interpersonal networks is widespread, like rumor spreading.[10−13] Zanette analysed the effects of structural parameters in a WS-SWN,[10] while Li discussed how the shortcut might influence spreading in different spatial dimensions with a NW-SWN.[13]

In this paper, considering that individuals are in their rebellious period and some may be disconnected or out of touch with others, just like orphan clusters, we promote our model based on a WS-SWN. The majority of relative researches are constrained in a single SWN, which cannot perfectly characterize

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the features of those social organization structures, in which communications are common. As a special group, which is sensitive and susceptible to rumor propagation, we share our own zones within subordinate organizations. There are several patterns of organizational communication: "Chain", "Wheel", and "Star".[14] The Chain can readily be seen to represent the hierarchical pattern which characterizes strictly formal information flow, "from the top down", in military and some types of business organizations. The Wheel can be compared with a typical autocratic organization, meaning one-man rule and limited employee participation. Complex organization communication can be simulated from macro organization structure to micro small-world groups. For simplicity, we aimed at building up a connecting multi-network which consists of the three groups: SWN1, SWN2, and SWN3 to simulate propagation. Comparison with two and four groups will be discussed in Section 2.

As for the rules of self-evolution, the general principle in rumor propagation employing a susceptible-infective-refractory (SIR) model, an epidemiology model introduced by Daley and Kendall to information spreading,[15,16] is as follows: at the beginning, only one individual is infected and the remnant population is susceptible. At each time step, an individual i is randomly chosen as the infected individual. This individual contacts one of her neighbours, say j. If j is in the susceptible state, i transmits the rumor and j becomes infected. If, on the other hand, j is already infected or refractory, then i loses her interest in the rumor and becomes refractory.[16] However, this rule meets some limitations: 1) random choosing may leave some nodes unchosen; 2) non-directional spreading is, to some extent, not consistent with a real-world situation, which tends to be more pertinent and targeted. Thus, getting some insights from classical opinion dynamics models, we consider two ergodic principles, Conformity and Obeying, to better delineate the phenomena. The former principle says that a certain adolescent's opinion mostly depends on the popular standpoint of the vast majority. By contrast, the obeying principle means a certain "peripheral" individual pays more attention to a "Star" node when making decisions. A specific description of these two principles is given in Section 3.

2. Model construction

As previously mentioned in the Introduction, we build our CM-SWN model (Fig. 1) based on a WS-SWN, then for each SWN: consider that a network of N nodes is arranged in a ring, where every node is connected to its neighbours ($K/2$ nodes on its left and right). K is an even number and also the degree of a node. The average of all the nodes' degree is denoted as K. Then, every edge E in the network will break and rewire with a probability p. In other words, for every edge, one of the ends remains while the other end will randomly reconnect to another node. Assume one edge connects any two nodes at most and every node is forbidden to connect to itself. A centralized network exists when information is funneled through a small number of individuals within an organization. During propagation, the hub node plays an important role and the simplified organization communication structure seems to be a "pyramid" as shown in Fig. 1.

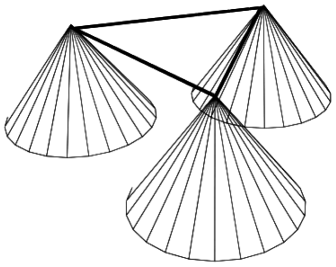


Fig. 1. Structure of Connecting Multi-SWN. It is the starting phase of a CM-SWN consisting of 3 single but mutually connecting SWNs (the SWN1, the SWN2, and the SWN3).

However complicated the structure is, it can be simplified as a double-floor connecting pyramid model as shown in Fig. 1. The planar view of the connected organization structure in Fig. 2 can be likened to small-world network, where the "Chain" communication mode is represented by the interaction of information of the hub node and other nodes; meanwhile, the "Wheel" is represented by a rewiring process in a small-world network. And for every SWN, the network is a regulated network when $p = 0$ or a random network with $p = 1$. It becomes practical to control the transition from regulated network to random network by adjusting the reconnecting probability p.[8]

As for rumor spreading, like an epidemic, there are basically two models: a susceptible-infected-susceptible (SIS) model, in which the susceptible one gets infected and can recover and return to a vulnerable state; a susceptible-infected-refractory (SIR)

model,[13,17,18] which introduces a new status of refractory to describe the immunized nodes. In light of the phenomena that people get bored with or lose faith in rumor and no longer spread the news, we adopt an SIR model.

