Describing spatiotemporal couplings in ultrashort pulses using coupling coefficients*

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Three coupling coefficients are defined to describe spatiotemporal coupling in ultrashort pulses. With these coupling coefficients, the first-order spatiotemporal couplings of Gaussian pulse and beam are described analytically. Also, the first-order and the second-order spatiotemporal couplings caused by angular dispersion elements are studied using these coupling coefficients. It can be shown that these coupling coefficients are dimensionless and normalized, and readily indicate the severity of spatiotemporal coupling.

Keywords: spatiotemporal coupling, ultrashort pulse, angular dispersion, spatial chirp

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1. Introduction

In the design of optical systems using broadband optical pulses, the manipulation of spatiotemporal couplings is one of the most important issues.[1] The diffraction and the refraction of broadband optical pulses propagating through the systems are frequency-dependent, leading to a coupling of spatial and temporal effects. Making use of spatiotemporal couplings of ultrashort pulses allows some applications, such as pulse compression[2] and shaping,[3] which are not possible to be realized in the absence of the spatiotemporal couplings. However, spatiotemporal couplings can also be detrimental, for example, to the focusing of an ultrashort pulse.[4] Consequently, a benchmark to quantify spatiotemporal coupling is critical and such a benchmark should be used to evaluate the severities of all possible spatiotemporal couplings.

Unfortunately, such a general benchmark does not currently exist. In Ref. [1], a degree of spatiotemporal uniformity ρ was defined to quantify spatiotemporal coupling, which contains both amplitude coupling and phase coupling. In Ref. [3], the spatiotemporal couplings in femtosecond pulse shaping were quantified by the space-time coupling constant $\Delta x/\Delta t$ or the frequency-to-space mapping $\Delta\Omega/\Delta x$. In Ref. [5], Akturk *et al.* classified spatiotemporal couplings into amplitude coupling and phase coupling. The amplitude coupling includes spatial chirp,[6] angular dispersion,[7] pulse-front tilt,[8] and "time versus angle", while the phase coupling includes "wavefront rotation", "wave-front-tilt dispersion", "angular frequency chirp" and "angular temporal chirp".[5] In Refs. [9] and [10], several correlation coefficients were defined to evaluate the severities of first-order and second-order amplitude couplings, respectively. However, to our knowledge, there is lack of literature about how to evaluate the severity of phase couplings and arbitrary order amplitude couplings.

In this paper, based on the definition of the degree of spatiotemporal uniformity,[1] three coupling coefficients are defined to estimate the severity of arbitrary amplitude couplings, phase couplings and total couplings, respectively. The coupling coefficients of the first-order spatiotemporal coupling of Gaussian ultrashort pulse with Gaussian spatial profile are derived analytically. Furthermore, the spatiotemporal coupling caused by angular dispersion elements is studied with these coupling coefficients.

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2. Definitions of coupling coefficients

In this paper, we mainly consider spatiotemporal couplings in the x–ω domain. Extension to the x–t, k–ω, and k–t domains is straightforward. In the x–ω domain, the amplitude coupling is spatial chirp and the phase coupling is the so-called "wave-front-tilt dispersion".[5] The electric field of the pulse in the x–ω domain can be expressed as

$$\tilde{E}(x,\omega) = \left|\tilde{E}(x,\omega)\right| \mathrm{e}^{\mathrm{i}\,\Phi(x,\omega)}, \tag{1}$$

where $\left|\tilde{E}(x,\omega)\right|$ denotes the spectral amplitude and $\Phi(x,\omega)$ is the spectral phase. The total coupling depends not only on $\left|\tilde{E}(x,\omega)\right|$, but also on $\Phi(x,\omega)$. Taking advantage of the definition of the degree of spatiotemporal uniformity in Ref. [1], the total coupling coefficient μ_{t} can be defined as

$$\mu_{\mathrm{t}} = 1 - \frac{\iint \mathrm{d}\omega_1 \mathrm{d}\omega_2 \left|\int \tilde{E}(x,\omega_1)\tilde{E}^*(x,\omega_2)\mathrm{d}x\right|^2}{\left[\iint \left|\tilde{E}(x,\omega)\right|^2 \mathrm{d}x\mathrm{d}\omega\right]^2}. \tag{2}$$

