Sideband entanglement in collective resonance fluorescence*

Zhang Xue-Hua(张雪华) and Hu Xiang-Ming(胡响明)†

School of Physical Science and Technology, Central China Normal University, Wuhan 430079, *China*

(Received 11 April 2011; revised manuscript received 11 May 2011)

We examine the quantum correlation between the Mollow sidebands in the collective resonance fluorescence from a strongly driven ensemble of two-level atoms. By using the criterion proposed by Shchukin and Vogel, we show that non-Gaussian entanglement exists between the two separated sidebands. The responsible mechanism is traced to the spontaneous parametric process, in which the nonclassical correlation is established. This suggests that the collective resonance fluorescence provides a continuous source for the non-Gaussian entangled light and thus has great potentials for various applications in quantum information and quantum computation.

Keywords: collective resonance fluorescence, non-Gaussian entanglement, spontaneous parametric interaction

PACS: 42.50.Dv, 03.65.Ud, 03.67.Bg, 32.50.+d **DOI:** 10.1088/1674-1056/20/11/114205

1. Introduction

Recently much interest has been focused on searching for entangled sources of light.[1−3] A great deal of progress has been made in the creation of continuous variable entanglement for the Gaussian sources.[4−8] Among various mechanisms is the four-wave mixing, by which optical fields are parametrically amplified and become entangled with each other.[9−14] Most recently, it has become clear that non-Gaussian sources (non-Gaussian states and non-Gaussian operations) and their applications are not only advantageous but also at times necessary for quantum information tasks.[15] The non-Gaussian states have non-Gaussian photon distribution. A diverse range of non-Gaussian states of light have been successfully generated, including sub-Poissonian states,[16] Fock states,[17,18] the superposition of Fock states,[19] single-photon subtracted or added states,[20,21] and squeezed two-photon states.[22] The non-Gaussian operations are required for the entanglement distillation[23,24] and their use along with the non-Gaussian states may improve quantum teleportation[25,26] and quantum cloning.[27] In particular, non-Gaussianity is necessary for universal quantum computation with continuous variables[28] and the proof of quantum nonlocality through the violation of Bell's inequality.[29] These advantages stimulate the experimental de-Gaussification of Gaussian states.[30,31]

Resonance fluorescence has been one of the most important subjects in quantum optics and laser physics because of its pioneering role in exploring the quantum nature of light[32,33] and its potential applications in various tasks.[30,31] The resonance fluorescence has the non-Gaussian photon statistics with respect to the photon number and is a typical non-Gaussian source. The non-Gaussianity is reflected by the non-zero odd-order fluctuation in the field amplitudes. The third-order correlations have been studied with an emphasis on their contributions to the intensity correlations.[34,35] Most recently, there has been an increasing interest in their non-Gaussianity. Gerber *et al.*[36] performed an experimental investigation of the amplitude-intensity correlation. Castro-Beltrán[37] showed that for the resonance fluorescence from two-level atoms, the third-order fluctuation is responsible for the non-Gaussianity of the fluorescence. In particular, Shchukin and Vogel (SV)[38] proposed a hierarchy of conditions to identify the inseparability of all orders, but there has been no specific application of such criteria to the resonance fluorescence and other optical systems. Although Grünwald and Vogel[39] predicted the entanglement of the fluores-

*Project supported by the National Natural Science Foundation of China (Grant No. 11074086) and the Natural Science Foundation of Hubei Province, China (Grant No. 2010CDA075).

†Corresponding author. E-mail: xmhu@phy.ccnu.edu.cn

© 2011 Chinese Physical Society and IOP Publishing Ltd

cent radiation propagating in two different directions, they used only the second-order criteria, which was met by locating atoms regularly and fixing the phase factors associated with different atoms. It is a big challenge to regularly locate many atoms. As shown recently, the second-order criteria are sufficient and necessary for judging the entanglement of the Gaussian states, and are sufficient but not necessary for the non-Gaussian states. In other words, the entanglement of the non-Gaussian states can sometimes be reflected by the second-order moments but sometimes cannot. For the latter, a higher-order criterion is required as shown in Ref. [15], where such entanglement is referred to as the genuine non-Gaussian entanglement.