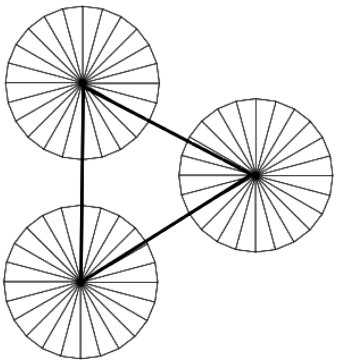


Fig. 2. Planar view of organization communication "pyramid", which represents a connecting multi-small-world network.

Thus, employing an SIR model, we find that our CM-SWN model (as shown in Fig. 1) possesses three of the above constructed single SWNs, and regarding that rumor propagation is not just confined to one single network, we introduce our hub bridges and assume that they never break down. We have the following equations:

$$\begin{array}{ccccccc} N1_S & + & N1_I & + & N1_R & = & N1 \\ + & & + & & + & & + \\ N2_S & + & N2_I & + & N2_R & = & N2 \\ + & & + & & + & & + \\ N3_S & + & N3_I & + & N3_R & = & N3 \\ \| & & \| & & \| & & \| \\ N_S & + & N_I & + & N_R & = & N, \end{array} \tag{1}$$

where, $N1_S$, $N1_I$, $N1_R$ denote the number of nodes in SWN1 in the state of S, I, and R respectively, while $N1$, $N2$, $N3$ indicate the total nodes in SWN1, SWN2, and SWN3 respectively; N_S, N_I, N_R implies the number of nodes in all SWNs which is in the state of S, I, and R respectively; and N is the number of nodes in all SWNs.

Along with the propagation processes, all nodes will diverge into two states: 1) those who have heard of the rumor and consequently turn into R; 2) those who never heard of the opinions and consequently remain S. Assume that in SWN1 the number of nodes $N1$ tends to an infinite number, $r = N1_R/N1$ fluctuates around an equilibrium state r^*, namely, in a single SWN there will be $1 - r^*$ people who have never heard of the rumor. It had been proved[19] that in an SWN with k nearest neighbours, r finally decays to zero when $p < p_c$, that is the rumor dies in a small neighbourhood of its origin under that threshold, otherwise it corresponds to the following equation:

$$r \propto |p - p_c|^{\gamma}, \tag{2}$$

where the critical probability gradually decays along with an increase in k, $\gamma = 2.2$.

3. Principles

As previously discussed in Section 1, the general rules for evolution meet some limitations. Though missing the fantasy of a complex network, the three classical models in opinion dynamics (the missionary model by Sznajd,[18] which changes states according to two neighbours; the opportunism model by Krause–Hegselmann,[20] which consults every neighbour's opinion; and the negotiator model by Deffuant *et al.*,[21] which chooses an interlocutor) all provide some insights.

We design two spreading principles: conformity and obeying principles. In the beginning, the three single networks SWN1, SWN2, and SWN3 will be assigned $N1_I(0)$, $N2_I(0)$, $N3_I(0)$ respectively for the number of nodes which are in the state of I, and the rest with the state of S (note: the edges connecting the three hub nodes never break).

3.1. Model 1: conformity principle

As introduced, the SWN is employed as a heterogeneous network due to the evolution and rewiring of edges and node state changes in a certain orientation. But it suffers the uncertainty of state transitions because of the rewiring probabilities and uncertainty of choosing nodes. Most previous studies focused on how many nodes experienced state-transition, like the spread of disease, the most common method is the mean-field rate equation.[22] And the majority numerically simulated the transition probability of $S \to I$ and $I \to R$. In rumor propagation research by Moreno *et al.*,[23] the spreading process evolves by directed contacts of the infected node, say the state of I with others. When an infected node meets a susceptible one, the state of S, the latter turns into I with probability λ. However, this determination process, through numerical simulation, is not based on the quantification of realistic rumor propagation. Thus, in view of the conformity, a process by which an individual's

