Tight focus of a radially polarized and amplitude-modulated annular multi-Gaussian beam*

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The focusing of a radially polarized beam without annular apodization ora phase filter at the entrance pupil of the objective results in a wide focus and low purity of the longitudinally polarized component. However, the presence of a physical annular apodization or phase filter makes some applications more difficult or even impossible. We propose a radially polarized and amplitude-modulated annular multi-Gaussian beam mode. Numerical simulation shows that it can be focused into a sharper focal spot of $0.125\lambda^2$ without additional apodizations or filters. The beam quality describing the purity of longitudinally polarized component is up to 86%.

Keywords: focusing, polarization, laser beam

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1. Introduction

Sharp focusing of light beams has been an academically interesting topic and practically very significant issue. A large number of optical instruments and devices demand much smaller focus spots. For example, highly focused light beams can increase the imaging resolution and enhance the localized electric field at the nanometer scale metal tip in a variety of microscopes and spectroscopes. For optical data storage, the sharply focused beam implies a much denser recording to increase the storage capacity. Also in optical manipulation, well-focused beams can increase the trapping force and provide the possibility of locating and interacting with single molecules and atoms. It has long been known that the shape of pupil apodization, the polarization states of the incident beams, and the amplitude and the phase distributions as well as the numerical aperture of the objective determine the transverse distribution and axial extension of the focal spot.[1−10] It is shown that for clear aperture geometry and plane-wave illumination, linear or circular polarization is preferable over radial polarization for spot size reduction. However, with an annular aperture and a high NA objective, the radial polarization illumination gives a smaller spot size and reaches the limit of scalar diffraction theory. For sub-wavelength and super-resolution focusing, the polarization properties of the electromagnetic field play a dominant role. Thus the vector diffraction theory needs to be adopted for obtaining the exact field profile in the focal area. Vector analysis shows that the radially polarized beam can have a narrow central peak due to the appearance of the strong longitudinal field component that is sharply centred on the optical axis. It is verified that the longitudinal component generated by radially polarized illumination produces the narrow spot size for wide range of geometries, and for annular illumination the total intensity profile of longitudinal component and radial component in the focal area usually has the smallest spot size because the radial components cancel out each other in the focal area. For a longitudinally polarized beam in the focal area of the objective, many attractive applications such as particle acceleration, fluorescent imaging, second harmonic generation, photo-thermal therapy, detection of single molecule dipole moment and z-polarized confocal microscopy have been found.[11−19]

To produce annular illumination, one may insert a physical annular mask in the pupil plane of

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the objective.[20] However, it is not ideal since most of the incident optical field is blocked resulting in a low efficient use of the available light. An alternative approach uses a conical lens or an axicon.[21,22] Although this is a light efficient method to produce ring illumination, it has some drawbacks for microscopy applications because of the presence of an additional physical element which makes the system more complicated. To avoid these drawbacks, a light efficient binary phase-only diffraction optical element, which converts a laser beam into an annular ring, has been designed for laser-scanning two-photon fluorescence microscopy application.[23]

In focusing radially polarized beam which mainly produces longitudinally polarized component in focal area, there is a slight residual radially polarized component. To suppress this residual component, an annular five-belt binary phase filter was recently designed.[24] The binary optical element here acts like a special polarization filter, which diffracts the radial field component away from beam centre more than the longitudinal field, thus making the beam in the focal region substantially longitudinally polarized. With Bessel–Gaussian beam being incident beam and numerical aperture NA being 0.95, a full-width at half-maximum (FWHM) of 0.43λ is achieved. The corresponding spot size reaches $0.14\lambda^2$. The longitudinal field accounts for 81% of the total power. This is the theoretical prediction. Experimentally, the best result reported thus far[20] is $0.16\lambda^2$ which is obtained with a plane wave blocked by an annulus and numerical aperture NA 0.9. The power contained in the longitudinal field is calculated to be 49.6% of the total beam power.

Generally, those specially designed apodizers are quite complex in phase and amplitude transmittance, so fabricating them is difficult and their efficiency is low. Moreover, the presence of a physical annular apodization or phase filter makes some applications more complicated or even impossible. In this paper, we propose a radially polarized and amplitude-modulated annular multi-Gaussian beam mode for illuminating the pupil plane of the objective. It is shown that with numerical aperture NA 0.95, it is capable of achieving much sharper focus of $0.125\lambda^2$ and that the longitudinal field accounts for up to 86% of the total power. For the proposed illumination, there is no need to design a complex physical apodizer and insert it in the pupil plane of the objective. The radial amplitude modulation is implemented at the beam generation stage with computer-generated hologram method.

