Stationary properties of a single-mode laser system with non-Gaussian and Gaussian noise

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(Received 28 November 2010; revised manuscript received 19 June 2011)

A single-mode laser system with non-Gaussian and Gaussian noise is investigated. The stationary mean value and the normalized variance of the laser intensity are numerically calculated under the condition that the stationary probability distribution function (SPDF) is derived. The SPDF as a function of the laser intensity exhibits a maximum. The maximum becomes smaller with the increase of the correlation intensity or the non-Gaussian parameter, where the later is a measure of the deviation from the Gaussian characteristic. The maximum becomes larger as the correlation time increases. The laser intensity stationary mean value decreases with the increase of the correlation intensity or the non-Gaussian parameter while increases with the correlation time increasing. The laser intensity normalized variance increases with the increase of the correlation intensity or the non-Gaussian parameter while decreases as the correlation time increases.

Keywords: single-mode laser, non-Gaussian noise, Gaussian noise

PACS: 42.55.Ah, 05.40.–a, 42.60.Mi **DOI:** 10.1088/1674-1056/20/11/114207

1. Introduction

In recent years, the effect of noise on a nonlinear system has been widely investigated in many fields. Bistable systems,[1−6] biological systems,[7−9] optical bistable systems,[10,11] and laser systems[12−17] have been studied. The study of the laser system has revealed that the simultaneous consideration of the additive and the multiplicative noise is important to deeply understand the statistical properties of the single-mode laser system.[18−20] In 2001, Luo *et al.*[21] studied the effects of the coloured noise on the intensity and the phase in a laser system. Cao and Wu[22] proposed a single-mode laser noise model and derived the approximate Fokker–Planck equation for the laser phase and the laser amplitude. In all those work, the Gaussian noise was assumed. However, some recent experimental and theoretical results for one kind of crayfish and the rat skin offered strong indications that there could be non-Gaussian noise sources in the stochastic systems.[23−27]

The stochastic systems with the non-Gaussian noise need to be investigated because of their potential applications. Fuentes and Toral[28] analysed the phenomenon of the stochastic resonance in a double-well potential driven by the coloured non-Gaussian noise. Goswami *et al.*[29] studied the barrier crossing dynamics in the presence of the non-Gaussian noise and observed that the multiplicative coloured non-Gaussian noise could induce the resonant activation. Wu and Zhu[30] investigated the phenomenon of the stochastic resonance in a bistable system driven by the non-Gaussian noise with a time-delayed feedback by using the small time delay approximation, the path-integral approach and the unified coloured noise approximation. Zhang *et al.*[31] studied a stochastic system with the non-Gaussian and the Gaussian coloured noise. Guo *et al.*[32] investigated a stochastic dynamical system with double singularities driven by the non-Gaussian noise.

In the present paper, the stationary mean value and the normalized variance of the laser intensity for a single mode laser system with a coupling between the coloured multiplicative non-Gaussian and the additive Gaussian white noise are studied.

2. Steady-state probability density of single-mode laser system

The complex field-amplitude E of the cubic model of a singe-mode laser system can be described by the

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following Langevin equation:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = a_0 E - A\left|E\right|^2 E + \tilde{\xi}(t)E + \tilde{p}(t), \tag{1}$$

where a_0 and A are real values and represent the net gain and the self-saturation coefficients, respectively, $\tilde{\xi}(t)$ is the pump noise (multiplicative noise) and $\tilde{p}(t)$ is the quantum noise (additive noise). Via the polar coordinate transform $E = r\,\mathrm{e}^{\mathrm{i}\varphi}$, equation (1) can be transformed into two coupled Langevin equations of field amplitude r and phase φ. By decoupling them, the Langevin equation of field amplitude r can be obtained as[33]

$$\frac{\mathrm{d}r}{\mathrm{d}t} = a_0 r - Ar^3 + \frac{D}{2r} + r\xi(t) + p(t), \tag{2}$$

and the laser intensity ($I = r^2$) is described by

$$\frac{\mathrm{d}I}{\mathrm{d}t} = (2a_0 - AI)I + D + 2I^{1/2}\xi(t) + 2Ip(t), \tag{3}$$

where $\xi(t)$ is the non-Gaussian noise

$$\dot{\xi} = -\frac{1}{\tau}\frac{\mathrm{d}}{\mathrm{d}\xi}V_{\mathrm{p}}(\xi) + \frac{1}{\tau}\varepsilon(t). \tag{4}$$

and $V_{\mathrm{p}}(\xi)$ is given by

$$V_{\mathrm{p}}(\xi) = \frac{Q}{\tau(q-1)}\ln\left[1 + \frac{\tau}{Q}(q-1)\frac{\xi^2}{2}\right], \tag{5}$$