In this paper, we use a higher-order criterion to show that the non-Gaussian entanglement exists between the Mollow sidebands of the collective resonance fluorescence. The Mollow sidebands are new spectral components in driven atomic and molecular systems. In particular, for two-level atoms and at high laser intensities, the fluorescence spectrum consists of three well-separated components,[40,41] the incident frequency ω_0 and two sidebands as new components at $\omega_0 \pm \bar{\Omega}$, with $\bar{\Omega}$ being the generalized Rabi frequency and much larger than their width γ. We can use filters centred at any one of these components and having a width $\Delta\omega$ such that $\gamma \ll \Delta\omega \ll \bar{\Omega}$. Aspect *et al.*[42] demonstrated the time correlations between the two sidebands. The two-mode squeezing of the sidebands was predicted.[43,44] An experimental progress was made in the observation of the squeezing in the resonance fluorescence for a sample of many atoms.[45] When a sample of N excited atoms is confined in a region of the order of a typical optical wavelength, the fluorescence intensity is proportional to N^2.[46,47] Essentially, the sidebands are generated via the spontaneous parametric interaction, via which the nonclassical correlations are formed. The purpose of the present paper is to show that the spontaneous parametric process of collective atoms causes the non-Gaussian entanglement.

2. Model and equation

Our analysis begins with the separation of the fluorescence sidebands for the strong excitation. A collection of N two-level atoms are driven by an external field. By $|1\rangle$ and $|2\rangle$ we denote the ground and the excited states of the atoms, respectively. The Hamiltonian of the system in the interaction picture takes the form $H = H_0 + H_1$, and

$$\begin{aligned} H_0 &= \frac{\hbar}{2}\Delta J_3 + \frac{\hbar}{2}\Omega(J_{12} + J_{21}) + \sum_\lambda \hbar\delta_\lambda a_\lambda^\dagger a_\lambda, \\ H_1 &= \mathrm{i}\hbar \sum_\lambda g_\lambda (a_\lambda^\dagger J_{12} - a_\lambda J_{21}), \end{aligned} \tag{1}$$

where $J_{ij} = \sum_{\mu=1}^{N} |i^\mu\rangle\langle j^\mu|$ $(i, j = 1, 2)$ are the atomic spin-flip $(i \neq j)$ and the projection $(i = j)$ operators for the collective atoms, $J_3 = J_{22} - J_{11}$, a_λ and $a_\lambda^\dagger$ are the radiation field annihilation and creation operators for mode λ, $\hbar$ is the reduced Planck constant, $\Delta = \omega_\mathrm{a} - \omega_0$, $\delta_\lambda = \omega_\lambda - \omega_0$, with ω_a, ω_0 and ω_λ being the atomic resonant, the applied field and the radiation field frequencies, respectively, Ω is the Rabi frequency and g_λ is the atom–radiation coupling constant.

It is convenient to express atomic operator J_{ij} in the Schwinger (boson) representation.[48] We write $J_{ij} = b_i^\dagger b_j$ $(i, j = 1, 2)$, where $b_i^\dagger$ and b_j obey the bosonic commutation relations and describe the creation and the annihilation of atoms in state $|i\rangle$ with the additional constrain of $b_1^\dagger b_1 + b_2^\dagger b_2 = N$. The atom–driving field part of Hamiltonian H_0 can be diagonalized using the canonical transformation[49]

$$\begin{aligned} b_1 &= q_1 \cos\phi + q_2 \sin\phi, \\ b_2 &= -q_1 \sin\phi + q_2 \cos\phi, \end{aligned} \tag{2}$$

where $\tan(2\phi) = \Omega/\Delta$. This gives the Hamiltonian in terms of the dressed states

$$H_0 = \frac{1}{2}\hbar\bar{\Omega}R_3 + \sum_\lambda \hbar\delta_\lambda a_\lambda^\dagger a_\lambda, \tag{3}$$

where $R_{ij} = q_i^\dagger q_j$ $(i, j = 1, 2)$ are the atomic spin-flip $(i \neq j)$ and the projection $(i = j)$ operators for the collective dressed atoms, $R_3 = R_{22} - R_{11}$ and $\bar{\Omega} = \sqrt{\Omega^2 + \Delta^2}$ is the generalized Rabi frequency. In the dressed state representation, $q_i^\dagger$ and q_i obey the bosonic commutation relations and represent the creation and the annihilation of atoms in state $|\tilde{i}\rangle$, respectively. We define the second interaction picture as $\tilde{H}_1 = U^\dagger(t) H_1 U(t)$, where $U(t) = \exp(-\mathrm{i} H_0 t/\hbar)$. The collective dressed atomic operators and the field operators in this picture exhibit the following time dependence:

