Mobile user forecast and power-law acceleration invariance of scale-free networks*

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(Received 18 October 2010; revised manuscript received 30 June 2011)

This paper studies and predicts the number growth of China's mobile users by using the power-law regression. We find that the number growth of the mobile users follows a power law. Motivated by the data on the evolution of the mobile users, we consider scenarios of self-organization of accelerating growth networks into scale-free structures and propose a directed network model, in which the nodes grow following a power-law acceleration. The expressions for the transient and the stationary average degree distributions are obtained by using the Poisson process. This result shows that the model generates appropriate power-law connectivity distributions. Therefore, we find a power-law acceleration invariance of the scale-free networks. The numerical simulations of the models agree with the analytical results well.

Keywords: mobile user forecast, power-law accelerating growth, complex networks, scale-free networks

PACS: 89.75.–k, 89.75.Hc, 89.65.–s, 89.70.–a **DOI:** 10.1088/1674-1056/20/11/118902

1. Introduction

Tremendous changes have taken place in mobile communications in China over the past two decades. The explosive growth of mobile users is therefore of interest from a network science perspective, as it offers a source of data. In mainland China, the mobile users were less than 3000 by the end of 1988, while over 640 million by the end of 2008. The contact among the mobile users can be studied by using networks. In fact, most of the interactions between individuals in the real world can be described by networks, including the Internet, cell interaction networks, supply chain networks and social networks. Complex networks lie in everywhere in our daily life and have attracted the great interest of scholars.[1−11] It is necessary to study the topology of complex networks.

From the 1960's to the end of the twentieth century, studies on complex networks were based on the random graph theory. However, empirical results show that the topology of most real networks is different from that of the random graph. Real networks, such as technological and social networks, show various degree distributions,[1,3] and many networks have some common properties, such as the power-law degree distribution and the small-world effect. In order to depict real networks, Watts and Strogatz proposed a small-world model in 1998 and Barabási and Albert (BA) proposed a scale-free network in 1999. From then on, a new era of complex network studies began

Barabási and Albert pointed out that the growth and the preferential attachment are two important mechanisms for scale-free networks and proposed a model with the two mechanisms. The growth of networks includes two aspects, the growth of nodes and the addition of new edges. Dorogovtsev *et al.* studied a model with an accelerating growth of edges. At every step, a new node was added to the network, which received n incoming edges from random nodes in the network. Additionally, $c_0 t^{\theta}$ new edges were distributed, each of them being directed from a randomly selected node to a node with a high incoming degree with asymptotically linear preferential attachment $\Pi(k_{\rm in}) \propto A + k_{\rm in}$.[1] Although Dorogovtsev *et al.* showed that the accelerating growth did not change the scale-free nature of the degree distribution, and its degree exponent was controlled by exponent θ. We know that edges of new nodes have an impact on the scale-free property.[11] So far, networks with an accelerating growth of nodes are rarely studied. In the real

*Project supported by the National Natural Science Foundation of China (Grant No. 70871082) and the Shanghai Leading Academic Discipline Project, China (Grant No. S30504).

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world, there are many networks with an accelerating growth of nodes. A large directed network has been constructed from the mobile telephone call patterns, where nodes are the phone numbers and every completed phone call is an edge directed from the caller to the receiver. We can prove that this is a network with an accelerating growth of nodes. Then we may ask: what effect does the accelerating growth have on the structure of the network? Which kind of acceleration can produce a scale-free network? What is the form of the degree distribution in such a network?

In this paper, some concepts related to complex networks are first introduced. In Section 2, we collect data on mobile phone users from 1997 to 2010 in mainland China and use those data to study the growth law of the mobile phone users. The regression method is applied to predict the development trend of the mobile phone users in mainland China. We find that the number of the mobile phone users grows with a power-law acceleration. In Section 3, we propose a network model with a power-law accelerating growth on the basis of the growth law of the mobile phone users and study analytically the effect of the accelerated growth on the degree distribution using the Poisson process. In Section 4, we give the simulation results. Our findings from the numerical simulations and the analytical arguments conclude that this scale-free network follows the power-law acceleration invariance. The conclusions are given in Section 5.

Let $(V(t), E(t))$ be a complex network, where $V(t)$ denotes a set of nodes at time t, $E(t)$ denotes a set of edges. The $N(t)$ is the number of nodes at time t, $E[N(t)]$ denotes the average number of nodes at time t, $k_i(t)$ denotes the degree of node i at time t, and $P\{k_i(t) = k\}$ $(k = 0, 1, 2, \ldots)$ denotes the degree distribution of node i at time t. Related definitions and theorems are as follows (see Ref. [8]).