with parameter q related to the Tsallis entropy,[23,34] and τ is the correlation time of $\xi(t)$. The $\varepsilon(t)$ and $q(t)$ are the Gaussian white noise terms,

$$\langle\varepsilon(t)\rangle = \langle p(t)\rangle = 0, \tag{6}$$

$$\langle\varepsilon(t)\varepsilon(t')\rangle = Q\delta(t-t'), \tag{7}$$

$$\langle p(t)p(t')\rangle = D\delta(t-t'), \tag{8}$$

$$\langle p(t)\varepsilon(t')\rangle = \langle\varepsilon(t)p(t')\rangle = \lambda\sqrt{QD}\delta(t-t'), \tag{9}$$

with Q and D the intensities of noise $\varepsilon(t)$ and $p(t)$ respectively, and λ the correlation intensity between $\varepsilon(t)$ and $p(t)$ with $|\lambda| \leq 1$. In the limit of $q \to 1$, process $\xi(t)$ coincides with the Gaussian coloured noise with self-correlation time τ (Ornstein–Uhlenbeck process[35]). If $q \neq 1$, it is a non-Gaussian noise term. Using the path integral approach,[36] we have

$$\dot{\xi} = -\frac{1}{\tau_{\mathrm{eff}}}\xi + \frac{1}{\tau_{\mathrm{eff}}}\varepsilon_{\mathrm{eff}}(t), \tag{10}$$

with

$$\langle\varepsilon_{\mathrm{eff}}(t)\varepsilon_{\mathrm{eff}}(t')\rangle = 2Q_{\mathrm{eff}}\delta(t-t'), \tag{11}$$

$$\langle p(t)\varepsilon_{\mathrm{eff}}(t')\rangle = \langle p(t')\varepsilon_{\mathrm{eff}}(t)\rangle = 2\lambda\sqrt{Q_{\mathrm{eff}}D}\delta(t-t'), \tag{12}$$

the effective noise correlation time is $\tau_{\mathrm{eff}} = 2(2-q)\tau/(5-3q)$, and the associated effective noise intensity is $Q_{\mathrm{eff}} = [2(2-q)/(5-3q)]^2 Q$. It is obvious that $\tau_{\mathrm{eff}} \to \tau$ and $Q_{\mathrm{eff}} \to Q$ when $q \to 1$.

Using the Novikov theorem[37] and the Fox's approach,[38] the Fokker–Planck equation corresponding to Eq. (3) reads

$$\frac{\partial P(I,t)}{\partial t} = L_{\mathrm{FP}}P(I,t), \tag{13}$$

$$L_{\mathrm{FP}} = -\frac{\partial}{\partial I}f(I) + \frac{\partial^2}{\partial I^2}G(I), \tag{14}$$

where the drift coefficient $f(I)$ and the diffusion coefficient $G(I)$ are given respectively by

$$f(I) = 2(a_0 - AI + \frac{Q_{\mathrm{eff}}}{1+2a_0\tau_{\mathrm{eff}}})I + \frac{3\lambda\sqrt{DQ_{\mathrm{eff}}}I^{1/2}}{1+2a_0\tau_{\mathrm{eff}}} + 2D, \tag{15}$$

$$G(I) = 2\frac{Q_{\mathrm{eff}}}{1+2a_0\tau_{\mathrm{eff}}}I^2 + \frac{4\lambda\sqrt{DQ_{\mathrm{eff}}}I^{3/2}}{1+2a_0\tau_{\mathrm{eff}}} + 2DI. \tag{16}$$

It should be pointed out that the above Fokker–Planck equation is valid only for the case of $1 + 2a_0\tau_{\mathrm{eff}} > 0$. We only consider the stationary state, thus the stationary probability distribution function (SPDF) of Eq. (13) can be obtained as

$$P_{\mathrm{st}} = P_{\mathrm{st}}(I) = N\frac{1}{G(I)}\exp\left[\int\frac{f(I)}{G(I)}\right]\mathrm{d}I, \tag{17}$$

where N is the normalization constant.

3. Effects of non-Gaussian and Gaussian noise on single-mode laser system

According to the SPDF expression in Eq. (17), the effects of λ, q and τ on P_{st} can be studied by using the numerical computation. The curves of P_{st} versus I with different values of λ, q and τ are plotted in Figs. 1–3. It is clear that there is a maximum on the P_{st}–I curve. We can also find that λ and q have the same effects on P_{st}, and the peak of P_{st} becomes lower as λ or parameter q increases (Figs. 1 and 2). In other words, λ and q both have negative effects on the stability of the system. However, the peak becomes higher with the increase of correlation time τ of the non-Gaussian noise (Fig. 3). That is, τ has a positive effect on the stability of the system.