$$\begin{aligned} \tilde{R}_{21}(t) &= R_{21}\exp(\mathrm{i}\,\bar{\Omega}t), \\ \tilde{R}_{12}(t) &= R_{12}\exp(-\mathrm{i}\,\bar{\Omega}t), \\ \tilde{a}_\lambda(t) &= a_\lambda \exp(-\mathrm{i}\,\delta_\lambda t), \\ \tilde{a}_\lambda^\dagger(t) &= a_\lambda^\dagger \exp(\mathrm{i}\,\delta_\lambda t). \end{aligned} \tag{4}$$

In the following, we drop the tilde on the interaction picture operators without confusion. In the second interaction picture, the interaction Hamiltonian $\tilde{H}_1$ takes the form

$$\begin{aligned}\tilde{H}_1 =& \mathrm{i}\hbar\sum_{\lambda} g_\lambda a_\lambda^\dagger[\cos\phi\sin\phi R_3\,\mathrm{e}^{\mathrm{i}\,\delta_\lambda t}\\ &+\cos^2\phi R_{12}\,\mathrm{e}^{\mathrm{i}(\delta_\lambda-\bar{\Omega})t}\\ &-\sin^2\phi R_{21}\,\mathrm{e}^{\mathrm{i}(\delta_\lambda+\bar{\Omega})t}]+\mathrm{H.c.},\end{aligned}\tag{5}$$

where H.c. denotes the Hermitian conjugate. Using the density of states of the vacuum, we derive the master equation for density operator ρ in terms of the dressed states and in the Born–Markov approximation[46,47]

$$\begin{aligned}\dot{\rho} =& \frac{A_0}{2}(R_3\rho R_3-\rho R_3^2)\\ &+\frac{A_1}{2}(R_{12}\rho R_{21}-\rho R_{21}R_{12})\\ &+\frac{A_2}{2}(R_{21}\rho R_{12}-\rho R_{12}R_{21})+\mathrm{H.c.},\end{aligned}\tag{6}$$

where $A_0=\gamma(\omega_0)\cos^2\phi\sin^2\phi$, $A_1=\gamma(\omega_0+\bar{\Omega})\cos^4\phi$, $A_2=\gamma(\omega_0-\bar{\Omega})\sin^4\phi$, and $\gamma(\nu)=\wp^2\nu^3/3\pi\varepsilon_0\hbar c^3$ is the decay rate that depends on transition frequency ν. The $\wp$ is the dipole moment of the atom, ε_0 is the dielectric constant, c is the speed of light in vacuum. Since $\omega_0\gg\bar{\Omega}$, we can take $\gamma(\omega_0\pm\bar{\Omega})=\gamma(\omega_0)=\gamma$. In deriving Eq. (6), we have used the secular approximation for the strong applied field with $\bar{\Omega}\gg N\gamma$. That is, we have neglected the fast oscillating terms with frequencies $\bar{\Omega}$ and $2\bar{\Omega}$. Physically, it means that the three Mollow spectral components are well separated and that the overlap between them is negligible.

3. Quantum correlation

Now we show the quantum correlation between the two sidebands. Recently, Shchukin and Vogel[38] presented a hierarchy of necessary and sufficient conditions by using the negativity of the partial transposition of the quantum state of a two-party system. The criteria are constructed by arranging a matrix $\boldsymbol{M}$ with elements

$$M_{\mu\nu}=\langle a_1^{\dagger m_1}a_1^{n_1}a_1^{\dagger k_1}a_1^{l_1}a_2^{\dagger l_2}a_2^{k_2}a_2^{\dagger m_2}a_2^{n_2}\rangle,\tag{7}$$

where exponents $k=(k_1,k_2)$ and $l=(l_1,l_2)$ correspond to row index μ, while $m=(m_1,m_2)$ and $n=(n_1,n_2)$ to column index ν according to a prescribed ordering.[38] The SV criteria then indicate that a quantum state has a positive partial transpose if and only if all principal minors of $\boldsymbol{M}$ are non-negative. The principal minors are the determinants of all submatrices that are constructed by picking out the same rows and columns from $\boldsymbol{M}$, i.e., whenever a row j is chosen, the corresponding column j is included as well. To show the existence of the entanglement, it is sufficient to prove that just one of the principal minors of $\boldsymbol{M}$ is negative.