Definition 1 If $\lim_{t\to\infty} P\{k_i(t) = k\} = P_i(k)$ exists for $k = 0, 1, 2, \ldots$, and $\sum_{k=0}^{\infty} P_i(k) = 1$, the limit is said to be a stationary degree distribution of node i.

Definition 2 $P(t,k) = \frac{1}{E[N(t)]} \sum_{i\in V(t)} P\{k_i(t) = k\}$ is the average degree distribution of the network at time t. If $\lim_{t\to\infty} P(t,k) = P(k)$ exists for $k = 0, 1, 2, \ldots$, and $\sum_{k=0}^{\infty} P(k) = 1$, the limit is said to be a stationary average degree distribution of the network.

Theorem The $N_k(t)$ denotes the number of nodes with degree k at time t in the network, then

$$P(t,k) = \frac{E[N_k(t)]}{E[N(t)]}.$$

2. Growth law of mobile phone users

In recent years, mobile communications develope rapidly, and the number of mobile phone users in China is increasing at a high speed.[12–16] The popularity rate of mobile phone in China reaches over 41.6%, which shows a great growth potential. The data in Table 1 are taken from Refs. [12]–[16].

Table 1. Number of mobile phone users in mainland China.

Sequence No.	Year	Size/10^3
1	1997	13100
2	1998	24980
3	1999	43300
4	2000	85300
5	2001	144812
6	2002	206616
7	2003	268693
8	2004	334824
9	2005	393428
10	2006	461082
11	2007	547286
12	2008	641245
13	2009	747384
14	2010	859004

We try to fit mobile phone users in mainland China by using the power regression method and then assume that the relationship between the mobile phone users and the time is given by

$$z = dt^b, \tag{1}$$

where d and b are two fitting parameters. Let $y = \lg z$, $x = \lg t$, and $a = \lg d$, then

$$y = a + bx. \tag{2}$$

From the linear regression, we obtain

$$a \approx 4, \quad b \approx 1.69, \quad r_{xy} \approx 0.98,$$

where r_{xy} is the correlation coefficient. The correlation coefficient measures the linear correlation between variables x and y. Since $r_{xy} \approx 0.98$ is very close to 1, y and x are strongly linearly correlated. Thus, the power regression function is given by

$$z = 10000t^{1.69}. \tag{3}$$

Figure 1 is a fitting with Eq. (3) shown in the double-logarithmic coordinates. It shows that the growth law of the mobile phone users approximates to a power law.

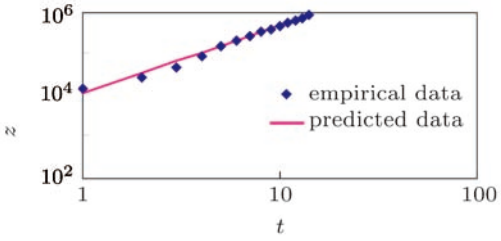


Fig. 1. (colour online) Comparison between Eq. (3) and the original data in the double-logarithmic coordinates.

As Table 2 shows, the relative errors between the predicted and the actual data are acceptable. In particular, the errors after the year of 2000 are very small, and the relative error is only 0.006795 in 2010. Due to the Markov property of the growth of the mobile phone users, we can use the power regression function to forecast the number of mainland China's mobile phone users in the future. From Eq. (3), we can forecast the number of China's mobile phone users, which should be 971.838 million in 2011.

Table 2. Comparison between actual and predicted data.

Year	Actual users/10^3	Predicted users/10^3	Relative error
1997	13100	10000.000	0.310000
1998	24980	32265.670	0.225803
1999	43300	64022.838	0.323679
2000	85300	104107.348	0.180653
2001	144812	151795.643	0.046007
2002	206616	206573.978	0.000203
2003	268693	268049.924	0.002399
2004	334824	335909.339	0.003231
2005	393428	409892.376	0.040168
2006	461082	489778.819	0.058591
2007	547286	575378.511	0.048824
2008	641245	666524.789	0.037928
2009	747384	763069.799	0.020556
2010	859004	864881.047	0.006795

A large directed network has been constructed from the mobile phone call patterns, where nodes are the mobile phone numbers and every completed phone call is an edge. As noted in the previous result, the network is in the accelerated growth. Based on Eq. (3), at time t, $10^4 t^{1.69}$ new nodes will be added into the network. We will discuss this kind of power-law growth network in the following section.