2. Mathematical model

The mathematical model for a radially polarized and amplitude-modulated annular multi-Gaussian beam can be written as

$$E(\theta) = \left(\frac{\theta}{\theta_0}\right)^m \sum_{n=-N}^{N} \exp\left[-\left(\frac{\theta - \theta_c - nw_0}{w_0}\right)^2\right]. \tag{1}$$

Here, θ is the converging semi-angle. We denote the maximum converging semi-angle as $\theta_{\max}$ which is related to objective numerical aperture by $\theta_{\max} = \arcsin(NA)$. θ_0 is an angle which, along with integer m, determines the shape of the modulation function. θ_0 is usually chosen to be slightly smaller than $\theta_{\max}$. θ_c determines the radial position translation of the $E(\theta)$. Here we take $\theta_c = \theta_{\max}/2$. w_0 is the waist width of single Gaussian beam which is calculated by the following formula:

$$w_0 = \frac{1}{2}\frac{\theta_{\max}}{N + \left\{1 - \ln\left[\sum_{n=-N}^{N} \exp\left(-n^2\right)\right]\right\}^{1/2}}. \tag{2}$$

Equation (1) describes an object beam. It is suitable to convert Eq. (1) from a function of angle into a function of radial polar coordinate. One may substitute θ with $\arcsin(r/f)$, where f is the focal distance of the objective. In Eq. (1), the factor $(\theta/\theta_0)^m$ ensures that the most of light energy is located on the annular edge of the pupil. Increasing the integer m will concentrate more energy into the annular edge area in which the converging semi-angle is more than θ_0. The sum of $(2N+1)$ spatially equally spaced Gaussian beams ensures that the amplitude of the constructed annular multi-Gaussian beam decreases suddenly when reaching the outer edge of the pupil. For comparison, we plot one-dimensional transverse distribution of Bessel–Gaussian beam, radially polarized and amplitude-modulated annular multi-Gaussian beam, as described by Eq. (1), and all individual Gaussian beams together in Fig. 1(a). When $NA = 0.95$, $\theta_{\max} = 71.8^\circ = 1.2531$ rad, $\theta_c = 0.6265$ rad. Letting $N = 20$, according to Eq. (2), $w_0 = 0.0303$ rad. Other two parameters are $m = 10$, $\theta_0 = 65^\circ = 1.1345$ rad. It is shown that the incident light energy distributions of two beams differ from each other greatly. It should be noted that the individual Gaussian beams in Fig. 1(a) are not radially polarized strict Gaussian beam as the amplitude of each Gaussian beam is also amplitude-modulated. Figures 1(b) and 1(c)

are the two-dimensional intensity contour and polarization distribution of the radially polarized and amplitude-modulated annular multi-Gaussian beam in the pupil plane of the objective with focal distance $f = 3.684$ mm. The maximum radius of this pupil is assumed to be 3.5 mm. $N = 20$, $w_0 = 0.0303$ rad. In Fig. 1(b), $m = 8$, $\theta_c = 1.1868$ rad. In Fig. 1(c), $m = 2$, $\theta_c = 1.1868$ rad. It is verified that with small m, the incident beam energy distribution tends to become far away from outer edge area.