First, we examine the second-order correlations for the present fluorescence sidebands. Up to the second order, $\boldsymbol{M}$ takes the form

$$\boldsymbol{M}_2=\begin{pmatrix}1 & \langle a_1\rangle & \langle a_1^\dagger\rangle & \langle a_2^\dagger\rangle & \langle a_2\rangle\\ \langle a_1^\dagger\rangle & \langle a_1^\dagger a_1\rangle & \langle a_1^{\dagger2}\rangle & \langle a_1^\dagger a_2^\dagger\rangle & \langle a_1^\dagger a_2\rangle\\ \langle a_1\rangle & \langle a_1^2\rangle & \langle a_1a_1^\dagger\rangle & \langle a_1a_2^\dagger\rangle & \langle a_1a_2\rangle\\ \langle a_2\rangle & \langle a_1a_2\rangle & \langle a_1^\dagger a_2\rangle & \langle a_2^\dagger a_2\rangle & \langle a_2^2\rangle\\ \langle a_2^\dagger\rangle & \langle a_1a_2^\dagger\rangle & \langle a_1^\dagger a_2^\dagger\rangle & \langle a_2^{\dagger2}\rangle & \langle a_2a_2^\dagger\rangle\end{pmatrix}.\tag{8}$$

Several popular second-order separability conditions[50,51] are based on the combination of the determinants of minors of $\boldsymbol{M}_2$. As an example, Simon's criterion[51] simply takes the SV form $\det\boldsymbol{M}_2\geq 0$. A violation of the inequality indicates the two-mode entanglement. However, for the present case, we find that the determinants of all second-order submatrices of $\boldsymbol{M}_2$ are non-negative. It can be seen that the second-order criteria fail to identify the non-Gaussian entanglement. Although the second-order criteria are sometimes applicable to the non-Gaussian states,[39,50,51] but they do not hold for the present case. The genuine non-Gaussian entanglement should be identified by using a higher-order criterion as shown in Ref. [15].

Then we turn to the forth-order criteria. To the forth order, two subdeterminants are chosen,[38]

$$D_1=\begin{vmatrix}1 & \langle a_1a_2\rangle\\ \langle a_1^\dagger a_2^\dagger\rangle & \langle a_1^\dagger a_1a_2a_2^\dagger\rangle\end{vmatrix}\tag{9}$$

and $D_2=D_1(a_j\to a_j^\dagger, a_j^\dagger\to a_j)$. When $D_1<0$ or $D_2<0$, we have the bipartite non-Gaussian entanglement.

The above criteria can be generalized to the fluorescence sidebands. It can be seen from the interaction Hamiltonian (Eq. (5)) that the higher and the lower sidebands $\omega_0\pm\bar{\Omega}$ and the central component ω_0 of the Mollow triplet are related to atomic operators R_{12}, R_{21} and R_3, respectively. For the present case, instead of the annihilation operators, we introduce the photon detection operators[52,53]

$$d_1=g_1\cos^2\phi R_{12},\quad d_2=-g_2\sin^2\phi R_{21},\tag{10}$$

which correspond to the detection of radiation from the higher and the lower sidebands, respectively. Here we have assumed that $g_{1,2} = g_{\pm\Omega}$. When we substitute photon detection operators $d_{1,2}$ for $a_{1,2}$ and $d_{1,2}^{\dagger}$ for $a_{1,2}^{\dagger}$ in the above forth-order criteria, we need to treat the dressed atomic operators R_{12}, R_{21} and R_3. It is convenient to use the dressed state ket vector $|n\rangle = |N-n, n\rangle$ to denote a symmetrized N-atom state, in which $N-n$ atoms are in the lower dressed state $|\tilde{1}\rangle$ and n atoms are excited to the upper dressed state $|\tilde{2}\rangle$. According to the harmonic oscillator property of the Schwinger bosons,[48] we have

$$\begin{aligned} R_{12}|n\rangle &= \sqrt{n(N-n+1)}|n-1\rangle, \\ R_{21}|n\rangle &= \sqrt{(n+1)(N-n)}|n+1\rangle, \\ R_3|n\rangle &= (2n-N)|n\rangle. \end{aligned} \tag{11}$$