3. Networks with power-law growth

3.1. Model

From the previous section, we know that the network of the mobile phone users grows following a power law. A node in the network, a phone number, represents a mobile phone user. A node receives an incoming edge only if there is a new call to this phone number from a different mobile user, and sends an outgoing edge only if there is a new call from this phone number to another user. Thus, the mobile phone users form a directed network with a power-law growth. Motivated by the social network, we can establish a power-law growth model. The algorithm of the model is as follows. (i) Initial state: At $t = 0$, the network is a directed graph with m_0 nodes. (ii) Random growth: The arrivals of nodes occur in accordance with a Poisson process $N(t)$ having rate $\lambda = 1$. When $N(t)$-th batch nodes arrive at the system at time t, ct^θ new nodes are added to the system, where $c > 0$, and $\theta \geq 0$. All the new nodes are not connected to each other, and each new node is individually linked to m ($m \leq m_0$) different existing nodes in the system. The incoming and the outgoing edges of the new nodes respectively obey the binomial distributions $B(m, q)$ and $B(m, 1-q)$, with $q \in (0, 1)$. (iii) Preferential attachment: When choosing old node i to which a new node connects, we assume that the probability that node i receives an incoming edge is proportional to the incoming degree I_i of node i, while the probability that node i gets a new outgoing edge is proportional to the outgoing degree O_i of node i.

3.2. Degree distributions of nodes

Next, we analyse the transient degree distribution of a node. Let t_i denote the time that the i-th batch nodes are added to the system. I_{ij} the incoming degree of the j-th node in the i-th batch at time t. We assume that I_{ij} is a continuous real variable, the rate at which I_{ij} changes is expected to be proportional to incoming degree I_{ij}. Consequently, I_{ij} satisfies the dynamical equation

$$\frac{\partial I_{ij}}{\partial t} = (1-q)mct^\theta \frac{I_{ij}}{\sum\limits_{i,j} I_{ij}}. \tag{4}$$

According to the Poisson process theory,[17] we know $E[N(t)] = t$, thus

$$\sum_{i,j} I_{ij} = \int_0^t mct^{\theta}\,\mathrm{d}t = \frac{1}{1+\theta} mct^{1+\theta},$$

and

$$\frac{\partial I_{ij}}{\partial t} = \frac{(1-q)(1+\theta)I_{ij}}{t}. \tag{5}$$

The solution of the above equation for the initial condition that the i-th batch nodes at their introduction have $I_{ij}(t_i)$ is

$$I_{ij}(t) = I_{ij}(t_i)\left(\frac{t}{t_i}\right)^{(1-q)(1+\theta)}. \tag{6}$$

From Eq. (6), we obtain

$$\begin{aligned} & P\{I_{ij}(t) < k\} \\ &= P\left\{t_i > \left(\frac{I_{ij}(t_i)}{k}\right)^{\frac{1}{(1-q)(1+\theta)}} t\right\} \\ &= \sum_{l=0}^{m} \binom{m}{l} q^l (1-q)^{m-l} P\left\{t_i > \left(\frac{l}{k}\right)^{\frac{1}{(1-q)(1+\theta)}} t\right\}. \end{aligned} \tag{7}$$

The nodes batch arrival process is a Poisson process having rate $\lambda = 1$, therefore the nodes at time t_i follow the Γ distribution. Thus

$$\begin{aligned} & P\{I_{ij}(t) < k\} \\ &= \sum_{l=0}^{m} \binom{m}{l} q^l (1-q)^{m-l} \\ &\quad \times \mathrm{e}^{-(\frac{l}{k})^{\frac{1}{(1-q)(1+\theta)}} t} \sum_{h=0}^{i-1} \frac{1}{h!}\left[\left(\frac{l}{k}\right)^{\frac{1}{(1-q)(1+\theta)}} t\right]^h. \end{aligned} \tag{8}$$

Further, the incoming degree distribution of the j-th node in the i-th batch at time t is given by

$$\begin{aligned} & P\{I_{ij}(t) = k\} \\ &= \frac{\partial P\{I_{ij}(t) < k\}}{\partial k} \\ &= \frac{t k^{-\left[1+\frac{1}{(1-q)(1+\theta)}\right]}}{(1-q)(1+\theta)} \sum_{l=0}^{m} \binom{m}{l} l^{\frac{1}{(1-q)(1+\theta)}} q^l (1-q)^{m-l} \\ &\quad \times \mathrm{e}^{-(\frac{l}{k})^{\frac{1}{(1-q)(1+\theta)}} t} \frac{\left[\left(\frac{l}{k}\right)^{\frac{1}{(1-q)(1+\theta)}} t\right]^{i-1}}{(i-1)!}, \\ & i = 1, 2, \ldots, \quad t > t_i. \end{aligned} \tag{9}$$

