Linear and nonlinear excitations in complex plasmas with nonadiabatic dust charge fluctuation and dust size distribution*

Zhang Li-Ping(张丽萍)[a)†], Xue Ju-Kui(薛具奎)[b)], and Li Yan-Long(李延龙)[a)]

[a)] *School of Sciences, Lanzhou University of Technology, Lanzhou* 730050, *China*

[b)] *College of Physics and Electronic Engineering, Northwest Normal University, Lanzhou* 730070, *China*

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Both linear and nonlinear excitation in dusty plasmas have been investigated including the nonadiabatic dust charge fluctuation and Gaussian size distribution dust particles. A linear dispersion relation and a Korteweg–de Vries–Burgers equation governing the dust acoustic shock waves are obtained. The relevance of the instability of wave and the wave evolution to the dust size distribution and nonadiabatic dust charge fluctuation is illustrated both analytically and numerically. The numerical results show that the Gaussian size distribution of dust particles and the nonadiabatic dust charge fluctuation have strong common influence on the propagation of both linear and nonlinear excitations.

Keywords: dusty plasmas, nonadiabatic dust charge fluctuation, dust size distribution

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1. Introduction

Recently, there has been a great deal of interest in understanding wave propagation in dusty plasmas because of its vital role both in space and in laboratory experiments.[1−3] Dusty plasmas consist of ions, free electrons and micro-sized negatively charged dust particles. In reality, the dust grains have variable charge and mass due to fragmentation and coalescence. Therefore, the dust charge becomes a new dynamic variable and affects the characteristics of the collective motion of the plasmas. Its dynamic nature is of crucial importance in understanding dusty plasma waves. In the linear theory, it is known that the non-adiabatic ($\tau_{\rm ch}/\tau_{\rm d}$ is small but finite) dust charge variation results in the damping of the linear wave mode,[4−8] where $\tau_{\rm ch}$ is the charging time scale indicating the speed of charging and $\tau_{\rm d}$ is the hydrodynamical time scale indicating the speed of the change of the hydrodynamical character. However, in the nonlinear analysis, the non-adiabatic dust charge variation leads to an anomalous dissipation in plasmas, which causes the generation of shock waves in dusty plasmas.[9−11] The effect of dust charge fluctuation on the dust acoustic waves or instabilities in dusty plasmas has been studied by several researchers previously. For example, Xue[12] investigated the nonlinear dust acoustic wave governed by the Kadomtsev–Petviashvili (KP)–Burgers equation and proved that the dust acoustic shock waves can exist in the KP–Burgers equation due to the non-adibatic dust charge variation. Ghosh *et al.*[13] studied the properties of nonlinear dust acoustic waves in magnetized dusty plasmas with variable charges. The combined effects of the non-adiabatic dust charge fluctuation and nonthermally distributed ions on three-dimensional (3D) dust acoustic shock waves in dusty plasmas have been investigated.[14]

On the other hand, in a real case, the dust grains have many different sizes both in space plasmas and in laboratory experiments. It has been widely accepted that the dust size distribution can be described by a power law distribution in space plasmas.[15−18] Recently, Duan and Shi[19] studied the dust size distributions in an unmagnetized dusty plasma. They found that the soliton velocity is greater than that of monosized dusty plasmas. Duan and Parkes[20] studied the dust size distribution for dust acoustic waves (DAWs) in a magnetized multi-dimensional dusty plasma. In the laboratory, the dust size distribution is usually different for different cases. Usually, it has been

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†Corresponding author. E-mail: zhanglp@lut.cn

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widely accepted that the dust size distribution can be described by a Gaussian distribution in laboratory plasmas.[21] For charged dust grains with radii in a given range $[a_{\min}, a_{\max}]$, the differential Gaussian distribution is of the form