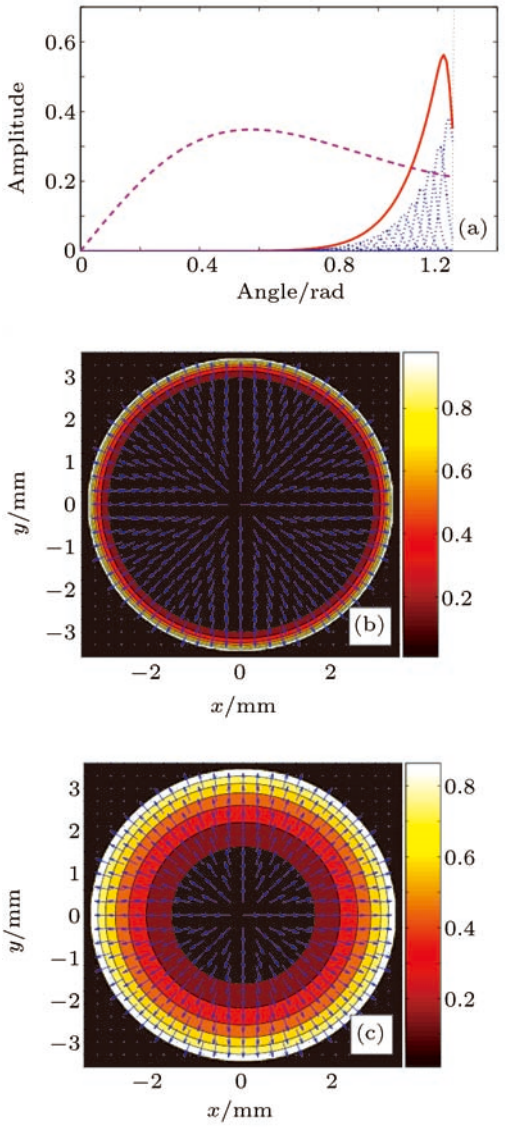


Fig. 1. (colour online) (a) Angle distribution of the radially polarized and amplitude-modulated annular multi-Gaussian beam (solid line) and Bessel–Gaussian beam (dashed line) as well as radially polarized and amplitude-modulated individual Gaussian beams (dotted line); panels (b) and (c) for the two-dimensional intensity contour and polarization distribution of the radially polarized and amplitude-modulated annular multi-Gaussian beam in the pupil plane of the objective with parameters (b) $m = 8$, $\theta_c = 1.1868$ rad, and (c) $m = 2$, $\theta_c = 1.1868$ rad.

3. Simulation of focusing property

To probe into the focusing property, one can follow Richards and Wolf's theory of focusing polarized beams[2] and write the radial component and the longitudinal component of electric field in the focal area when a radially polarized and amplitude-modulated annular multi-Gaussian beam is incident onto the pupil of the objective as follows:[2,6]

$$E_r(r,z) = E_0 \int_0^{\theta_{\max}} \cos^{1/2}\theta \sin(2\theta) E(\theta) \times \mathrm{J}_1(kr\sin\theta) \exp(\mathrm{i}kz\cos\theta)\,\mathrm{d}\theta, \quad (3)$$

$$E_z(r,z) = 2\mathrm{i}E_0 \int_0^{\theta_{\max}} \cos^{1/2}\theta \sin^2\theta E(\theta) \times \mathrm{J}_0(kr\sin\theta) \exp(\mathrm{i}kz\cos\theta)\,d\theta, \quad (4)$$

where E_0 is a constant; $\mathrm{J}_0(x)$ and $\mathrm{J}_1(x)$ denote zero order and first order Bessel functions of the first kind; k is the wave number and $k = 2\pi/\lambda$, with λ being the wavelength of illuminating beam. The one-dimensional radial intensity distribution, two-dimensional transverse intensity contour distribution in the focal plane of the objective and two-dimensional longitudinal intensity contour distribution in a plane containing optical axis of the objective are shown in Fig. 2. The parameters used in simulating Fig. 2 are the same as those in simulating Fig. 1 excepting that $m = 30$ and $\theta_0 = 68°$. Figures 2(a), 2(c), and 2(e) are for Bessel–Gaussian beam; figures 2(b), 2(d), and 2(f) are for radially polarized and amplitude-modulated annular multi-Gaussian beams. It can be seen that in Figs. 2(b) and 2(d), the radial polarized component is greatly suppressed and the longitudinally polarized component in turn is highly enhanced. Thus the spot size or full-width at half-maximum (FWHM) is substantially reduced. The calculation shows that the full-width at half-maximum is 0.4λ, which is much smaller than the diffraction limit of this objective $\lambda/(2NA) = 0.526\lambda$. The corresponding spot size area is as small as $0.125\lambda^2$. Similar to the definition described in Ref. [24], the beam quality is characterized by $\eta = \phi_z/(\phi_z + \phi_r)$ in which $\phi_z = 2\pi\int_0^{r_0} |E_z(r,0)|^2 r\,\mathrm{d}r$ and $\phi_r = 2\pi\int_0^{r_0} |E_r(r,0)|^2 r\,\mathrm{d}r$. Here r_0 is the radius of zero point of the total radial intensity distribution. In the case of Bessel–Gaussian beam focusing, the beam quality is $\eta = 44.9\%$. Thus the parasitic radial component accounts for 55.1% of the total energy. If a radially polarized and amplitude-modulated annular multi-Gaussian beam is focused