Considering that the amplitude coupling depends only on $\left|\tilde{E}(x,\omega)\right|$, the amplitude coupling coefficient μ_{a} can be defined as

$$\mu_{\mathrm{a}} = 1 - \frac{\iint \mathrm{d}\omega_1 \mathrm{d}\omega_2 \left[\int \left|\tilde{E}(x,\omega_1)\right| \left|\tilde{E}^*(x,\omega_2)\right| \mathrm{d}x\right]^2}{\left[\iint \left|\tilde{E}(x,\omega)\right|^2 \mathrm{d}x\mathrm{d}\omega\right]^2}. \tag{3}$$

Further, as total coupling is the combined action of amplitude coupling and phase coupling, the phase coupling coefficient μ_{p} can be defined as

$$\mu_{\mathrm{p}} = 1 - \frac{\iint \mathrm{d}\omega_1 \mathrm{d}\omega_2 \left|\int \tilde{E}(x,\omega_1)\tilde{E}^*(x,\omega_2)\mathrm{d}x\right|^2}{\iint \mathrm{d}\omega_1 \mathrm{d}\omega_2 \left[\int \left|\tilde{E}(x,\omega_1)\right| \left|\tilde{E}^*(x,\omega_2)\right| \mathrm{d}x\right]^2}. \tag{4}$$

In Eq. (3), μ_{a} is normalized, and $\mu_{\mathrm{a}} = 0$ corresponds to a pulse free of amplitude coupling, while $0 < \mu_{\mathrm{a}} < 1$ indicates that some amplitude couplings are present. Moreover, an increased value of μ_{a} indicates an increase in the magnitude of amplitude coupling. μ_{p} and μ_{t} also exhibit the similar properties. As shown in Fig. 1(a), the "spatial dispersion (SPD)[6]" is zero, i.e., $\partial x_0/\partial\omega = 0$, where x_0 is the mean beam position for a given frequency ω; whereas SPD has a nonzero value in each of Figs. 1(b) and 1(c). The larger the value of SPD, the larger the spatial chirp is. Thus, an increased value of μ_{a} indicates an increase in the magnitude of spatial chirp.