$$n(a)\,\mathrm{d}a = D\,\mathrm{e}^{-\mu'(a-\bar{a})^2}\,\mathrm{d}a, \tag{1}$$

where $a_{\min} < a < a_{\max}$ and D can be determined by

$$N_{\rm tot} = \int_{a_{\min}}^{a_{\max}} n(a)\,\mathrm{d}a, \tag{2}$$

where $N_{\rm tot}$ is the total number density of dust grains. Outside the limits $a < a_{\min}$ and $a > a_{\max}$, we use $n(a) = 0$; μ' can be found by requiring that $n(a_{\min}) = n(a_{\max}) = \epsilon' n(\bar{a})$, where $\bar{a}$ is the average radius of dust size, for some small values of ϵ'. We usually use $\epsilon' = 0.01$. Duan *et al.*[22] investigated the effect of the Gaussian dust size distribution on the propagation of dust acoustic solitary waves in dusty plasmas; however, their studies did not consider dust charge fluctuation. In general, the combined effects of non-adiabatic dust charge fluctuation and the Gaussian dust size distribution on the dust acoustic waves are not studied in previous investigations.

In this paper, under the assumption that $\tau_{\rm ch}/\tau_{\rm d}$ is small but finite, we investigate the combined effects of the non-adibatic dust charge fluctuation and Gaussian dust size distribution on linear and nonlinear excitation in hot dusty plasmas. We obtain the linear dispersion relation and a Korteweg–de Vries (KdV) Burgers equation governing the dust acoustic shock waves. It is discussed in great detail how the dust charge fluctuation and Gaussian dust size distribution modify the properties of dust acoustic shock structures.

2. Physical model

We consider a three-component dusty plasma system, which consists of extremely massive, micron-sized, negatively charged hot dust fluid, nonthermally distributed ions and Boltzmann-distributed electrons. Charge neutrality at equilibrium requires that $n_{\rm i0} = \sum_{j=1}^{N} Z_{{\rm d}j} n_{{\rm d}0j} + n_{\rm e0}$, where $n_{\rm i0}$, $n_{{\rm d}0j}$ and $n_{\rm e0}$ are the unperturbed ion, the jth dust grain and electron densities, respectively, and $Z_{{\rm d}j}$ is the number of electrons residing on the jth dust grain. The dust acoustic shock wave dynamics can be described by the following set of continuity, momentum, pressure, and Poisson's equations:

$$\begin{cases} \dfrac{\partial n_{{\rm d}j}}{\partial t} + \dfrac{\partial}{\partial x}(n_{{\rm d}j} u_{{\rm d}j}) = 0, \\ \dfrac{\partial u_{{\rm d}j}}{\partial t} + u_{{\rm d}j}\dfrac{\partial u_{{\rm d}j}}{\partial x} + \dfrac{\sigma_{\rm d}}{n_{{\rm d}j}}\dfrac{\partial n_{{\rm d}j}}{\partial x} = (1-q_{{\rm d}j})\dfrac{\partial \phi}{\partial x}, \\ \dfrac{\partial^2 \phi}{\partial x^2} = \sum_{j=1}^{N}(1-q_{{\rm d}j}) n_{{\rm d}j} + n_{\rm e} - n_{\rm i}, \end{cases} \tag{3}$$

where $u_{{\rm d}j}$ is the dust fluid velocity normalized to the dust acoustic speed $C_{\rm d} = \sqrt{\overline{Z}_{\rm d0} T_{\rm i}/\overline{m}_{\rm d}}$, with $T_{\rm i}$ being the ion-temperature (in energy units), $\overline{m}_{\rm d} = \sum_{j=1}^{N} m_{{\rm d}j} n_{{\rm d}0j}/N_{\rm tot}$ being the average mass of negatively charged dust particles, and $\overline{Z}_{\rm d0} = \sum_{j=1}^{N} Z_{{\rm d}0j} n_{{\rm d}0j}/N_{\rm tot}$ being the average charged number residing on a dust grain. The dust grain number density is normalized by $N_{\rm tot}$. the ion density is normalized by $n_{\rm i0}$. $\sigma_{\rm i} = T_{\rm i}/T_{\rm e}$, with $T_{\rm e}$ being the electron-temperature (in energy units); $\mu = n_{\rm e0}/n_{\rm i0}$. $\sigma_{\rm d} = T_{\rm d}/(\overline{Z}_{\rm d0} T_{\rm i})$, with $T_{\rm d}$ being the dust-temperature (in energy unit); ϕ is the electrostatic potential normalized to $T_{\rm i}/e$, with e being the magnitude of the electron charge. The time and space variables are normalized by the dust plasma period $\omega_{\rm pd}^{-1} = \sqrt{\overline{m}_{\rm d}/4\pi N_{\rm tot} \overline{Z}_{\rm d0}^2 e^2}$ and the Debye length $\lambda_{\rm Dd} = C_{\rm d}/\omega_{\rm pd}$.