The outgoing degrees (O_{ij}) are similar to the incoming ones (I_{ij}), so we obtain the outgoing degree distributions of the j-th node in the i-th batch at time t as

$$\begin{aligned} P\{O_{ij}(t) = k\} &= \frac{t k^{-\left[1+\frac{1}{q(1+\theta)}\right]}}{q(1+\theta)} \\ &\quad \times \sum_{l=0}^{m} \binom{m}{l} l^{\frac{1}{q(1+\theta)}} (1-q)^l q^{m-l} \\ &\quad \times \mathrm{e}^{-(\frac{l}{k})^{\frac{1}{q(1+\theta)}} t} \frac{\left[\left(\frac{l}{k}\right)^{\frac{1}{q(1+\theta)}} t\right]^{i-1}}{(i-1)!}, \\ & i = 1, 2, \ldots, \quad t > t_i. \end{aligned} \tag{10}$$

3.3. Stationary average degree distributions of the network

The previous section discusses the transient degree distributions of a node in the network. We next describe a method to calculate analytically the stationary average degree distribution, allowing us to determine exactly the scaling exponent. From Eqs. (9) and (10), at time t, we obtain the transient average incoming and outgoing degree distributions of the network as

$$\begin{aligned} & P\{I(t) = k\} \\ &= \frac{k^{-\left[1+\frac{1}{(1-q)(1+\theta)}\right]}}{(1-q)t^{\theta}} \sum_{l=0}^{m} \binom{m}{l} l^{\frac{1}{(1-q)(1+\theta)}} q^l (1-q)^{m-l} \\ &\quad \times \mathrm{e}^{-(\frac{l}{k})^{\frac{1}{(1-q)(1+\theta)}} t} \sum_{i=0}^{t-1} \frac{1}{i!} t_{i+1}^{\theta} \left[\left(\frac{l}{k}\right)^{\frac{1}{(1-q)(1+\theta)}} t\right]^i, \end{aligned} \tag{11}$$

$$\begin{aligned} & P\{O(t) = k\} \\ &= \frac{k^{-\left[1+\frac{1}{q(1+\theta)}\right]}}{q t^{\theta}} \sum_{l=0}^{m} \binom{m}{l} l^{\frac{1}{q(1+\theta)}} (1-q)^l q^{m-l} \\ &\quad \times \mathrm{e}^{-(\frac{l}{k})^{\frac{1}{q(1+\theta)}} t} \sum_{i=0}^{t-1} \frac{1}{i!} t_{i+1}^{\theta} \left[\left(\frac{l}{k}\right)^{\frac{1}{q(1+\theta)}} t\right]^i. \end{aligned} \tag{12}$$

Since $\lambda = 1$, we assume that node i is added to the network at time $t_i = i$. If $\theta = n$ is a non-negative integer, we obtain the stationary average incoming degree distribution as

$$\begin{aligned} & P(k) \\ &= \lim_{t\to\infty} \frac{1}{(1-q)k} \sum_{l=0}^{m} \binom{m}{l} \left(\frac{l}{k}\right)^{\frac{1}{(1-q)(1+n)}} q^l (1-q)^{m-l} \\ &\quad \times \mathrm{e}^{-(\frac{l}{k})^{\frac{1}{(1-q)(1+n)}} t} \frac{1}{t^n} \sum_{i=0}^{t-1} \frac{(i+1)^n \left[(\frac{l}{k})^{\frac{1}{(1-q)(1+n)}} t\right]^i}{i!} \\ &= \lim_{t\to\infty} \frac{1}{(1-q)k^{1+\frac{2}{(1-q)(1+n)}}} \sum_{l=0}^{m} \binom{m}{l} l^{\frac{2}{(1-q)(1+n)}} \\ &\quad \times q^l (1-q)^{m-l} \mathrm{e}^{-(\frac{l}{k})^{\frac{1}{(1-q)(1+n)}} t} \frac{1}{t^{n-1}} \\ &\quad \times \sum_{i=0}^{t-1} \frac{(i+1)^{n-1} \left[\left(\frac{l}{k}\right)^{\frac{1}{(1-q)(1+n)}} t\right]^i}{i!} \end{aligned}$$

$$\approx \frac{1}{(1-q)k^{1+\frac{1}{(1-q)}}} \sum_{l=0}^{m} \binom{m}{l} l^{\frac{1}{1-q}} q^l (1-q)^{m-l}. \quad (13)$$

