Acoustic carpet invisibility cloak with two open windows using multilayered homogeneous isotropic material

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We present a method for designing an open acoustic cloak that can conceal a perturbation on flat ground and simultaneously meet the requirement of communication and matter interchange between the inside and the outside of the cloak. This cloak can be constructed with a multilayered structure and each layer is an isotropic and homogeneous medium. The design scheme consists of two steps: firstly, we apply a conformal coordinate transformation to obtain a quasi-perfect cloak with heterogeneous isotropic material; then, according to the profile of the material distribution, we degenerate this cloak into a multilayered-homogeneous isotropic cloak, which has two open windows with negligible disturbance on its invisibility performance. This may greatly facilitate the fabrication and enhance the applicability of such a carpet-type cloak.

Keywords: transformation method, acoustic wave, carpet cloak, multilayered structure

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1. Introduction

Since the ground-breaking works on optical invisibility cloaks by Pendry *et al.*[1] and Leonhardt,[2] people have had an intense interest in realizing such a cloak based on the transformation method. A variety of optical cloaks have been proposed in recent literatures. Along with the extensive report of cloaks with diverse closed geometrical shapes based on analytical or numerical methods,[3−9] cloaks with open windows have also been investigated,[10−12] which emphasize communication and the matter interchange between the interior and the exterior of the cloak, together with its invisible functionality. These open cloaks widen our view on such novel device design.

Until now, most studies of the transformation method focus on the optical domain, i.e., transformation optics.[3] This idea is also applicable to acoustic waves and gives rise to transformation acoustics, which has been extensively adopted in the design of acoustic cloaks.[13−29] However, the resultant acoustic materials are generally anisotropic and inhomogeneous, which brings about a great challenge for their engineering realization. To solve this problem, layered systems were first proposed by Cheng *et al.* to design acoustic cloaks both in two dimensions (2D)[20] and three dimensions (3D).[21] This is an attractive approach for its realizable parameter and multilayered structure.[16,22−25] Another way to overcome this problem is by using the conformal mapping technique to obtain isotropic acoustic devices. Although it is well known that such scheme has limited potential for complete cloaking,[2,8] it is competent for the design of a practical carpet cloak. In fact, using the conformal or quasiconforml mapping to minimize the anisotropy of the cloak has been discussed in designing optical cloaks both theoretically and experimentally.[8−10] However, designing an acoustic carpet cloak with isotropic material still needs further investigation.

In this paper, we proposed an open acoustic carpet cloak with easily realizable materials. This cloak is designed in two steps: firstly, based on the embedded conformal coordinate transformation,[29,30] we obtain a quasiperfect carpet cloak with inhomogeneous isotropic material; then, it can be degenerated into a petal-shaped layered structure, and every layer is composed of homogenous isotropic material. In addition, we find that the simplified cloak has two open windows, through which the sound wave communication and the matter interchange can be carried out, which are difficult tasks by the closed cloak.

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2. Conformal transformation acoustics and the carpet cloak

In this study, we focus our concentration on a 2D carpet cloak based on the conformal transformation method[2,29] that allows the design of such a novel device consisting of isotropic material. Let $\zeta=u+\mathrm{i}\,v$ and $z=x+\mathrm{i}\,y$ be complex, and suppose that an analytic function $\zeta(z)$ is introduced for the coordinate transformation, it is well known that Laplace operator obeys

$$\boldsymbol{\nabla}^2=4\frac{\partial^2}{\partial z\partial\bar{z}}=4\frac{\mathrm{d}\zeta}{\mathrm{d}z}\frac{\mathrm{d}\bar{\zeta}}{\mathrm{d}z}\frac{\partial^2}{\partial z\partial\bar{z}}=\left|\frac{\mathrm{d}\zeta}{\mathrm{d}z}\right|^2\boldsymbol{\nabla}'^2, \quad (1)$$

where the upper bar symbolized the complex conjugation. Thus the acoustic equation (Helmholtz equation)

$$\left(\boldsymbol{\nabla}^2+\rho_0w^2/\kappa_0\right)p=0 \quad (2)$$

in the virtual space can be transformed into

$$\left(\boldsymbol{\nabla}'^2+\rho'w^2/\kappa'\right)p'=0 \quad (3)$$

in the physical space.