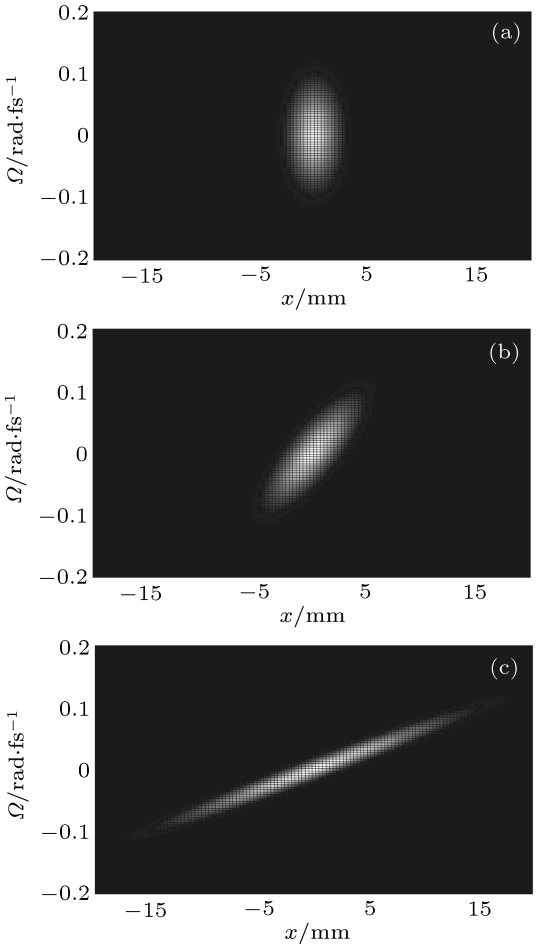


Fig. 1. Profiles of an ultrashort pulse with amount of spatial chirp increasing, and hence with value of μ_{a} increasing, where $\Omega = \omega - \omega_0$, with ω_0 being the central angular frequency. (a) $\mu_{\mathrm{a}} = 0.00$, (b) $\mu_{\mathrm{a}} = 0.40$, and (c) $\mu_{\mathrm{a}} = 0.80$.

3. Spatiotemporal couplings in the x–ω domain

3.1. First-order spatiotemporal coupling of a Gaussian pulse and beam

A Gaussian ultrashort pulse with Gaussian spatial profile with first-order spatiotemporal coupling in the x–ω domain can be expressed in the form[5]

$$\begin{aligned} E(x,\Omega) &\propto \exp\left\{R_{xx}x^2 + 2R_{x\omega}x\Omega - R_{\omega\omega}\Omega^2\right\} \\ &= \exp\left\{ax^2 + 2bx\Omega - c\Omega^2\right\} \\ &\quad \times \exp\left\{\mathrm{i}(ex^2 + 2fx\Omega - g\Omega^2)\right\}. \end{aligned} \tag{5}$$

In Eq. (5), $a = R^{\mathrm{R}}_{xx}$, $b = R^{\mathrm{R}}_{x\omega}$, $c = R^{\mathrm{R}}_{\omega\omega}$, $e = R^{\mathrm{I}}_{xx}$, $f = R^{\mathrm{I}}_{x\omega}$, $g = R^{\mathrm{I}}_{\omega\omega}$, $\Omega = \omega - \omega_0$, and ω_0 is the central

angular frequency, where we have used a shortcut notation, superscripts "R" and "I", for the real and the imaginary parts of the parameter respectively. b and f are the amplitude coupling parameter and phase coupling parameter respectively. Because a and c are related to beam spot size and bandwidth, respectively,[5] a is always negative, and c is always positive. Substituting Eq. (5) into Eqs. (2)–(4), the coupling coefficients for an ultrashort pulse described by Eq. (5) can be deduced as

$$\mu_{\rm a} = 1 - \left(\frac{b^2 + ac}{ac}\right)^{1/2}, \tag{6}$$

$$\mu_{\rm t} = 1 - \left(\frac{b^2 + ac}{ac - f^2}\right)^{1/2}, \tag{7}$$

$$\mu_{\rm p} = 1 - \left(\frac{ac}{ac - f^2}\right)^{1/2}. \tag{8}$$

With Eqs. (6)–(8), for a monochromatic light beam ($c \to \infty$), there is $\mu_{\rm a} = \mu_{\rm p} = \mu_{\rm t} = 0$. So the amplitude coupling, the phase coupling and the total coupling do not exist in a monochromatic light Gaussian beam. With the increase of the pulse bandwidth $\Delta\omega(\propto 1/c)$, $\mu_{\rm a}$, $\mu_{\rm p}$, and $\mu_{\rm t}$ increase, and the magnitude of the first-order spatiotemporal coupling in a Gaussian pulsed beam increases. From Eqs. (6)–(8), we can see that the amplitude coupling parameter b and the phase coupling parameter f are the origin of the first-order spatiotemporal coupling. In the case of $b = 0$, equation (6) yields $\mu_{\rm a} = 0$, indicating that amplitude coupling does not exist. Also, if $f = 0$, equation (8) yields $\mu_{\rm p} = 0$, indicating that phase coupling does not exist.