For any given $\theta \geq 0$,

$$\begin{aligned} & P(k) \\ & \geq \lim_{t\to\infty} \frac{1}{(1-q)k} \sum_{l=0}^{m} \binom{m}{l} \left(\frac{l}{k}\right)^{\frac{1}{(1-q)(1+\theta)}} q^l (1-q)^{m-l} \\ & \quad \times \mathrm{e}^{-(\frac{l}{k})^{\frac{1}{(1-q)(1+\theta)}} t} \frac{1}{t^{\lceil\theta\rceil}} \\ & \quad \times \sum_{i=0}^{t-1} \frac{(i+1)^{\lceil\theta\rceil} \left[\left(\frac{l}{k}\right)^{\frac{1}{(1-q)(1+\theta)}} t\right]^i}{i!} \\ & \approx \frac{1}{(1-q)k^{1+\frac{1}{1-q}}} \sum_{l=0}^{m} \binom{m}{l} l^{\frac{1}{1-q}} q^l (1-q)^{m-l}. \end{aligned}$$

On the other hand,

$$\begin{aligned} & P(k) \\ & \leq \lim_{t\to\infty} \frac{1}{(1-q)k} \sum_{l=0}^{m} \binom{m}{l} \left(\frac{l}{k}\right)^{\frac{1}{(1-q)(1+\theta)}} q^l (1-q)^{m-l} \\ & \quad \times \mathrm{e}^{-(\frac{l}{k})^{\frac{1}{(1-q)(1+\theta)}} t} \frac{1}{t^{\lfloor\theta\rfloor}} \\ & \quad \times \sum_{i=0}^{t-1} \frac{(i+1)^{\lfloor\theta\rfloor} \left[\left(\frac{l}{k}\right)^{\frac{1}{(1-q)(1+\theta)}} t\right]^i}{i!} \\ & \approx \frac{1}{(1-q)k^{1+\frac{1}{1-q}}} \sum_{l=0}^{m} \binom{m}{l} l^{\frac{1}{1-q}} q^l (1-q)^{m-l}. \end{aligned}$$

Thus, for any given $\theta \geq 0$, we obtain the stationary average incoming degree distribution of the network as

$$P(k) \approx \frac{1}{(1-q)k^{1+\frac{1}{1-q}}} \times \sum_{l=0}^{m} \binom{m}{l} l^{\frac{1}{1-q}} q^l (1-q)^{m-l}. \quad (14)$$

Similarly, we can also obtain the stationary average outgoing degree distribution of the model as

$$P(k) \approx \frac{1}{qk^{1+\frac{1}{q}}} \sum_{l=0}^{m} \binom{m}{l} l^{\frac{1}{q}} (1-q)^l q^{m-l}. \quad (15)$$

It can be concluded that the incoming and the outgoing degree distributions of the network with the power-law growth are two power-law distributions with power-law exponents $\gamma_{\text{in}} = 1 + \frac{1}{(1-q)}$ and $\gamma_{\text{out}} = 1 + \frac{1}{q}$, respectively. The results show that the network is a scale-free network and the degree exponent is independent of power-law growth parameter θ. The property is said to be the power-law acceleration invariance of the scale-free networks.

4. Simulations

4.1. Undirected network model and numerical simulation

Above we discussed the degree distribution of directed networks with a power-law accelerating growth. For an undirected network with a power-law accelerating growth, we establish the following model: (i) Initial state: At $t = 0$, the network is an undirected graph with m_0 nodes. (ii) Random growth: The arrivals of nodes occur in accordance with a Poisson process $N(t)$ having rate $\lambda = 1$. When $N(t)$-th batch nodes arrive at the system at time t, ct^θ new nodes are added to the network, where $c > 0$, and $\theta \geq 0$. All the new nodes are not connected to each other, and each new node is individually linked to m ($m \leq m_0$) different nodes already present in the system. (iii) Preferential attachment: When choosing the nodes to which the new node connects, we assume that probability Π with which a new node will be connected to node i depends on the degree k_i of node i, i.e., $\Pi(k_i) = \frac{k_i}{\sum_j k_j}$.