In addition, the acoustic impedance Z should be considered by the relation $Z=\sqrt{\rho_0\kappa_0}=\sqrt{\rho'\kappa'}$ while the impedance of the transformed media matches with the impedance of the background. Finally, the transformed material parameters are obtained from previous relations as the mass density $\rho'=\rho\rho_0$ and the bulk modulus $\kappa'=\kappa\kappa_0$, where ρ and κ are spatially varying scale factors with respect to the parameters of the background material, and

$$\kappa=1/\rho=\left|\zeta'(z)\right|. \quad (4)$$

In this paper, the background material is chosen as the water with the mass density $\rho_0=998\ \mathrm{kg/m^3}$ and the bulk modulus $\kappa_0=2.19$ GPa.

To create an isotropic carpet cloak, consider the conformal coordinate transformation described in Fig. 1. Such mapping transforms a rectangular region $ABCD$ into the design region $A'B'C'D'c'b'a'$, which becomes a carpet cloak. To be more specific, the optimal mapping transforms the line segment ab lying on the x-axis in Fig. 1(a) into the triangular $a'b'c'$ in Fig. 1(b), and ensures three boundaries AB, BC, CD between the cloak and the background medium to be identical with their images $A'B'$, $B'C'$, $C'D'$. As a result, the triangular region can be regarded as a virtual empty space expanded from the original line. When an object is located in such a triangular region, it will be concealed from the outer probe acoustically.

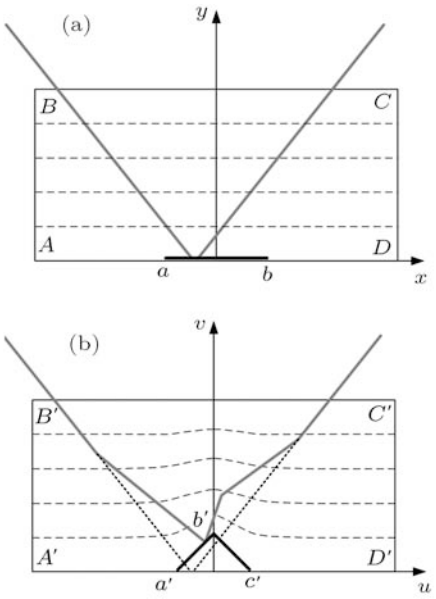


Fig. 1. Conformal transformation for the 2D carpet cloak design: (a) the original space, (b) the image space.

Unfortunately, under this mapping constraint on boundaries, it has been proved that in general such a conformal transformation cannot be obtained rigorously.[2,7] Therefore, in this work, we choose to use the Schwartz–Christoffel (SC) transformation to construct an approximate mapping, which can not only guarantee the conformality within the cloak, but also introduces negligible mismatch on three outer boundaries. By just using the well-known SC formula for a half-plane[31] and map $z_1=-1$, $z_2=0$, $z_3=1$ in the original space into $\zeta_1=-0.6$, $\zeta_2=0.6\,\mathrm{i}$, $\zeta_3=0.6$ in the transformed space, respectively, the resultant mapping function can be obtained by

$$\zeta(z)=\frac{1}{1.67}\left[\mathrm{i}+\frac{\sqrt{\pi}\,(1-\mathrm{i})\,z^{3/2}}{2\Gamma(3/4)\,\Gamma(7/4)}\times F\left(1/4,3/4,7/4,z^2\right)\right], \quad (5)$$

where $-5\ \mathrm{m}<\mathrm{Re}(z)<5\ \mathrm{m}$, $0<\mathrm{Im}(z)<5\ \mathrm{m}$, $\mathrm{i}^2=-1$; Γ denotes the Gamma function and F the hypergeometric function.