Because observation of space plasmas very often indicates the existence of ion distribution that is far from thermodynamic equilibrium, the occurrence of nonthermal ions is considered to be a very common feature in such environments. Nonthermal ions from the Earth's bow-shock have been observed by the Vela satellites.[23] The distribution function that was chosen by Cairns *et al.*[24] is chosen in order to model an ion distribution with a population of nonthermal particles. Therefore, the ion density $n_{\rm i}$ is directly given by

$$n_{\rm i} = \frac{1}{1-\mu}[1 + \beta(\phi + \phi^2)]\exp(-\phi), \tag{4}$$

where $\beta = 4\alpha/(1+3\alpha)$, α is a parameter determining the number of fast (nonthermal) ions,

$$n_{\rm e} = \frac{\mu}{1-\mu}\exp(\sigma_{\rm i}\phi). \tag{5}$$

When the charging time scale indicating the speed of charging is much larger than the hydrodynamical time scale indicating the speed of the change of the hydrodynamical character, the dust charge is assumed to be a constant. However, when the charging time scale and the hydrodynamical time scale are comparable, the dust charge fluctuation should be considered. The dust charge $Q_{{\rm d}j} = -Z_{{\rm d}0j} e + q_{{\rm d}j}$, where $q_{{\rm d}j}$ is the

fluctuating part of the charge associated with the jth species of the dust grain, is normalized in units of the equilibrium dust charge $Z_{\mathrm{d}0j}e$. To determine the normalized charge variable $q_{\mathrm{d}j}$, we consider the following orbital motion-limited[25] charge current balance equation. In the normalized form, it reads

$$\frac{\tau_{\mathrm{ch}}}{\tau_{\mathrm{d}}}\left(\frac{\partial q_{\mathrm{d}j}}{\partial t}+u_{\mathrm{d}j}\frac{\partial q_{\mathrm{d}j}}{\partial x}\right)=\frac{\tau_{\mathrm{ch}}}{Z_{\mathrm{d}0}e}(I_{\mathrm{e}}+I_{\mathrm{i}}), \quad (6)$$

where I_{e} and I_{i} are the electron and ion currents, respectively. We assume that the dust grain radius (a) $\ll$ the electron gyroradius (ρ_{e}). The charging characteristics are not significantly influenced by the existence of external magnetic field, since for $a \ll \rho_{\mathrm{e}}$, the curvature effect of the trajectory of an electron (ion) impinging on a dust grain of radius a can be neglected.[26,27] Thus, considering the nonthermally distributed ions, the normalized expressions for the electron and ion currents of spherical dust grains with radius a are given by[28]

$$I_{\mathrm{e}} = -\pi a^2 e\sqrt{\frac{8T_{\mathrm{e}}}{\pi m_{\mathrm{e}}}}n_{\mathrm{e}0}\exp(\sigma_{\mathrm{i}}\phi)\exp[z(q_{\mathrm{d}j}-1)], \quad (7)$$

$$I_{\mathrm{i}} = \pi a^2 e\sqrt{\frac{8T_{\mathrm{i}}}{\pi m_{\mathrm{i}}}}\frac{n_{\mathrm{i}0}}{(1+3\alpha)}\left[\left(\left(1+\frac{24\alpha}{5}\right)+\frac{16\alpha}{3}\phi+4\alpha\phi^2\right)-\frac{z(q_{\mathrm{d}j}-1)}{\sigma_{\mathrm{i}}}\times\left(\left(1+\frac{8\alpha}{5}\right)+\frac{8\alpha}{3}\phi+4\alpha\phi^2\right)\right]\exp(-\phi), \quad (8)$$

where $z = z_{\mathrm{d}0}e^2/4\pi\varepsilon_0 aT_{\mathrm{e}}$, $4\pi\varepsilon_0 a$ is the capacitance of the spherical dust grain of radius a; $\tau_{\mathrm{d}}(\approx \omega_{\mathrm{pd}}^{-1})$ is the dust hydrodynamical time scale, and $\tau_{\mathrm{ch}}[\approx (\mathrm{d}Q_{\mathrm{d}}/\mathrm{d}t)^{-1}]$ is the charging time scale given as