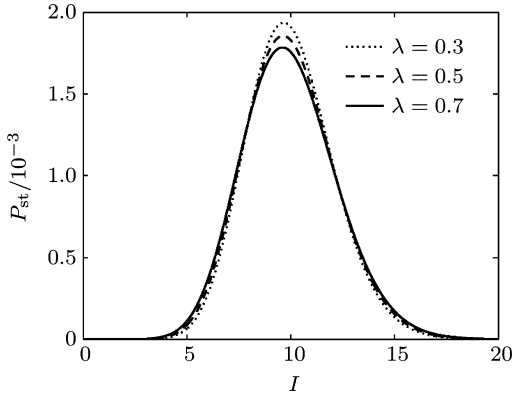


Fig. 1. The $P_{\rm st}$ versus I with different values of λ, $a_0 = 3$, $A = 0.3$, $D = 0.25$, $Q = 0.25$, $q = 0.9$ and $\tau = 0.5$.

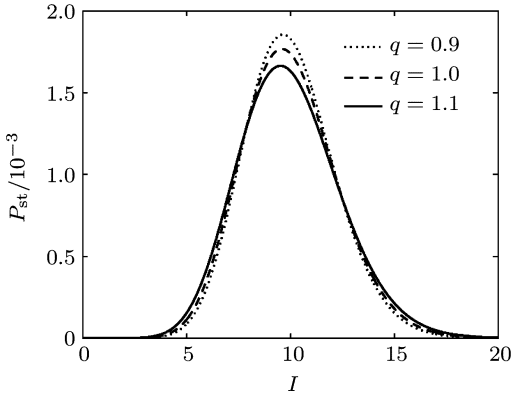


Fig. 2. The $P_{\rm st}$ versus I with different values of q, $a_0 = 3$, $A = 0.3$, $D = 0.25$, $Q = 0.25$, $\tau = 0.5$ and $\lambda = 0.5$.

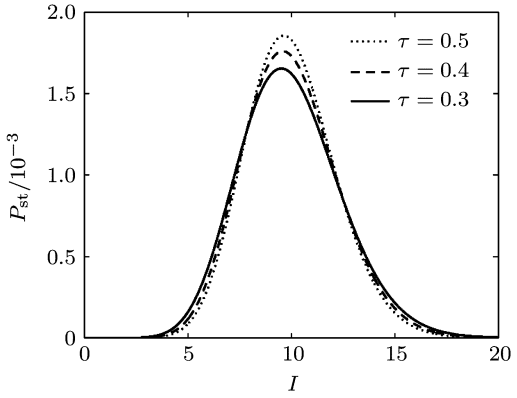


Fig. 3. The $P_{\rm st}$ versus I with different values of τ, $a_0 = 3$, $A = 0.3$, $D = 0.25$, $Q = 0.25$, $q = 0.9$ and $\lambda = 0.5$.

In order to quantitatively investigate the stationary properties of the single-mode laser system, we calculate the stationary mean value of the laser intensity

$$\langle I\rangle_{\rm st} = \int_0^{+\infty} IP_{\rm st}(I){\rm d}I. \tag{18}$$

According to the laser intensity mean value expression in Eq. (18), the curves of $\langle I\rangle_{\rm st}$ versus correlation time τ with different values of correlation intensity λ are plotted in Fig. 4, showing that $\langle I\rangle_{\rm st}$ increases with the correlation time τ increasing. We can also find that at large values of τ, $\langle I\rangle_{\rm st}$ does not change with correlation time τ and correlation intensity λ has an opposite effect on $\langle I\rangle_{\rm st}$, that is, $\langle I\rangle_{\rm st}$ decreases with the increasing λ.

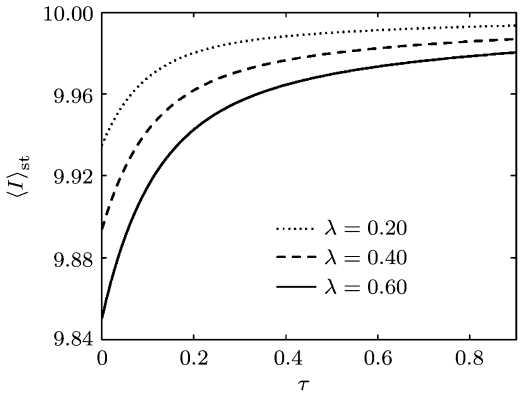


Fig. 4. The $\langle I\rangle_{\rm st}$ versus τ with different values of λ, $a_0 = 3$, $A = 0.3$, $D = 0.25$, $Q = 0.25$ and $q = 0.9$.