Substituting Eq. (5) into Eq. (4), we can calculate the isotropic constitutive parameters and plot the profile of material parameters κ in Fig. 2(a). It is evident that three corners of the triangle are singular points, which have zero or infinite parameter values. However, these singularities can be circumvented if we replace them with those at a small distance away from

the corners and limit the value of κ between 0.25 and 2.28. Consequently, the mass density ρ takes the value from 0.44 to 4 according to Eq. (4). With these non-singular and slight-variation parameters, it may facilitate the implementation. In the above discussion, for the purpose of reflectionless on the outer boundary of the quasi-perfect cloak, we take the idea of the embedded transformation method and adopt larger mapping domain in order to guarantee all the three boundaries nearly nondeformed with the expense of the large cloak volume.

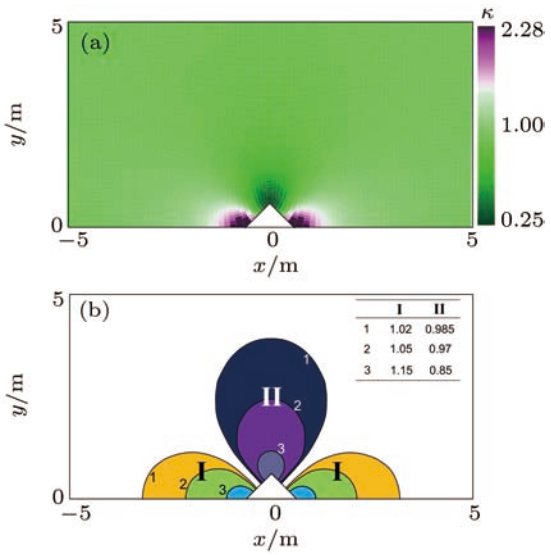


Fig. 2. (colour online) Distribution profile of the material parameters κ of the quasiperfect cloak. (b) Layered structure of the carpet cloak with two windows (the insert Table gives the values of κ for different regions).

3. Reduced petal-shaped cloak with open windows

The cloak obtained in Section 2 can be further improved to facilitate its implementation and add some additional functions. This is achieved by replacing its continuously distributed parameters with discrete finite-size cells. The resultant petal-shaped cloak is not only easy to fabricate but also natural to have two open windows, which facilitates the communication and the matter interchange without measurably affecting its invisible functionality. The details are discussed as follows.

As shown in Fig. 2(a), most regions inside the cloak have almost the same material properties as the background material. It is understood that these regions, where κ and ρ are close to 1, will not play important roles in manipulating acoustic wave, i.e., only a very small part of the cloak is essential to the invisibility. Therefore, it is safe to remove these unimportant regions from the cloak, resulting in a new cloak with small volume.

The isolines of the parameter κ can be obtained by using Eqs. (4) and (5) (see Fig. 2(b)). In this specific example, we round the material parameters κ within $(0.985, 1.02)$ to 1, and remove these regions from the original rectangular cloak. Thus, the rectangular cloak is degenerated into a petal-shaped one, which includes three separated parts from two kinds of petals, i.e., the petal-I (corresponding to $1.02 \leq \kappa \leq 2.28$) and petal-II (corresponding to $0.25 \leq \kappa \leq 0.985$) as denoted in Fig. 2(b). Here, petal-I and petal-II are simplified by three layers of isotropic and homogenous materials. It is noteworthy that parameter κ monotonically decreases from the outer to the inner regions in petal-I, being opposite to the case in petal-II. In addition, we notice that between petal-I and petal-II there are two opening windows in approximate 0.035-m width due to the above reduction procedure.

Although Fig. 2(b) gives a concise illustration of our idea, to ensure the good performance of the reduced cloak, we need fine structure with more layers. For this purpose, we constructed the reduced open cloak with M layers of petal-I and N layers of petal-II and assessed the performance of the cloak with different numbers of layers. It should be expected that the cloak will exhibit better cloaking performance as we increase N and M. However, in practice, a limited number of layers which can meet our requirement are expected. Table 1 presents the parameter κ for both cloaks and various layers.

Table 1. Parameter used for both layered cloaks in Figs. 3(c) and 3(d).