3.2. First-order spatiotemporal coupling caused by angular dispersion

The dispersive nature of an angular dispersion component can be described approximately by Martinez's model.[11,12] By using the Kirchhoff–Fresnel integral, an expression of the electric field of a Gaussian ultrashort pulse with Gaussian spatial profile at an arbitrary distance from an angular disperser can be expressed in the frequency domain as[11–13]

$$E(x,\Omega) = b_3 \exp\left(-\frac{\tau^2\Omega^2}{4}\right)\exp\left(-{\rm i}\,\frac{k_{\rm c}x^2}{2z}\right) \times \exp\left[\frac{{\rm i}\,k_{\rm c}}{2z}\frac{\tilde{q}(d)}{\tilde{q}(d+\alpha^2 z)}(x+\beta_1\Omega z)^2\right], \tag{9}$$

where b_3 is an amplitude constant, τ the Gaussian pulse parameter (full width at half maximum (FWHM) of intensity $T_{\rm FWHM} = \sqrt{2\ln 2}\tau$), $k_{\rm c}$ the wavenumber of the central frequency, d the distance from the Gaussian beam waist to the angular dispersion component, z the propagation distance of the laser beam from angular dispersion component (see Fig. 1 in Ref. [14]), $\tilde{q}$ a complex parameter of the Gaussian beam, $\alpha = \partial\theta/\partial\gamma$ the factor of angular magnification of the beam, with θ being the emerging angle and γ the angle of incidence, and $\beta_1 = \partial\theta/\partial\omega$ the first-order angular dispersion factor of the angular dispersion component.

It is easy to reduce Eq. (9) into the form of Eq. (5), and corresponding parameters $a \sim g$ can be obtained, such as

$$a = -\frac{k z_{\rm R}\alpha^2}{2\left[(d+\alpha^2 z)^2 + z_{\rm R}^2\right]}, \tag{10}$$

$$b = -\frac{k_{\rm c} z_{\rm R}\alpha^2\beta_1 z}{2\left[(d+\alpha^2 z)^2 + z_{\rm R}^2\right]}, \tag{11}$$

$$c = \frac{k_{\rm c} z_{\rm R}\alpha^2\beta_1^2 z^2}{2\left[(d+\alpha^2 z)^2 + z_{\rm R}^2\right]} + \frac{\tau^2}{4}, \tag{12}$$

$$f = \frac{k\beta_1[d(d+\alpha^2 z) + z_{\rm R}^2]}{2\left[(d+\alpha^2 z)^2 + z_{\rm R}^2\right]}, \tag{13}$$

where $z_{\rm R} = \pi s^2/\lambda_0$ is the Rayleigh length of the Gaussian beam, s beam waist, and λ_0 the central wavelength.

Substituting Eqs. (10)–(13) into Eqs. (6)–(8), the coupling coefficients for the first-order spatiotemporal coupling caused by an angular disperser can be written as

$$\mu_{\rm a} = 1 - \frac{\tau}{2}\left(\frac{1}{c}\right)^{1/2}, \tag{14}$$

$$\mu_{\rm t} = 1 - \left[\frac{z_{\rm R}\alpha^2\tau^2}{2k_{\rm c}\beta_1^2(z_{\rm R}^2 + d^2) + z_{\rm R}\alpha^2\tau^2}\right]^{1/2}, \tag{15}$$

$$\mu_{\rm p} = 1 - 2\left[\frac{z_{\rm R}\alpha^2 c}{2k_{\rm c}\beta_1^2(z_{\rm R}^2 + d^2) + z_{\rm R}\alpha^2\tau^2}\right]^{1/2}. \tag{16}$$

Equations (14)–(16) imply that the first-order spatiotemporal coupling caused by an angular disperser depends on angular dispersion parameters (α, β_1), pulse width parameter τ, Rayleigh length $z_{\rm R}$ and propagation distance z intimately. With the increase of τ or β_1, $\mu_{\rm a}$, $\mu_{\rm p}$ and $\mu_{\rm t}$ decrease, and the magnitude of the first-order spatiotemporal coupling caused by an angular disperser decreases. It follows from Eq. (12) that $\partial c/\partial z > 0$. Then, it can be readily shown from Eqs. (14) and (16) that with the increase of propagation distance, the amplitude coupling increases, while phase coupling decreases. These properties can also be seen clearly in Fig. 2. As shown in Fig. 2, at a small propagation distance from the disperser ($z \ll z_{\rm R}$), $\mu_{\rm a} = 0$, while $\mu_{\rm p}$ is almost 1. This can be explained