The curves of stationary mean value $\langle I\rangle_{\rm st}$ versus correlation time τ with different values of q are plotted in Fig. 5. We can find that $\langle I\rangle_{\rm st}$ decreases with the increase of parameter q. From Figs. 4 and 5, it is found that λ and q have the same effects on $\langle I\rangle_{\rm st}$. Contrasted to λ and q, τ has an opposite effect on $\langle I\rangle_{\rm st}$.

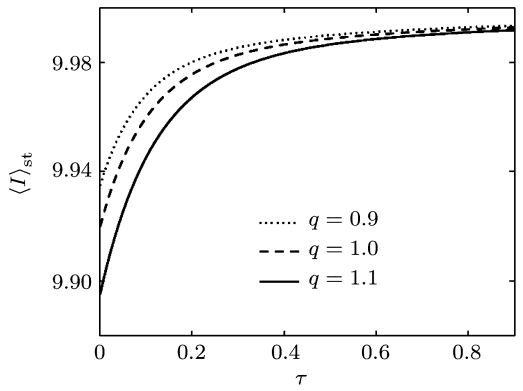


Fig. 5. The $\langle I\rangle_{\rm st}$ versus τ with different values of q, $a_0 = 3$, $A = 0.3$, $D = 0.25$, $Q = 0.25$ and $\lambda = 0.2$.

Next, we calculate the normalized variance of the laser intensity, which can give more information about the fluctuation of the system and is given by

$$\sigma_{\rm VAR}(I) = \frac{\left\langle (I - \langle I\rangle_{\rm st})^2\right\rangle_{\rm st}}{\langle I\rangle_{\rm st}^2}. \tag{19}$$

According to the normalized variance expression in Eq. (19), the curves of $\sigma_{\rm VAR}(I)$ versus correlation time τ with different values of λ and q are plotted in Figs. 6 and 7. In Fig. 6, we find that $\sigma_{\rm VAR}(I)$ decreases with the increase of non-Gaussian correlation time τ

while increases with the increasing λ. In Fig. 7, we find that $\sigma_{\rm VAR}(I)$ increases as q increases. That is, λ and q have the same effects on the laser intensity fluctuation. The larger the value of λ or q, the stronger the fluctuation of laser intensity I. Contrasted to λ and q, τ has an opposite effect on the laser intensity fluctuation. The larger the value of τ, the weaker the fluctuation of I.

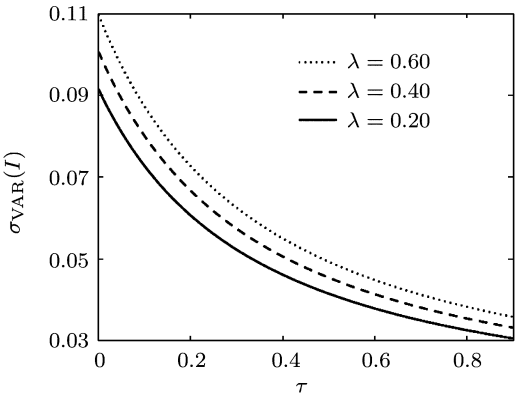


Fig. 6. The $\sigma_{\rm VAR}(I)$ versus τ with different values of λ, $a_0 = 3$, $A = 0.3$, $D = 0.25$, $Q = 0.25$ and $q = 0.9$.

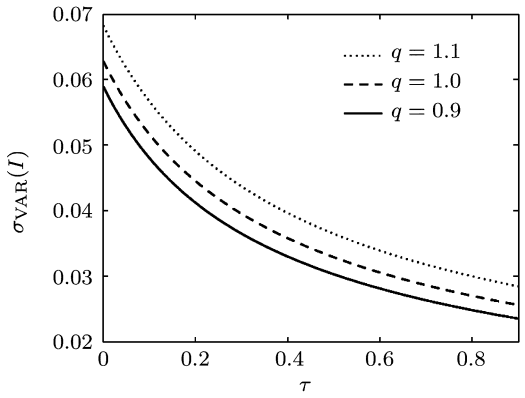


Fig. 7. The $\sigma_{\rm VAR}(I)$ versus τ with different values of q, $a_0 = 3$, $A = 0.3$, $D = 0.25$, $Q = 0.25$ and $\lambda = 0.2$.

4. Conclusion

In this paper, we have investigated the effects of the non-Gaussian and the Gaussian noise on a single mode laser system. The stationary mean value and the normalized variance are numerically calculated under the condition that the SPDF is derived. We find that correlation intensity λ and parameter q have the same effects on the system. The stationary mean value decreases as λ or q increases. The normalized variance increases with the increase of λ or q. Contrasted with λ and q, correlation time τ has an opposite effect on the system. The stationary mean value increases as τ increases. The normalized variance decreases with the increasing τ.

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