Coarse-layered			Fine-layered					
No.	κ		No.	κ		No.	κ	
	I	II		I	II		I	II
1	1.02	0.985	1	1.02	0.985	11	1.40	0.60
2	1.03	0.98	2	1.03	0.98	12	1.50	0.55
3	1.04	0.95	3	1.04	0.97	13	1.60	0.50
4	1.05	0.85	4	1.05	0.95	14	1.70	0.45
5	1.07	0.75	5	1.06	0.90	15	1.80	0.40
6	1.15	0.65	6	1.07	0.85	16	2.00	0.35
7	1.30	0.55	7	1.10	0.80	17	2.10	0.30
8	1.50	0.45	8	1.15	0.75	18	2.28	0.25
9	1.70	0.35	9	1.20	0.70	–	–	–
10	2.00	0.25	10	1.30	0.65	–	–	–

We used the ABAQUS finite-element package to simulate the performance of the designed cloaks. The

resulted pressure fields due to the acoustic wave beam at $\omega = 30/(2\pi)$ kHz incident from 45° are shown in Fig. 3. Figure 3(a) shows the result of a flat boundary without perturbation object, whereas Fig. 3(b) shows how the wave field is changed into two beams from one beam by the triangular rigid object acting as the perturbation.

In Fig. 3(c), the object together with the coarse cloak ($M = N = 10$) do mimic the flat boundary with distinct deviation, because the reflected beam undesirably focused into a narrow beam. When the layer numbers of petal-I and petal-II are equally increased up to 18, as shown in Fig. 3(d), the cloak shows a remarkable performance since the field outside the cloak is very close to that in Fig. 3(a).

To further evaluate the cloak performance, we take the normalized average power flow (NAPF) through the dot-dashed line AB as a universal measure, where point A is located at $(0, 6)$ and point B is located at $(6, 0)$ (see Fig. 3). The calculated distributions of the NAPF are plotted in Fig. 4. It can be seen that the NAPF of the reflected beam has a wide peak at the central region of AB, when only a flat plane is presented. When the object without the cloak is presented, the reflected beam is split into two different beams and the NAPF has two narrow peaks and a relatively wide valley between them. When the coarse cloak covers the object, the resulting NAPF is concentrated and features one steep peak. Finally, the fine cloak behaves considerably better and the relevant NAPF is very close to that of the flat plane.

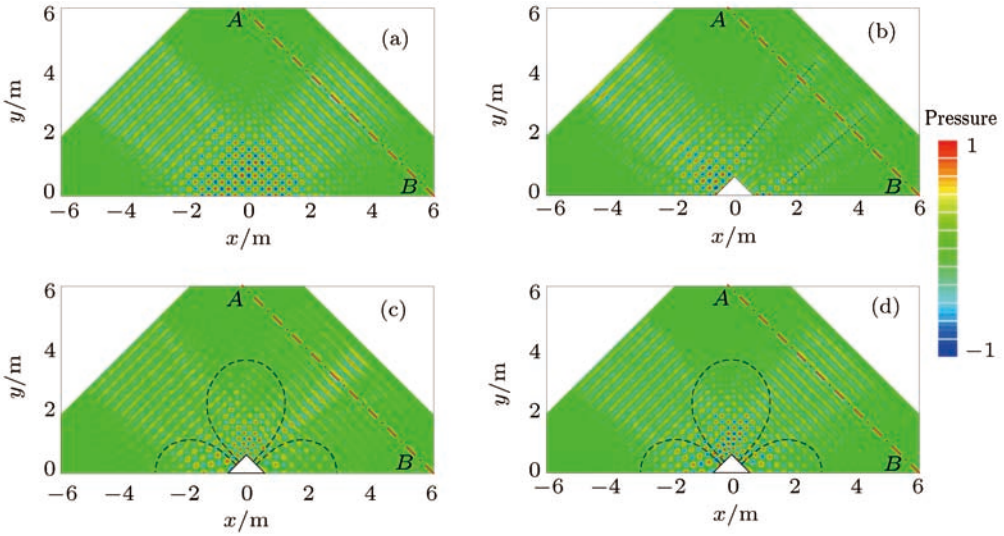


Fig. 3. (colour online) The pressure fields of the beam at $\omega = 30/(2\pi)$ kHz incident from 45°: (a) without the perturbation object; (b) with the perturbation object; (c) perturbation object with a coarse-layered cloak ($M = N = 10$); (d) perturbation object with a fine-layered cloak ($M = N = 18$). The reduced cloak is within the region bounded by the dashed petal-shaped curves.