Undirected networks with a power-law accelerating growth can be regarded as special cases of directed networks with a power-law accelerating growth if $q = \frac{1}{2}$. From Eq. (15), we can obtain the degree distribution of an undirected network with a power-law accelerating growth

$$P(k) \propto \frac{1}{k^3}. \quad (16)$$

For $m_0 = 60$, $m = 8$, and $c = 1$, if $\theta = 1,\ 2,\ 3$, we respectively simulate the degree distributions and compare them with the theoretical values obtained with $P(k) \propto \frac{1}{k^3}$. The results of the numerical simulations are shown in Figs. 2, 3 and 4.

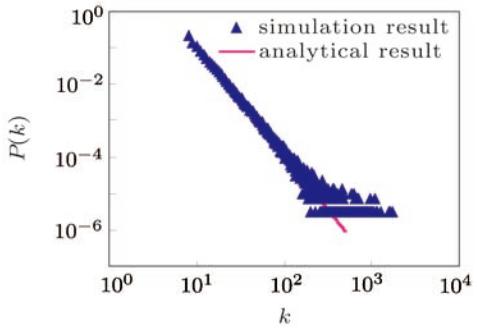


Fig. 2. (colour online) Degree distribution of the model with $\theta = 1$.

The theoretical results are in good agreement with the computer simulations (see Figs. 2, 3, and 4). From the simulation results, we find that the degree distribution of the undirected network model with a power-law accelerating growth follows a power-law distribution with degree exponent $\gamma = 3$.

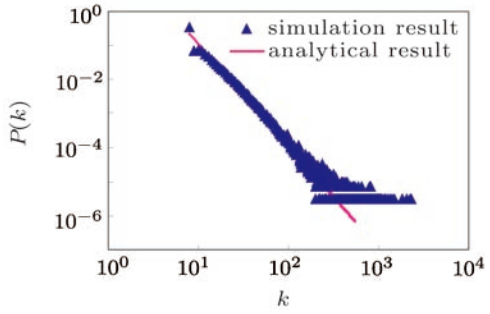


Fig. 3. (colour online) Degree distribution of the model with $\theta = 2$.

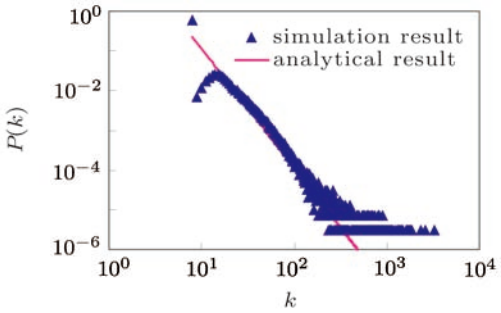


Fig. 4. (colour online) Degree distribution of the model with $\theta = 3$.

4.2. Numerical simulation of directed network model

For $m_0 = 6$, $m = 4$, and $c = 1$, if $\theta = 0$, $\frac{1}{2}$, $q = 0.25$, 0.35, 0.5, we respectively simulate the out- and in-degree distributions of the directed network model and compare them with the theoretical values obtain from Eqs. (14) and (15). To reduce the noise in the tail, we use the complementary cumulative distribution. The results of the numerical simulations are shown in Figs. 5–14.

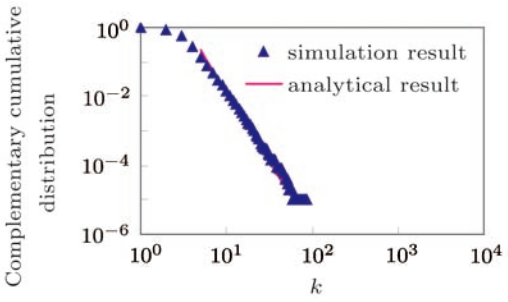


Fig. 5. (colour online) Log–log plot of complementary cumulative out-degree distribution for $\theta = 0$ and $q = 0.25$, which follows power-law form $k^{1-\gamma_{\rm out}}$ with $\gamma_{\rm out} = 5$.