by using Eq. (9) in Ref. [12]. Because the angular disperser introduces a phase coupling term $\exp(\mathrm{i}\, k\beta_1\Omega x)$, the μ_p changes from 0 to 1 suddenly; while the amplitude coupling parameter b is almost zero, i.e. $b \approx 0$ at a small propagation distance, resulting in $\mu_\mathrm{a} = 0$. Figure 2 also shows that at a large propagation distance ($z \gg z_\mathrm{R}$), μ_p tends to an asymptotic value 0.3, while μ_a keeps almost 1. This can be explained according to Eqs. (14) and (16). For the case of $z \to \infty$, μ_p tends to a minimum value 0.293, while μ_a tends to a maximum value 0.994. From another point of view, because the spectral lateral walk-off increases with propagation distance increasing and finally tend to a constant (see Fig. 2 in Ref. [14]), the amplitude coupling (i.e. spatial chirp) also increases with propagation distance increasing and finally becomes unchanged. Moreover, according to Eq. (15), μ_t has a constant value 0.996 and the total coupling does not vary with propagation distance.

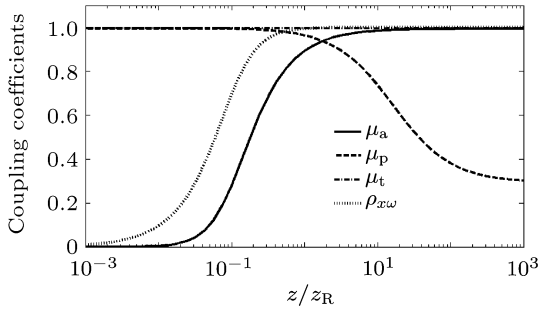


Fig. 2. Coupling coefficients and correlation coefficients versus propagation distance after an angular dispersion component. $\alpha = -0.27$, $\beta_1 = -0.29$ fs, $s = 0.5$ mm, $d = z_\mathrm{R} = 0.98$ m, $\lambda_0 = 800$ nm, and $\tau = 22.7$ fs.

It is also interesting to compare the amplitude coupling coefficient μ_a with another normalized parameter proposed for describing the first-order spatial chirp, i.e., the correlation coefficient $\rho_{x\omega}$.[5,9] An increased value of $\rho_{x\omega}$ indicates an increase in the magnitude of first-order spatial chirp. The correlation coefficient $\rho_{x\omega}$ is not very sensitive to the larger spatiotemporal coupling. As shown in Fig. 2, there is little change of $\rho_{x\omega}$ in the region of larger coupling ($\mu_\mathrm{a} \approx 1$). For example, when $z = z_\mathrm{R}$, $\rho_{x\omega} = 0.99$, and $\mu_\mathrm{a} = 0.89$.

3.3. Second-order spatiotemporal coupling caused by angular dispersion

It should be noted that the correlation coefficient defined in Refs. [5] and [9] can be used only for quantifying the first-order amplitude coupling. However, the coupling coefficients defined by Eqs. (2)–(4) can be used for quantifying arbitrary order spatiotemporal couplings, such as higher-order spatial chirp caused by high-order angular dispersion (naturally occurring after prisms and gratings). For example, when only the second-order angular dispersion of an angular disperser is considered, the electric field of Gaussian ultrashort pulse with Gaussian spatial profile at an arbitrary distance from an angular disperser can be expressed in the frequency domain as