with the same objective as the one used in Ref. [24], the beam quality is increased to 86%. This quality parameter is also better than the one presented in Ref. [24]. Figures 2(e) and 2(f) show that the axial light intensity distribution has been elongated when a radially polarized and amplitude-modulated annular multi-Gaussian beam is focused. The focal volume takes a shape of micro-bottle beam with two sharp ends. This is a surprising result which seems contradictory to conventional prediction. In scalar diffraction theory, the transverse size of the focal spot reaches minimum in the focal plane with two ends along the axis widening away from the focal plane. This property is useful in metal-tip electric field excitation of near-field optical scanning microscopy by longitudinally polarized component. In orientation determination of absorptive single molecule dipole moment, the elongated, purified, uniform and sharper longitudinal polarization component improves the precise positioning and deep interaction detection. We believe that this sharper focusing will also find its application in the second harmonic generation, Romann spectroscopic imaging, micro-particle manipulation, electron acceleration and photo-thermal therapy.

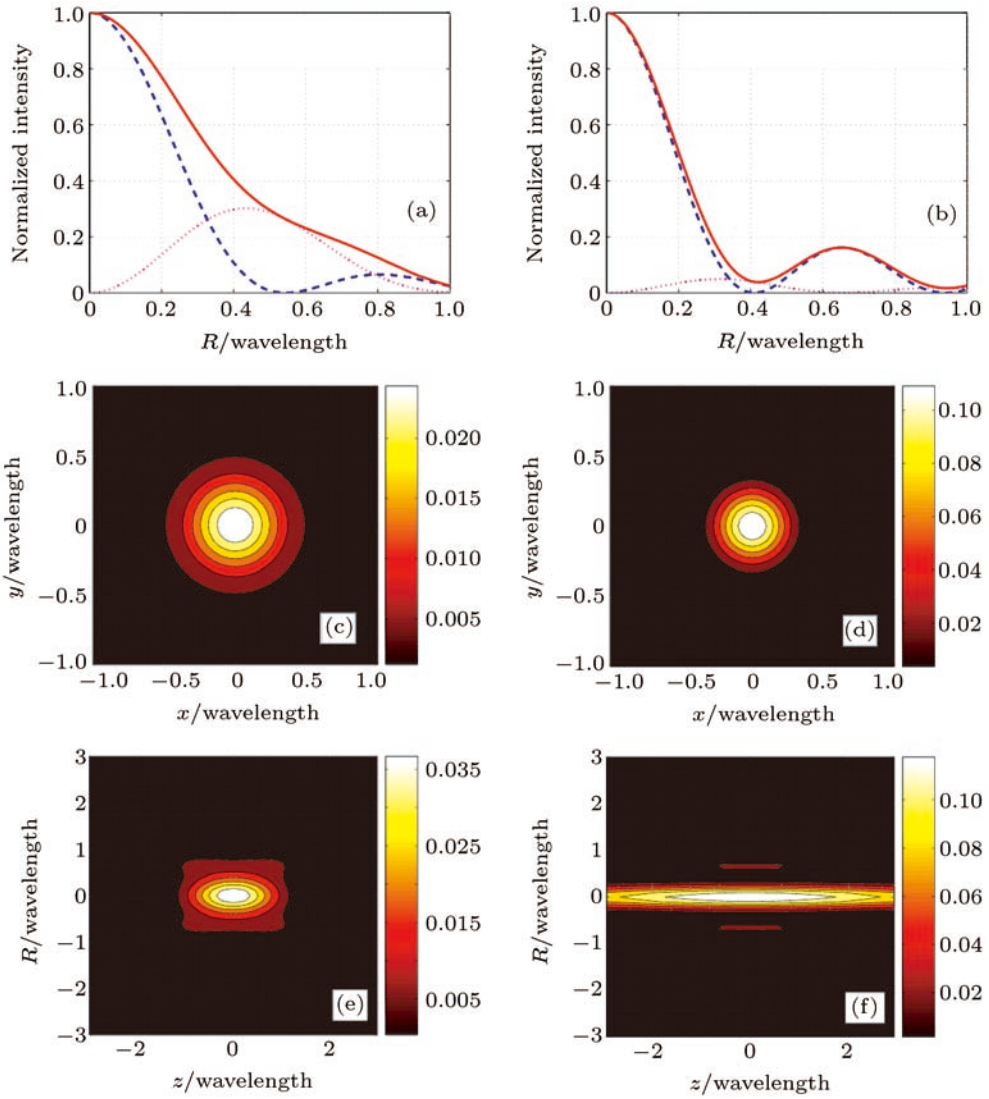


Fig. 2. (colour online) Intensity contour distribution of a Bessel–Gaussian beam [panels (a), (c) and (e)] and radially polarized and amplitude-modulated annular multi-Gaussian beam [panels (b), (d) and (f)]. Panels (a) and (b) show one-dimensional radial intensity distributions of longitudinal polarization component (dashed line), radial polarization component (dotted line), and the sum of two components (solid line); panels (c) and (d) exhibit the transverse intensity distribution in the focal plane; panels (e) and (f) display the longitudinal plane intensity distributions. The length is scaled with wavelength.