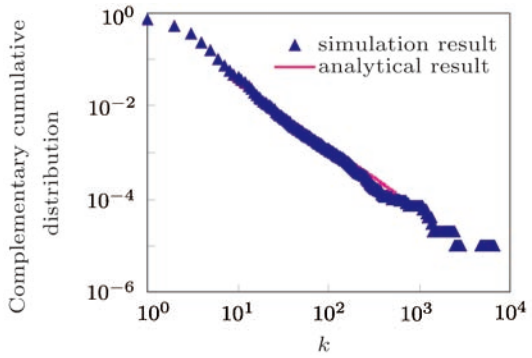


Fig. 6. (colour online) Log–log plot of complementary cumulative in-degree distribution for $\theta = 0$ and $q = 0.25$, which follows power-law form $k^{1-\gamma_{\rm in}}$ with $\gamma_{\rm in} = 2.34$.

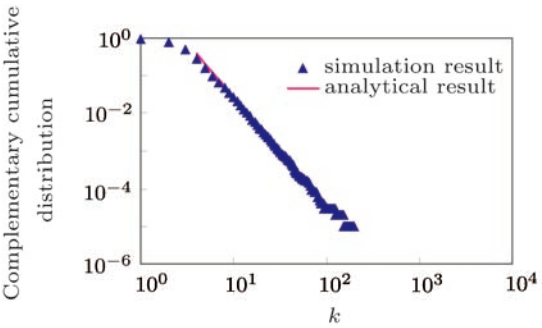


Fig. 7. (colour online) Log–log plot of complementary cumulative out-degree distribution for $\theta = 0$ and $q = 0.35$, which follows power-law form $k^{1-\gamma_{\rm out}}$ with $\gamma_{\rm out} = 3.86$.

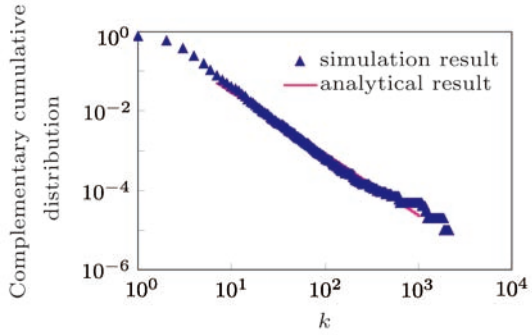


Fig. 8. (colour online) Log–log plot of complementary cumulative in-degree distribution for $\theta = 0$ and $q = 0.35$, which follows power-law form $k^{1-\gamma_{\rm in}}$ with $\gamma_{\rm in} = 2.54$.

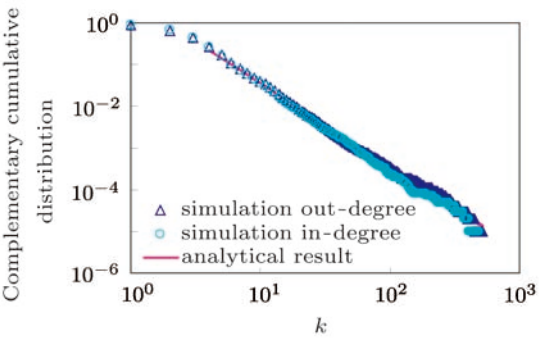


Fig. 9. (colour online) Log–log plot of complementary cumulative out-degree distribution for $\theta = 0$ and $q = 0.5$, which follows power-law form $k^{1-\gamma_{\rm out}}$ with $\gamma_{\rm out} = 3$, the log–log plot of complementary cumulative in-degree distribution follows power-law form $k^{1-\gamma_{\rm in}}$ with $\gamma_{\rm in} = 3$.

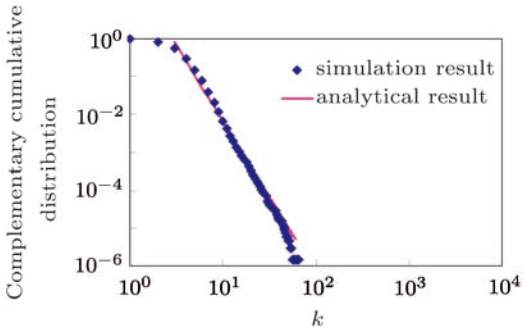


Fig. 10. (colour online) Log–log plot of complementary cumulative out-degree distribution for $\theta = 1/2$ and $q = 0.25$, which follows power-law form $k^{1-\gamma_{\rm out}}$ with $\gamma_{\rm out} = 5$.