$$\tau_{\mathrm{ch}} = \left[\frac{a}{\sqrt{2\pi}}\frac{\omega_{\mathrm{pi}}^2}{V_{\mathrm{thi}}}\frac{5+8\alpha}{5+15\alpha}(1+z+\frac{5+24\alpha}{5+8\alpha}\sigma_{\mathrm{i}})\right]^{-1}, \quad (9)$$

where ω_{pi} and V_{thi} are the ion plasma frequency and the ion thermal velocity, respectively.

3. Linear excitation

In order to investigate the linear excitation in dusty plasmas, we shall carry out a normal analysis. We first express the dependent variables $u_{\mathrm{d}j}$, ϕ and $n_{\mathrm{d}j}$ in terms of their equilibrium and perturbed parts as $n_{\mathrm{d}j} = n_{\mathrm{d}0j} + n'_{\mathrm{d}j}$, $u_{\mathrm{d}j} = 0 + u'_{\mathrm{d}j}$, $\phi = 0 + \phi'$ and the perturbed quantities are proportional to $\exp[\mathrm{i}(kx-\omega t)]$. Then, substituting Eqs. (4) and (5) into Eq. (3), and linearizing Eqs. (3) and (6) with these perturbations, we obtain the following linear dispersion relation:

$$\left(\frac{\omega^2}{k^2}-\frac{\sigma_{\mathrm{d}}}{\sum_{j=1}^{N}n_{\mathrm{d}0j}}\right)\left(\frac{1}{k^2}\frac{\beta_1}{1-\mathrm{i}\,\omega\lambda}+\frac{1}{k^2}\frac{\beta-\mu\sigma_{\mathrm{i}}-1}{1-\mu}-1\right)+1=0, \quad (10)$$

where $\beta_1 = \frac{\frac{40\alpha}{15+24\alpha}(2\sigma_{\mathrm{i}}+z)-(\delta\sigma_{\mathrm{i}}+z)(1+\sigma_{\mathrm{i}})}{z(1+z+\delta\sigma_{\mathrm{i}})}$ ($\delta = \frac{5+24\alpha}{5+8\alpha}$). Equation (10) represents a dispersion relation for linear excitation in dusty plasmas consisting of extremely massive, micron-sized, negatively charged dust fluid Gaussian distribution, non-thermally distributed ions and Boltzmann-distributed electrons. It is seen from Eq. (10) that the dispersion relation is dependent on the dust size distribution, the nonadiabatic dust charge fluctuation and non-thermally distributed ions.

We consider the continuous dust size distribution instead of the N-species of dust grain. As is widely accepted in laboratory plasmas, dust size distribution is described by a Gaussian distribution, as in by Eq. (1). It is obtained from Eq. (1) that $n(\bar{a}) = D$, $n(a_{\min}) = n(a_{\max})$, i.e., $\exp[-\mu'(a_{\max}-\bar{a})^2] = \exp[-\mu'(a_{\min}-\bar{a})^2] = \epsilon'$.[22] Then, $a_{\max}-\bar{a} = \bar{a}-a_{\min} = d_0$ and $\mu' = \frac{1}{d_0^2}\ln\frac{1}{\epsilon'}$ are gained. For the Gaussian distribution, we obtain the following quantities:

$$N_{\mathrm{tot}} = \int_{a_{\min}}^{a_{\max}} D\,\mathrm{e}^{-\mu'(a-\bar{a})^2}\,\mathrm{d}a, \quad (11)$$

$$\bar{a} = \frac{\int_{a_{\min}}^{a_{\max}} Da\,\mathrm{e}^{-\mu'(a-\bar{a})^2}\,\mathrm{d}a}{N_{\mathrm{tot}}}. \quad (12)$$