attitudes, beliefs, and behaviours are conditioned by what the majority might perceive, which occurs in both small groups and society as a whole, we provide the following equations to quantify the transition probabilities (take SWN1 as an illustration)

$$\begin{aligned}\lambda 1(t) &= \frac{N1_I(t)}{N1},\\ \mu 1(t) &= \frac{N1_I(t)+N1_R(t)}{N1},\end{aligned} \tag{3}$$

where, $\lambda 1(t)$ implies the infection probability of $S \to I$ in SWN1, and $\mu 1(t)$ denotes the refractory probability of $I \to R$ in SWN1, because if the infected node spreads the rumor towards other infected nodes or the refractory node at time t, the node will lose the desire to spread the rumor further and turn into a refractory node at time $t+1$.

Thus, the transition probabilities take the form of a matrix as shown in the following expression

$$\begin{array}{cccc} & S(t) & I(t) & R(t)\\ S(t+1) & 1-\lambda & 1-\mu & 1\\ I(t+1) & \lambda & 0 & 0\\ R(t+1) & 0 & \mu & 0. \end{array} \tag{4}$$

And because the hub node, the fixed communicating bridge between two SWNs, which owns the largest degree $\langle K\rangle$ in its SWN, improves the propagation efficiency, we introduce a positive deviation factor $\varepsilon \geq 1$ in our model, and obtain the following equations:

$$\begin{aligned}\frac{\mathrm{d}s(t)}{\mathrm{d}t} &= -\varepsilon\lambda(t)\langle K\rangle s(t)i(t),\\ \frac{\mathrm{d}i(t)}{\mathrm{d}t} &= \varepsilon\lambda(t)\langle K\rangle i(t)s(t) - \varepsilon\mu(t)\langle K\rangle i(t)[i(t)+r(t)],\\ \frac{\mathrm{d}r(t)}{\mathrm{d}t} &= \varepsilon\mu(t)\langle K\rangle i(t)[i(t)+r(t)],\end{aligned} \tag{5}$$

where $\langle K\rangle$ denotes the average number of nodes, to which each node is connected; $s(t)$, $i(t)$, and $r(t)$ suggest the density of the state of S, I, R respectively.

3.2. Model 2: obeying principle

The obeying principle means that one will obey or agree with a certain person due to his requirement or authority. Our ergodic algorithm first experiences all the edges in the network and examines the state of every edge's ends and then determines how the states convert. In the field of disease propagation, Rand[17] first put forward the formulation of pair-wise models to analyse the spreading. Then another more complicated numerical model[21] was raised, with the mean densities of states, but also with the densities of links connecting nodes with different states.

As for the CM-SWN, we introduce the number of nodes A, the number of connected pairs $[A-B]$, where A and B stand for any of the types S, I, R and take the following equation of dynamic evolution. Keeling *et al.*[24] have put up a novel triple method. Although the mutual effects of three or more than three nodes seem never to happen under the obeying principle, traverses of each edge in the whole network may lead to that happening sometimes. Therefore, the obeying rule is a simplified and applicable rule improved from the method discussed by Keeling *et al.*[24]

For the $S \to I$ process, the infected individual I directly affects its connecting nodes of susceptible individuals S with a probability of α which depends on one's curiosity and ability to distinguish authenticity of the rumor.

For the $I \to R$ process, when the infected individual I encounters another infected individual I or the refractory individual R, he loses interests in spreading the rumor further and becomes refractory with the probability of β_1 and β_2 respectively.

4. Application

We apply our CM-SWN model under two principles to rumor propagation among groups, to be specific, among regimes. And we conduct a simulation in the Matlab to see how the rumor transmits from one SWN to other SWNs and how the parameters might influence equilibrium. We employ a matrix $A = (a_{ij})$ to denote the connection, if node i is wired with node j then $a_{ij} = 1$, otherwise $a_{ij} = 0$. The Matlab is doing the simulation in a discrete time scale, say the running time.