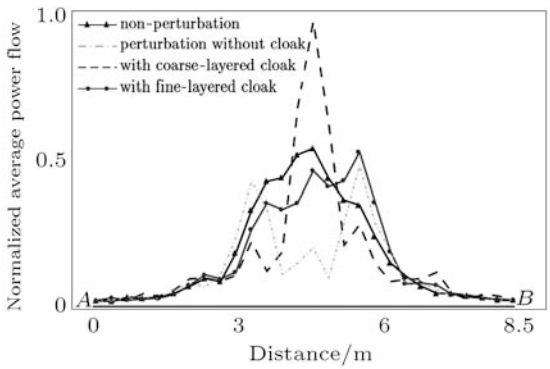


Fig. 4. Comparison of the normalized average power flow along AB.

Besides the above qualitative evaluation, we can also use the modal assurance criterion (MAC)[32], which is a general criterion to compare two vectors with the same number of components from a mathematical point of view, to measure the similarity between two NAPF curves quantitatively. Given a pair of curve lines represented by two real valued vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$, MAC_{XY} is defined as

$$\mathrm{MAC}_{XY} = \frac{\boldsymbol{X}^{\mathrm{T}}\boldsymbol{Y}}{\sqrt{(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X}) \cdot (\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Y})}}. \tag{6}$$

Generally, if such a value is more close to one, the two curves are more similar to each other. And usually, 0.95 can be used as the threshold of distinguishing two curves. In this case, in comparison with the same

curve from the flat plane (Fig. 3(a)), the MAC_{XY} equal 0.82, 0.88 and 0.96 corresponding to Fig. 3(b), Fig. 3(c) and Fig. 3(d), respectively. Based on this criterion, we have the confidence to say that the fine-layered cloak presented in Fig. 3(d) is an appropriate design.

To further verify the functional performance of the carpet cloak presented in Fig. 3(d) for incoming beams from any direction, we demonstrate another example, where the impinging beam incidents upon the object from left to right. As the carpet cloak ($M = N = 18$) together with the object are present, the beam is expected to deflect within the cloak and then return to its original trajectory. The pressure field is shown in Fig. 5(b) in contrast to the strongly scattered pattern in the absence of the cloak (see Fig. 5(a)).

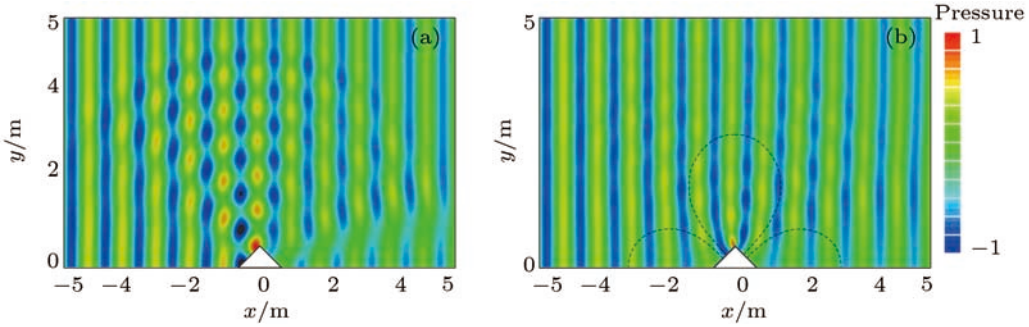


Fig. 5. (colour online) The pressure field illuminated by the left-to-right acoustic beam at $\omega = 10/(2\pi)$ kHz (a) without the cloak; (b) with the petal-shaped cloak.

Such a petal-shaped cloak composed of isotropic material could be easily engineered due to its simple layered homogeneous structure. Although the fine-layered cloak worked well based on the preceding two numerical examples, further improvement can be anticipated by optimizing the layer numbers and parameters of each layer, etc. In addition, such a cloak can operate well in a broad bandwidth because it does not use resonant elements.[8,9]

4. Conclusion

In conclusion, we firstly designed a rectangular acoustic carpet cloak made from heterogeneous isotropic materials through conformal transformation acoustics. Besides a slight variation of its material parameters, such a quasiperfect cloak has another attractive feature of having a large non-essential region with close-to-host parameters. Then, by removing the unimportant region and approximating the continuously changed parameters with those in a stepwise fashion, it can be degenerated into a layered-homogeneous petal-shaped cloak with slight disturbance on the cloak performance. Simultaneously, the reduced cloak naturally has two open windows, which are beneficial to offer the possibility of the free matter interchange and the communication between the inner and outer side of the cloak.