$$E(x,\Omega) = b_3 \exp\left(-\frac{\tau^2\Omega^2}{4}\right)\exp\left(-\mathrm{i}\,\frac{k_\mathrm{c}x^2}{2z}\right) \times \exp\left[\frac{\mathrm{i}\,k_\mathrm{c}}{2z}\frac{\tilde{q}(d)}{\tilde{q}(d+\alpha^2 z)}\left(x+\frac{\beta_2\Omega^2}{2}z\right)^2\right], \tag{17}$$

where $\beta_2 = \partial^2\theta/\partial\omega^2$ is the second-order angular dispersion factor of the angular dispersion component. From Eq. (17) it follows that the second-order spatiotemporal coupling exists. The corresponding coupling coefficients can be calculated by using Eqs. (2)–(4) and (17), and their properties are shown in Fig. 3. The trends of the curves in Fig. 3 are similar to those in Fig. 2. This can be explained as the fact that the factor $\beta_2\Omega^2/2$ plays a similar role to the factor $\beta_1\Omega$, when the pulsed beam propagates through and after an angular dispersion component. Also in Fig. 3, μ_a, μ_p, and μ_t are much smaller than those in Fig. 2 at a certain propagation distance z. It means that the first-order spatiotemporal coupling is dominant, and the spatiotemporal coupling beyond an angular disperser is mainly caused by the first-order angular dispersion of an angular disperser.

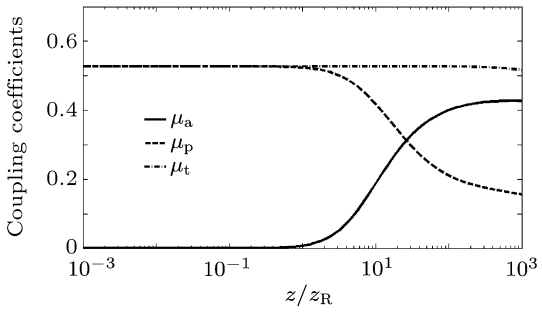


Fig. 3. Coupling coefficients versus propagation distance after an angular dispersion component. $\beta_1 = 0$ fs, $\beta_2 = 0.12$ fs^2 and other parameters are the same as those in Fig. 2.

4. Spatiotemporal coupling in the k–ω domain

In the k–ω domain, the amplitude coupling is angular dispersion and the phase coupling is angular frequency chirp.[5] Following Eqs. (2)–(4), the coupling coefficients in the k–ω domain can be defined by replacing x with k in Eqs. (2)–(4).

By using Eq. (2.133) of Ref. [11], an expression for the electric field of Gaussian pulse with Gaussian spatial profile at an arbitrary distance from an angular disperser can be expressed as

$$\begin{aligned}\tilde{E}(k_x,\Omega) = {} & b_4 \exp\left(-\frac{\tau^2\Omega^2}{4}\right) \\ & \times \exp\left[\frac{\mathrm{i}\,\tilde{q}(d)}{2k_\mathrm{c}\alpha^2}(k_x - k_\mathrm{c}\beta_1\Omega)^2\right] \\ & \times \exp\left[-\mathrm{i}\,k_\mathrm{c}\left(1-\frac{k_x^2}{2k_\mathrm{c}^2}\right)z\right], \end{aligned} \tag{18}$$

where b_4 is an amplitude constant and k_x is the spatial frequency. In a way similar to that in Subsection 3.2, in the k–ω domain, the amplitude coupling coefficient for the first-order spatiotemporal coupling caused by an angular disperser can be deduced by using Eq. (18) as follows:

$$\mu_\mathrm{a} = 1 - \left(\frac{\alpha^2\tau^2}{\alpha^2\tau^2 + 2k_\mathrm{c}z_\mathrm{R}\beta_1^2}\right)^{1/2}. \tag{19}$$

Equation (19) implies that angular dispersion caused by an angular disperser depends on angular dispersion parameters (α, β_1), pulse width parameter τ, and Rayleigh length z_R intimately, but does not depend on distances z and d. It means that the magnitude of angular dispersion caused by an angular disperser keeps constant as the beam propagates in the free space. For a monochromatic light beam ($\tau \to \infty$), equation (19) yields $\mu_\mathrm{a} = 0$, indicating that angular dispersion does not exist in a monochromatic light Gaussian beam. In the case of $\beta_1 = 0$, there is $\mu_\mathrm{a} = 0$, meaning that angular dispersion does not exist in the absence of angular disperser. With the increase of β_1, μ_a increases, and the magnitude of angular dispersion increases.