Now, we substitute Eq. (11) into Eq. (10) and investigate the effects of the non-adiabatic dust charge fluctuation and Gaussian dust size distribution on the frequency and the damping rate of the linear excitation, respectively. All results obtained here are for the fixed parameters $z = 2$, $\sigma_{\mathrm{i}} = 0.1$, $\sigma_{\mathrm{d}} = 0.01$, $\alpha = 0.5$ and $\mu = 0.3$. Figure 1 illustrates the numerical solutions of Eq. (10) for both the frequency ω_{r} and the damping rate ω_{i} for different d_0 against k. Figures 1(a) and 1(b) show the frequency and the damping rate of the linear excitation, respectively. It is obvious from Fig. 1(a) that the frequency decreases with d_0 but increases with k. It is seen from Fig. 1(b) that the damping rate is independent of d_0, but the variation of the damping rate with k is nonlinear. There exists a critical k, dented by k_{c}, and when $k < k_{\mathrm{c}}$, the damping rate decreases significantly. However, when $k > k_{\mathrm{c}}$, the damping rate increases slowly. In Fig. 2, we display the effect of λ on the frequency and the

damping rate. Figure 2(a) shows the frequency is independent of λ. It is illustrated from Fig. 2(b) that the damping rate decreases with λ. Figure 2(b) also shows that for smaller values of k, the damping rate decreases with λ slowly, but for larger values of k, it decreases significantly.

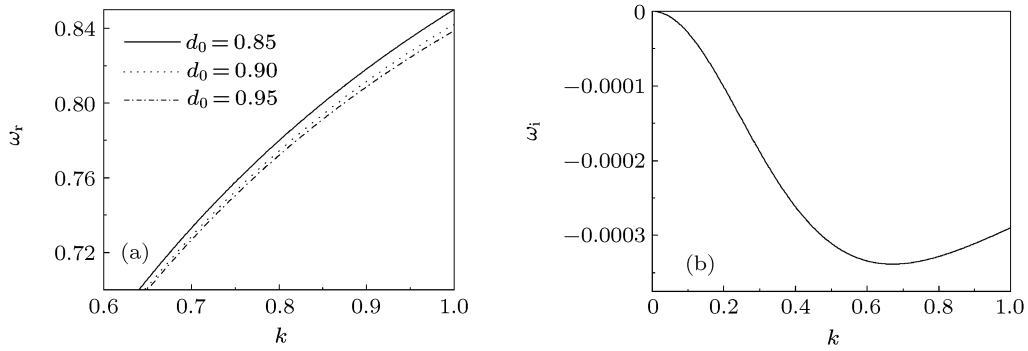


Fig. 1. The frequency (a) and the damping rate (b) aganinst k for different d_0.

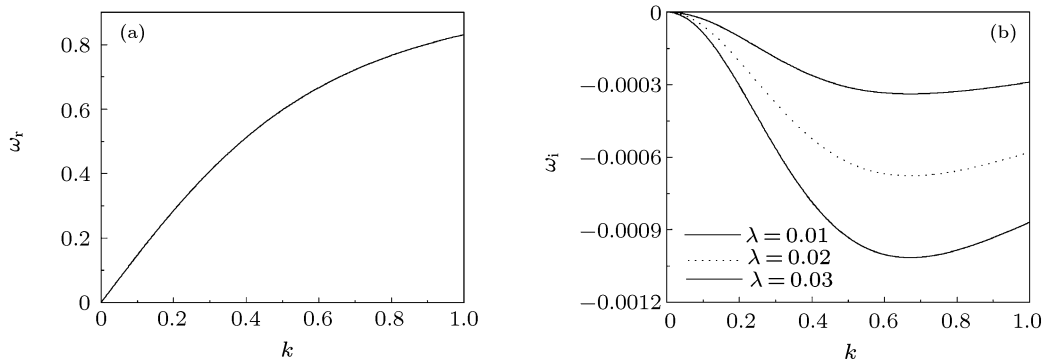


Fig. 2. The frequency (a) and the damping rate (b) aganinst k for different λ.