For an electric field, there is always a mutually coexistent magnetic field which is related to electric field by Maxwell's equation. Accompanied with a radially polarized electric field, the magnetic field is an azimuthally polarized field. After being focused, it will not produce radially polarized and longitudinally polarized components. It is still a purely azimuthally polarized field which can be obtained by $H_\varphi = -\mathrm{i}\left(\partial E_r/\partial z - \partial E_z/\partial r\right)/k$. Using Eqs. (3) and (4) yields

$$H_\varphi = 2E_0 \int_0^{\theta_{\max}} \cos^{1/2}\theta \sin\theta E(\theta) \times \mathrm{J}_1(kr\sin\theta)\exp(\mathrm{i}kz\cos\theta)\,\mathrm{d}\theta. \quad (5)$$

Figures 3(a) and 3(b) show the magnetic field transverse intensity contour distribution and longitudinal intensity contour distribution of radially polarized and amplitude-modulated annular multi-Gaussian beam in focal area, respectively. The parameters are as follows: $m = 10$, $\theta_0 = 68°$. Other parameters are the same as those in Figs. 1 and 2. This distribution will provide an insight into the forces exerted by magnetic field on the moving and accelerated charged particle when the longitudinally polarized electric field is used to accelerate charged particle.

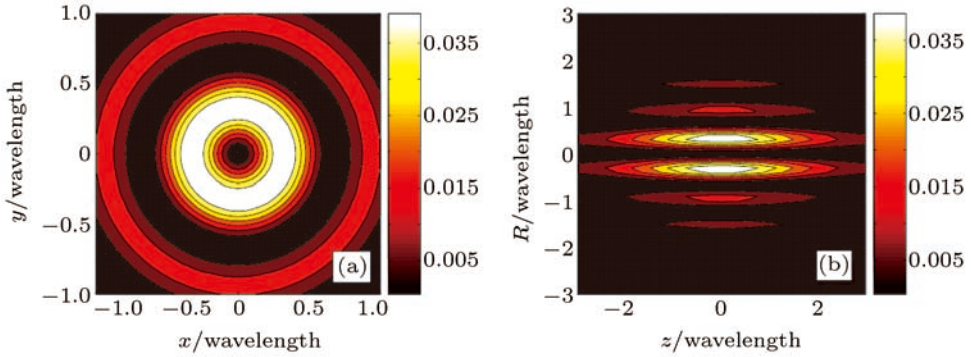


Fig. 3. (colour online) Magnetic field transverse intensity contour distribution and longitudinal intensity contour distribution of radially polarized and amplitude-modulated annular multi-Gaussian beam in focal area.

4. Conclusion

We proposed a radially polarized and amplitude-modulated annular multi-Gaussian beam mode for sharp focusing. This beam mode can be converted from a plane wave by using a computer-generation hologram. Without inserting a physical annulus or fabricating sophisticated multi-belt phase filter, the sharpest focal spot and the purest longitudinally polarized electric field thus far is feasible. The energy efficiency in the incident pupil plane of objective is almost perfect. The focused beam may improve the performance of wide range of applications.

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