Finally, to measure these coupling coefficients, one can use a self-referencing technique,[1,15,16] SEA TADPOLE technique[17] or SPIDER technique,[18–20] etc.

5. Conclusion

We defined three coupling coefficients to describe spatiotemporal coupling in ultrashort pulses. These coupling coefficients are dimensionless and normalized, and readily indicate the severity of spatiotemporal coupling. With these coupling coefficients, we studied the first-order spatiotemporal couplings of Gaussian pulses and beams, and corresponding analytical expressions are obtained. We found that the amplitude coupling parameter and phase coupling parameter are the origin of the first-order spatiotemporal coupling. We also studied the spatiotemporal coupling caused by an angular disperser and found that it depends on the temporal width, angular dispersion and propagation distance intimately. We also compared the first-order and the second-order spatiotemporal couplings caused by an angular dispersion component and showed that the first-order spatiotemporal coupling is dominant. We believe that these coupling coefficients can help us better understand the spatiotemporal couplings and their consequences, and may be used as a benchmark to evaluate the severities of all possible spatiotemporal couplings.

References

[1] Dorrer C and Walmsley I A 2002 *Opt. Lett.* **27** 1947

[2] Walmsley I, Waxer L and Dorrer C 2001 *Rev. Sci. Instrum.* **72** 1

[3] Frei F, Galler A and Feurer T 2009 *J. Chem. Phys.* **130** 1

[4] Tanabe T, Kannari F, Korte F, Koch J and Chichkov B 2005 *Appl. Opt.* **44** 1092

[5] Akturk S, Gu X, Gabolde P and Trebino R 2005 *Opt. Express* **13** 8642

[6] Gu X, Akturk S and Trebino R 2004 *Opt. Commun.* **242** 599

[7] Varju K, Kovacs A P and Osvay K 2002 *Opt. Lett.* **27** 2034

[8] Akturk S, Kimmel M, O'Shea P and Trebino R 2003 *Opt. Express* **11** 491

[9] Gabolde P, Lee D, Akturk S and Trebino R 2007 *Opt. Express* **15** 242

[10] Zeng S G, Dan Y Q and Zhang B 2009 *J. Opt. Soc. Am.* B **26** 1869

[11] Diels J C and Rudolph W 2006 *Ultrashort Laser Pulse Phenomena* 2nd edn. (London: Academic Press) p. 124

[12] Martinez O E 1986 *J. Opt. Soc. Am.* B **3** 929

[13] Li D, Lü X, Zeng S and Luo Q 2008 *Opt. Lett.* **33** 128

[14] Zeng S, Li D, Lü X, Liu J and Luo Q 2007 *Opt. Lett.* **32** 1180

[15] Dorrer C, Kosik E M and Walmsley I A 2002 *Opt. Lett.* **27** 548

[16] Dorrer C, Kosik E M and Walmsley I A 2002 *Appl. Phys.* B: *Lasers and Optics* **74** [Suppl.] S209

[17] Bowlan P, Gabolde P, Coughlan M A, Trebino R and Levis R J 2008 *J. Opt. Soc. Am.* B **25** 81

[18] Lei L, Wen J H, Jiao Z X, Lai T S and Lin W Z 2008 *Acta Phys. Sin.* **57** 307 (in Chinese)

[19] Wen J H, Liu J, Zhang H, Chen J L, Huang Z Z, Jiao Z X and Lai T S 2010 *Acta Phys. Sin.* **59** 370 (in Chinese)

[20] Liu W J, Ren S T and Qu S L 2010 *Acta Phys. Sin.* **59** 3286 (in Chinese)