4. Nonlinear excitation

In order to investigate the nonlinear propagation of dust acoustic shock waves in dusty plasmas, we employ the standard reductive perturbation technique[29] to obtain the KdV–Burger equation. The independent variables are $\xi = \epsilon^{1/2}(x - v_0 t)$ and $\tau = \epsilon^{3/2}t$, where ϵ is a small parameter measuring the strength of nonlinearity and v_0 is the phase velocity of wave. Substituting $n_{dj} = n_{d0j} + \epsilon n_{d1j} + \epsilon^2 n_{d2j} + \cdots, u_{dj} = \epsilon u_{d1j} + \epsilon^2 u_{d2j} + \cdots, \phi = \epsilon\phi_1 + \epsilon^2\phi_2 + \cdots, q_{dj} = \epsilon q_{d1j} + \epsilon^2 q_{d2j} + \cdots$, $\frac{\tau_{ch}}{\tau_d} = \nu\epsilon^{1/2}$ (for non-adiabatic charge variation, τ_{ch}/τ_d is small but finite[30]) into Eqs. (3) and (6), and collecting the terms in the different powers of ϵ to the lowest order in ϵ, we obtain $u_{d1j} = \frac{v_0}{n_{d0j}} B\phi_1$, $\sum_{j=1}^{N} n_{d1j} = B\phi_1$, $q_1 = \beta_1\phi_1$, $v_0^2 = \sigma_d - \frac{\sum_{j=1}^{N} n_{d0j}}{B}$, where

$$B = \beta_1 \sum_{j=1}^{N} n_{d0j} + \frac{\beta - \mu\sigma_i - 1}{1 - \mu}$$

and $\beta_1 = \frac{\frac{40\alpha}{15+24\alpha}(2\sigma_i + z) - (\delta\sigma_i + z)(1+\sigma_i)}{z(1+z+\delta\sigma_i)}$ ($\delta = \frac{5+24\alpha}{5+8\alpha}$). The variable β_1 stands for the dust charge fluctuation effect.

To the next higher order in ϵ, for the continuity equation, the momentum equation, Poisson's equation and charge balance equation, we obtain

$$\begin{cases} \dfrac{\partial n_{d1j}}{\partial\tau} - v_0\dfrac{\partial n_{d2j}}{\partial\xi} + n_{d0j}\dfrac{\partial u_{d2j}}{\partial\xi} + \dfrac{\partial n_{d1j}u_{d1j}}{\partial\xi} = 0, \\ \dfrac{\partial u_{d1j}}{\partial\tau} - v_0\dfrac{\partial u_{d2j}}{\partial\xi} + u_{d1j}\dfrac{\partial u_{d1j}}{\partial\xi} + \dfrac{\sigma_d}{n_{d0j}}\dfrac{\partial n_{d2j}}{\partial\xi} - \dfrac{1}{n_{d0j}^2} n_{d1j}\dfrac{\partial n_{d1j}}{\partial\xi} - \dfrac{\partial\phi_2}{\partial\xi} + q_{d1j}\dfrac{\partial\phi_1}{\partial\xi} = 0, \\ \dfrac{\partial^2\phi_1}{\partial\xi^2} - \sum\limits_{j=1}^{N} n_{d2j} + \sum\limits_{j=1}^{N} n_{d1j}q_{d1j} + \sum\limits_{j=1}^{N} n_{d0j}q_{d2j} - \dfrac{\mu\sigma_i^2 - 1}{2(1-\mu)}\phi_1^2 + \dfrac{\beta - \mu\sigma_i - 1}{1-\mu}\phi_2 = 0, \\ -\beta_1\phi_2 + q_{d2j} + \beta_2 q_{d1j}\phi_1 + \beta_3\phi_1^2 + \beta_4 q_{d1j}^2 - \nu v_0\dfrac{\partial q_{d1j}}{\partial\xi} = 0, \end{cases} \tag{13}$$

where

$$\beta_2 = \frac{(\delta\sigma_{\rm i}+z)-\frac{15-16\alpha}{15+24\alpha}}{\delta\sigma_{\rm i}+z+1},$$
$$\beta_3 = \frac{(z+\delta\sigma_{\rm i})(\sigma_{\rm i}^2-1)+\frac{40\alpha}{15+24\alpha}(\sigma_{\rm i}-z)}{2z(\delta\sigma_{\rm i}+z+1)},$$
$$\beta_4 = \frac{z(\delta\sigma_{\rm i}+z)}{2(\delta\sigma_{\rm i}+z+1)}.$$