4.1. Propagation among SWNs

Now, we examine an interesting phenomenon in the CM-SWN, that is, how a piece of news transmits from one SWN to other SWNs. To begin with, we set 50% of nodes in the SWN1 as believing the rumor and spreading it further. This is because when a rumor breaks out in the SWN2, such as, John's father is a criminal, the news will soon overwhelmingly be the topic of almost every student conversation.

As figure 3(a) shows, under the conformity principle, the soaring rumor in SWN1 does not enter the other two SWNs, and gradually fades away when $t = 10$. It is interesting that the rumor, with which

half the nodes in SWN1 are infected cannot spread to the other two networks. In SWN1, we find that the infected nodes decrease with time t until they vanish. The $N1_I(t)$ reduces slowly with a speed of 5 nodes/step as the speed with which susceptible nodes turn to infected nodes is larger than the speed with which infected nodes turn to refractory nodes in the first two steps. From $t = 3$, $N1_I(t)$ changes with a speed of 10 nodes/step, which is on average higher than the first two steps as there are not any susceptible pupils and only infected nodes turn to refractory nodes. The number of the infected individuals changes every timescale, based on the infected transition probability $\lambda(t-1)$ and the refractory transition probability $\mu(t-1)$. We simulate the propagation under the obeying principle as shown in Fig. 3(b) with $\alpha = 1$ and $\beta_1 = \beta l = P_{IR}$ for simplification. From Fig. 3(b), it is obvious that the rumor arrives at SWN2, then SWN3 with a time-lag with the help of hub nodes. As it propagates, the number of I in AN dumps to zero when $t = 1.5$, while those in SWN2 and SWN3 respectively climax at about 60 ($t = 2$ and $t = 2.5$), then dump.

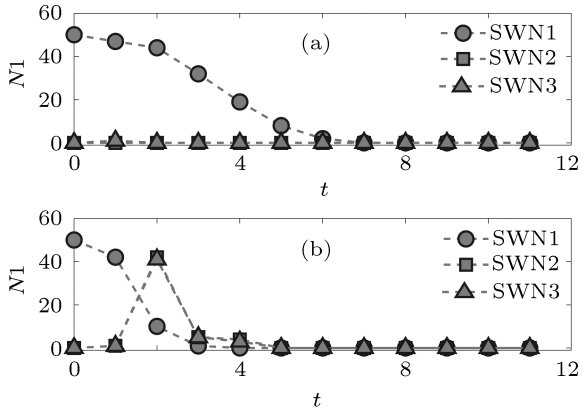


Fig. 3. (a) Evolution of the number $N_I(t)$ of infected individuals as a function of time under the conformity principle with the total number of pupils in each SWN $N1$ (circles) = $N2$ (squares) = $N3$ (triangles) = 100, $p = 0.1$, $K = 2$. (b) Evolution of the number $N_I(t)$ of infected individuals as a function of time under the obeying principle with the total number of pupils in each SWN $N1 = N2 = N3 = 100$, $p = 0.1$, $K = 2$, and $P_{IR} = 0.2$. For the initial date set, $N1_I(0) = 50$, $N1_S(0) = 50$, and all pupils in the other two schools are susceptible, namely $N2_S(0) = N3_S(0) = 100$. From left to right, the time t changes by a step of size =1.

Under the conformity principle, the rumor has very little probability λ of spreading from one school to the other two regimes. In our 100 simulations, the rumor cannot spread to other networks as the susceptible nodes are so rational that they have a larger probability λ of changing their attitudes to the rumor only when more nodes are infected with the rumor. One kind of rumor cannot grow out of nothing under the conformity principle. Thus we should hold a critical attitude, not the absolute conformity attitude to receive the rumor. However, under the obeying principle, the state transition of a node depends on the state of its connected nodes and the ergodic order. We find that given enough time, a rumor can certainly spread to the other networks, just like the "domino effect".