References

[1] Pendry J B, Schurig D and Smith D R 2006 *Science* **312** 1780
[2] Leonhardt U 2006 *Science* **312** 1777
[3] Chen H Y, Chan C T and Sheng P 2010 *Nature Mater.* **9** 387
[4] Cummer S A, Popa B I, Schurig D, Smith D R and Pendry J B 2006 *Phys. Rev.* E **74** 036621
[5] Ruan Z, Yan M, Neff C W and Qiu M 2007 *Phys. Rev. Lett.* **99** 113903
[6] Ma H, Qu S, Xu Z and Wang J 2008 *Opt. Express* **16** 15449
[7] Schmiele M, Varma V S, Rockstuhl C and Lederer F 2010 *Phys. Rev.* A **81** 033837
[8] Li J and Pendry J B 2008 *Phys. Rev. Lett.* **101** 203901
[9] Liu R, Ji C, Mock J J, Chin J Y, Cui T J and Smith D R 2009 *Science* **323** 366
[10] Chang Z, Zhou X, Hu J, *et al.* 2010 *Opt. Express* **18** 6089
[11] Ma H, Qu S, Xu Z and Wang J 2009 *Appl. Phys. Lett.* **94** 103501
[12] Ako T, Yan M and Qiu M 2010 *Opt. Express* **18** 27060
[13] Zhu X, Liang B, Kan W, *et al.* 2011 *Phys. Rev. Lett.* **106** 014301
[14] Cummer S A and Schurig D 2007 *New J. Phys.* **9** 45
[15] Chen H Y and Chan C T 2007 *Appl. Phys. Lett.* **91** 183518
[16] Torrent D and Sanchez-Dehesa J 2008 *New J. Phys.* **10** 063015

[17] Hu J, Zhou X M and Hu G K 2009 *Comput. Mater. Sci.* **46** 708
[18] Ma H, Qu S B, Xu Z and Wang J F 2009 *Chin. Phys.* B **18** 1123
[19] Norris A N 2009 *J. Acoust. Soc. Am* **125** 839
[20] Cheng Y, Yang F, Xu J Y and Liu X J 2008 *Appl. Phys. Lett.* **92** 151913
[21] Cheng Y and Liu X J 2009 *Appl. Phys.* A **94** 25
[22] Cheng Y and Liu X J 2008 *Appl. Phys. Lett.* **93** 071903
[23] Cheng Y and Liu X J 2008 *J. Appl. Phys.* **104** 104911
[24] Cheng Y and Liu X J 2009 *Chin. Phys. Lett.* **26** 014301
[25] Chen H Y, Yang T, Luo X D and Ma H R 2008 *Chin. Phys. Lett.* **25** 3696
[26] Greenleaf A, Kurylev Y, Lassas M and Uhlmann G 2008 *New J. Phys.* **10** 115024
[27] Farhat M, Guenneau S, Enoch S, Movchan A B, Zolla F and Nicolet N 2008 *New J. Phys.* **10** 115030
[28] Urzhumov Y, Ghezzo F, Hunt J and Smith D R 2010 *New J. Phys.* **12** 073014
[29] Ren C Y, Xiang Z H and Cen Z Z 2010 *Appl. Phys. Lett.* **97** 044101
[30] Rahm M, Cummer S A, Schurig D, Pendry J B and Smith D R 2008 *Phys. Rev. Lett.* **100** 063903
[31] Mathews J H and Howell R W 2006 *Complex Analysis for Mathematics and Engineering* (Sudbury, MA: Jones & Bartlett)
[32] Allemang R J and Brown D L 1982 *Proceedings of the 1st International Modal Analysis Conference* Society for Experimental Mechanics, Orlando