Now, one obtains the KdV–Burgers equation as

$$\frac{\partial\phi_1}{\partial\tau}+A\phi_1\frac{\partial\phi_1}{\partial\xi}+C\frac{\partial^3\phi_1}{\partial\xi^3}-E\frac{\partial^2\phi_1}{\partial\xi^2}=0, \tag{14}$$

where $\phi\equiv\phi_1$ and

$$A = \frac{3}{2}\frac{Bv_0}{\sum_{j=1}^{N}n_{{\rm d}0j}}-\frac{B}{v_0\sum_{j=1}^{N}n_{{\rm d}0j}} + \frac{\beta_1\sum_{j=1}^{N}n_{{\rm d}0j}}{2Bv_0}-\beta_1v_0+\frac{1}{2B}\frac{\mu\sigma_{\rm i}^2-1}{1-\mu} + \frac{v_0\sum n_{{\rm d}0j}}{B}(\beta_3+\beta_1\beta_2+\beta_1^2\beta_4), \tag{15}$$

$$C = -\frac{v_0}{2B}, \tag{16}$$

$$E = -\frac{v_0^2\beta_1\nu\sum_{j=1}^{N}n_{{\rm d}0j}}{2B}. \tag{17}$$

The terms β_1 and $\sum_{j=1}^{N}n_{{\rm d}0j}$ present in the expressions (15) and (17) are due to the dust charge variation and the dust size distribution, respectively. From Eq. (17), it is seen that the Burgers term is proportional to ν, arising due to the non-adiabatic dust charge variation. E is also modified by $\sum_{j=1}^{N}n_{{\rm d}0j}$. We can expect that the shock structures will be modified by the dust charge variation, the presence of nonthermally distributed ions, the temperatures of ion and dust, and the ratio of electron to ion density. As we expect, numerical solutions also show that α, ν, μ, $\sigma_{\rm i}$ and $\sigma_{\rm d}$ have significant effects on the shock structures.

The Burgers term in Eq. (14) due to non-adiabatic charge variation of the dust particles implies the possibility of the existence of shock structures. On transforming to the wave frame

$$\eta = V\tau-\xi, \tag{18}$$

the KdV–Burgers equation (14) reduces to

$$\frac{{\rm d}^2\phi}{{\rm d}\eta^2}=\frac{V}{C}\phi-\frac{A}{2C}\phi^2-\frac{E}{C}\frac{{\rm d}\phi}{{\rm d}\eta}. \tag{19}$$

The second-order equation (19) can be written as a system of two first-order equations

$$\frac{{\rm d}\phi}{{\rm d}\eta}=\Phi, \tag{20}$$

$$\frac{{\rm d}\Phi}{{\rm d}\eta}=\frac{V}{C}\phi-\frac{A}{2C}\phi^2-\frac{E}{C}\Phi. \tag{21}$$

The equations of the system (19) and (20) have two fixed points $(\phi_1^*,\Phi_1^*)=(0,0)$ and $(\phi_2^*,\Phi_2^*)=(\frac{2V}{A},0)$. It is easy to analyse that the first one (ϕ_1^*,Φ_1^*) is a saddle point while the second one (ϕ_2^*,Φ_2^*) is a stable focus or a stable node according to

$$E^2<4VC \quad \text{or} \quad E^2>4VC. \tag{22}$$

This means that there is a heteroclinic orbit connecting the saddle-focus or saddle-node point in Eq. (20) and (21). On the other hand, oscillatory shock wave or monotonic shock wave exists in the system according to $E^2<4VC$ or $E^2>4VC$.

To reveal the shock structures clearly, $\phi(\eta)$ is obtained by numerical integration of Eq. (19) subject to the boundary conditions $\phi\to0$ at $\eta\to-\infty$. Thus, for any x, the potential ϕ builds up from zero value at long pass $t\to\infty(\eta\to\infty)$ to a steady value $2V/A$. Figures 3 and 4 show how the structures of the dust acoustic shock waves change with parameters ν and d_0 for $\mu=0.3$, $\sigma_{\rm d}=0.01$, $\sigma_{\rm i}=0.1$, $z=2$ and $\alpha=0.5$. It is clear from Fig. 3 that the amplitude of the oscillatory shock waves does not vary with ν. Figure 3(a) shows that the oscillatory frequency and the steeping of the shock waves decreases when ν increases. It is shown from Fig. 3(b) that when the values of ν