4.2. Fraction of refractory nodes r

Apart from steady-reaching time, we also pay attention to the number of N_R of the final state, which directly reflects the seriousness of propagation. If $r = \langle N_R \rangle / N$ is large, spreading within the network is threatening; oppositely, we assume that rumor spreading is not activate. In this section we will discuss the relationship between r and the configuration parameter (rewiring probability p and degree K).

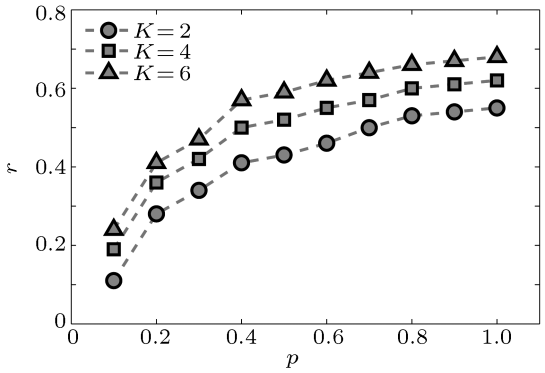


Fig. 4. Final fraction of refractory pupils $r = \langle N_R \rangle / N$ as a function of the "randomness" p on CM-SWNs with a total population $N = 100$, $P_{IR} = 0.2$, and $K = 2$, 4, 6 under the obeying principle. Results are obtained by averaging r over 1000 simulations for each p.

As figure 4 shows, the three curves are very close. All show that r increases along with p and when $p \approx 0$, the changing rate peaks then decreases as p increases. Consider that r linearly changes with p, we have $r \propto p^{0.686}$. When $p = 0$ at first, the network stays steady at $r = 0.1$. Along with the increase of p, the CM-SWN approximates the random network and r becomes larger, but with a diminishing margin. Our stimulating results coincide with the description of the conformity principle. In Ref. [13], a critical value $r^* = 0.8$ is obtained. In the CM-SWN, under the obeying principle, the propagating efficiency of rumor increases with r and finally r reaches 0.698

($K = 6$). Although under this principle the propagation traverses every node, we found that there are still some nodes avoiding a rumor's influence and staying in the S state. The reason is that these nodes have not been influenced when the rumor was quite popular. Then when most of the infected nodes turn into recovered nodes, they also avoided the refractory's influence.

As figure 5 shows, according to the description in Ref. [11] under the obeying principle, the final value of r depends on the average degree $\langle K\rangle$. In Fig. 6, r increases along with $\langle K\rangle$, and finally reaches 60%. According to Ref. [11], when $p = 0$ and $K = 24$, r peaks at about 60%. However, our simulation shows that r reaches its maximum when $p = 1$. As K increases, the influence of the hub nodes augments, causing the increase of the cluster coefficient and enlarging the propagating path of the whole network. At last, we gained a conclusion that r increases along with $\langle K\rangle$. During the propagation, according to formula,[17,20] more nodes in a single network turn into infected nodes after the hub nodes have been infected. Then the whole CM-SWN will be soon infected due to the connection of hub nodes from different single networks.

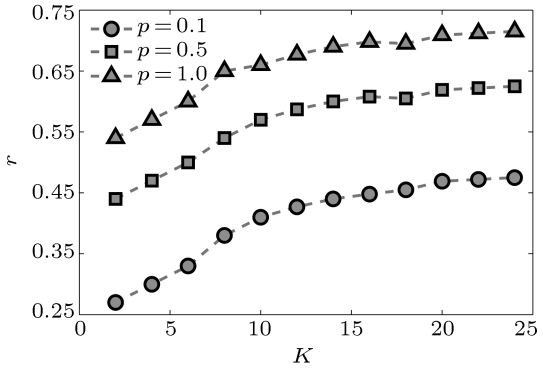


Fig. 5. Final fraction of refractory pupils $r = \langle N_R\rangle/N$ as a function of the degree K on CM-SWNs with a total population $N = 100$, $P_{IR} = 0.2$ and $p = 0.1$, 0.5, 1, under the obeying principle. Results are obtained by averaging r over 1000 simulations for each K.