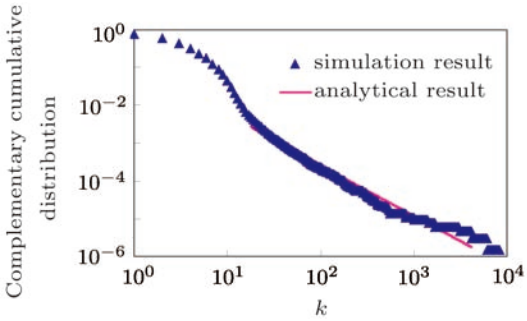


Fig. 11. (colour online) Log–log plot of complementary cumulative in-degree distribution for $\theta = 1/2$ and $q = 0.25$, which follows power-law form $k^{1-\gamma_{\rm in}}$ with $\gamma_{\rm in} = 2.34$.

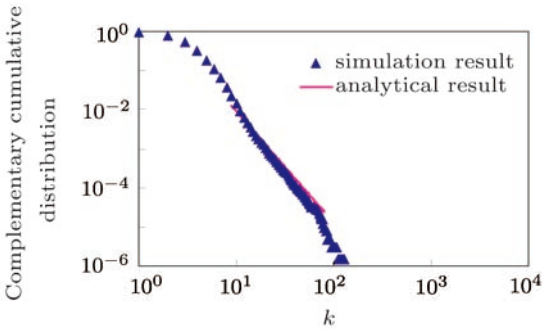


Fig. 12. (colour online) Log–log plot of complementary cumulative out-degree distribution for $\theta = 1/2$ and $q = 0.35$, which follows power-law form $k^{1-\gamma_{\rm out}}$ with $\gamma_{\rm out} = 3.86$.

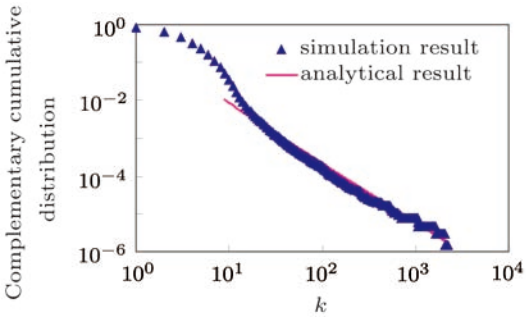


Fig. 13. (colour online) Log–log plot of complementary cumulative in-degree distribution for $\theta = 1/2$ and $q = 0.35$, which follows power-law form $k^{1-\gamma_{\rm in}}$ with $\gamma_{\rm in} = 2.54$.

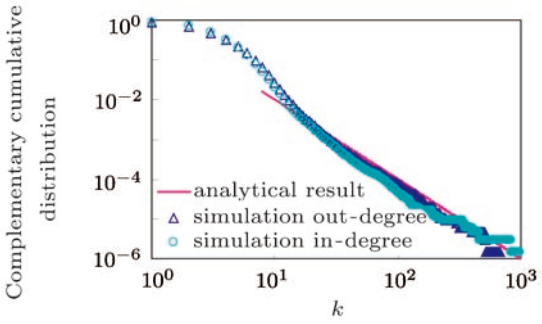


Fig. 14. (colour online) Log–log plot of complementary cumulative out-degree distribution for $\theta = 1/2$ and $q = 0.5$, which follows power-law form $k^{1-\gamma_{\rm out}}$ with $\gamma_{\rm out} = 3$, the log–log plot of complementary cumulative in-degree distribution follows power-law form $k^{1-\gamma_{\rm in}}$ with $\gamma_{\rm in} = 3$.

Computers can only simulate the finite time behaviour, while equations (14) and (15) are obtained at time $t \to \infty$. Although the front of the degree distribution curve is slightly curved in the simulation figures, the main body of the simulated curves agrees with the analytical results well.

5. Conclusion

We have collected the data of mobile phone users in mainland China from 1997 to 2010. We obtain a prediction function for the number of the mobile phone users and find that the growth of the mobile phone users follows a power law. According to the growth law and the rules of the batch arrival queue, we have proposed a directed network model with nodes of power-law accelerating growth and numbers of new incoming and outgoing edges being both random variables following the binomial distributions. We obtain analytical expressions for the transient degree distribution and the stationary average degree distribution. For any given $\theta \geq 0$, the in- and the out-degree exponents are $\gamma_{\rm in} = 1 + \frac{1}{(1-q)}$ and $\gamma_{\rm out} = 1 + \frac{1}{q}$, respectively. The main contribution of this paper is to offer an extension of the BA model for the power-law accelerating growth networks. We have found the power-law acceleration invariance of the scale-free networks. For both undirected and directed networks, our model is verified by means of simulations, and the invariance is also demonstrated by the numerical simulations.

Acknowledgement

We thank Gao L and Li C for reading our manuscript and helpful suggestions.

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