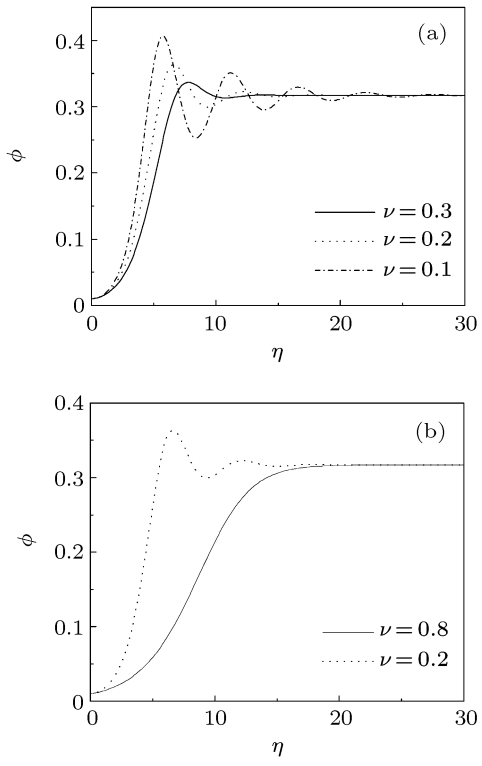


Fig. 3. Profile of the shock wave for different ν.

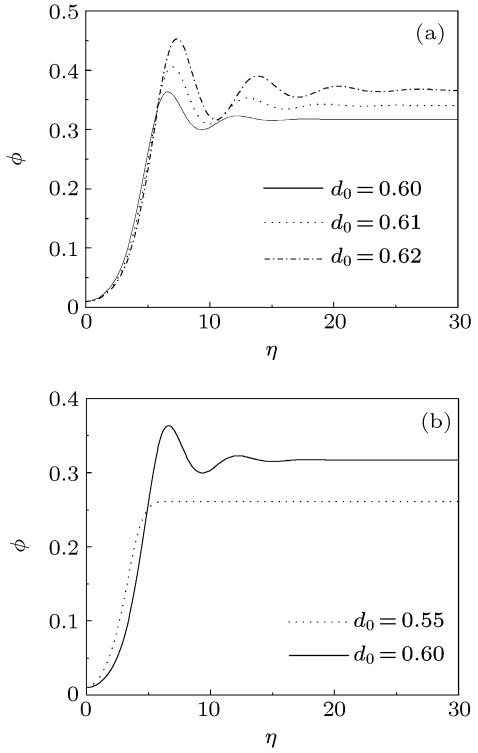


Fig. 4. Profile of the shock wave for different d_0.

increase to a fixed value, the oscillation of the shock waves vanishes, that is, the oscillatory shock waves are changed into the monotonic shock waves. Figure 4 illustrates that the amplitude of the oscillatory shock waves increases with d_0. Figure 4(a) shows that the oscillatory frequency of the shock waves increases with d_0, but the steeping of the shock waves varies with d_0 weakly. It is illustrated that when d_0 increases to a fixed value, the oscillatory shock waves are changed into the monotonic shock waves. The effects of the dust temperature ($\sigma_{\rm d}$), the ion temperature ($\sigma_{\rm i}$) and the number of the fast particles (α) on the structures of the shock waves is similar to the results in Ref. [14].

5. Conclusion

In summary, the linear dispersion relation and the KdV–Burgers equation describing the dust acoustic shock waves in dusty plasmas with non-adiabatic dust charge fluctuation and Gaussian dust size distribution are derived. We analysed mainly the effects of non-adiabatic dust charge fluctuation and Gaussian dust size distribution on the frequency and the damping rate of linear excitation and the shock structures in great detail. It is shown that the properties of shock structures and the transition between oscillatory shock waves and monotonic shock waves are influenced significantly by non-adiabatic dust charge fluctuation and Gaussian dust size distribution. With reference to the present study, we can obtain a deeper understanding of the properties of dust acoustic shock wave. Furthermore, we expect that the present investigation would be worthy for future experiments on dust acoustic shock waves.

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