4.3. Group numbers N

To examine how the number of CM-SWNs influences rumor propagation, we set two control groups, which consist of two and four SWNs. We simulate the propagation under the obeying principle as shown in Fig. 6 with the number of SWNs $n_1 = 2$, $n_2 = 3$, $n_3 = 4$ for comparison. It is interesting that the infected scaling $N_I(t)$ of communication with two or four connected networks disappeared in 8 steps, and is similar to the results of three SWNs in Fig. 1. Based on the obeying principle, the star node influences other "peripheral" nodes in a single SWN, and loses interest to spread the rumor as soon as other nodes are infected with the rumor. In a hierarchical model the star nodes are almost invariably located at the bottom of the hierarchy, they are strongly modulated and disconnected and they are connected mostly through a rewiring process. Similar hierarchical structures have also been observed in the social networks of open source communities.[25]

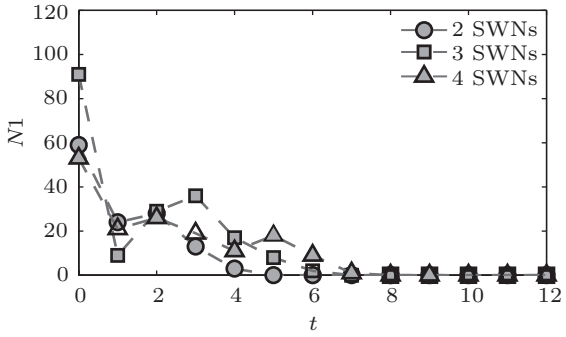


Fig. 6. Evolution of the number $N_I(t)$ of infected individuals as a function of time under the obeying principle with the total number of pupils in each SWN $n1$ (circles) = 2, $n2$ (squares) = 3, $n3$ (triangles) = 4, $p = 0.1$, $K = 2$.

5. Conclusion

Rumor spreading is one of the most basic mechanisms for information dissemination in networks. We discuss organization communication based on a preferential small-world model of Barabasi *et al.*[8,22] which is considered to be a valuable model for social networks, by focusing on group rumor interaction. As leadership plays an important role in group interaction, we introduce star nodes which handle all messages in the group and are respected by other people in the group, occupy key positions with regard to the number of messages they handle and the degree to which they exercise control over the flow of information and represents the hub of single group. In summary, we have studied analytically and numerically rumor propagation on centralized networks—the CM-SWN coupled with the SIR model. Also we have further investigated the dynamics on general growing networks and the role of immunization, which are important issues for realistic networks, under the conformity principle and obeying principle. As for the significance of our research, theoretically, we creatively detail the CM-SWN and two novel principles: conformity and obeying; practically, we employ the quantitative method and computer simulations to explore

rumor spreading and enrich the relative research on organization communication.

First, rumor propagating objects are not confined to one single group, but an interaction of multiple networks, which may be catalogued by people's workplace, personalities or other properties; therefore, our CM-SWN extends the traditional research on rumor spreading, especially in adolescents. Then, we set star nodes, the communicating bridges, to describe the mutual influence of multiple networks, which reflects the superiority of our model. In addition, by simulation we discover the rules of rumor propagation.

Based on a CM-SWN, we begin our simulation to examine the efficiency and reliability of rumor propagation among three regimes. At first, we examine how a rumor transmits from one SWN to the other SWNs under two principles. We find that under the conformity principle, the rumor has very little probability of spreading from one school to the other two regimes. All nodes are so rational that the rumor cannot grow out of nothing under the conformity principle. However, under the obeying principle, the fanatic rumor in SWN1 can easily spread to the other two networks in a "domino effect". Then, we conclude that r increases along with p and when $p \approx 0$, the changing rate peaks then decreases as p increases. The propagating efficiency of rumor increases with r and finally r reaches 0.698 ($K = 6$) due to some nodes avoiding the rumor's influence. As for the degree K, r increases along with $\langle K \rangle$, and finally reaches 60% when $p = 1$. The larger K makes the network denser while the larger p directly increases the propagating efficiency. At last, we find that in a hierarchical model the star nodes are almost invariably located at the bottom of the hierarchy, they are strongly modulated and disconnected and they are connected mostly through a rewiring process.

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