By using these rules and Eq. (6), we derive the equation for the diagonal elements of the density operator, $P_n = \langle n|\rho|n\rangle$, as

$$\begin{aligned} \dot{P}_n &= n(N-n+1)(A_2 P_{n-1} - A_1 P_n) \\ &\quad - (n+1)(N-n)(A_2 P_n - A_1 P_{n+1}). \end{aligned} \tag{12}$$

In the steady state, the off-diagonal elements of the density matrix vanish ($\dot{\rho} = 0$) and the diagonal elements are calculated according to the detailed balance in Eq. (12) as

$$P_n = P_0 \xi^n, \tag{13}$$

where $\xi = A_2/A_1$ is the ratio of the rate of population transfer between the dressed states and $P_0 = \frac{\xi-1}{\xi^{N+1}-1}$ is the normalization factor. Using atomic distribution function P_n, we derive the k-order moment as

$$\langle n^k \rangle = P_0 \sum_{n=0}^{N} \xi^n n^k = P_0 \left(\xi \frac{\partial}{\partial \xi}\right)^k P_0^{-1}. \tag{14}$$

In Fig. 1 we plot the two minors D_1 (solid lines) and D_2 (dotted lines) versus the normalized detuning Δ/Ω for small numbers of atoms: (a) $N = 1, 2$, (b) $N = 3, 4, 5$, (c) $N = 10$, 15, 20. For $N = 1, 2$, the minors are always positive. For $N \geq 3$, we have negative values for D_1 and D_2 when the driving field is tuned off resonance with the atomic transition. For $N \gg 1$, we can obtain conditions for negative value of the determinant. Approximately, we have the simple expression

$$D_1 \approx -\frac{3g^4 N^3 \sin^4(2\phi)}{8(\xi-1)}, \tag{15}$$

together with $D_2 = D_1(\xi \to \xi^{-1})$, where we have assumed $g_{12} = g$. Equation (15) indicates the entanglement for both $\Delta < 0$ and $\Delta > 0$. In this case, the entanglement exists between the sidebands so long as the driving field is off resonance with the atomic transition ($\Delta \neq 0$, i.e., $\tan^4\phi \neq 1$).

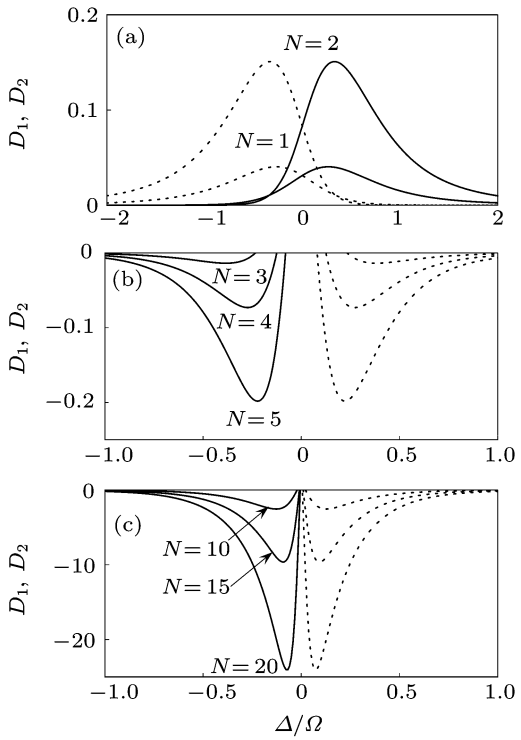


Fig. 1. Four-order minors D_1 (solid lines) and D_2 (dotted lines) versus the normalized detuning Δ/Ω for (a) $N = 1, 2$, (b) $N = 3, 4, 5$, and (c) $N = 10$, 15, 20.

It is seen from Eq. (7) that atomic spin-flip operator R_{12} (R_{21}) corresponds to the absorption of the higher (lower) sideband photons and the creation of the lower (higher) sideband photons. The two sidebands have a spontaneous parametric interaction.[54] This has also been shown by the Hamiltonian (Eq. (5)). The spontaneous parametric process is the intrinsic mechanism for the non-Gaussian entanglement.[30,31] The sideband entanglement occurs when

$$|\langle d_1 d_2 \rangle|^2 > \langle d_1^{\dagger} d_1 d_2 d_2^{\dagger} \rangle, \tag{16}$$

or

$$|\langle d_1 d_2 \rangle|^2 > \langle d_1 d_1^{\dagger} d_2^{\dagger} d_2 \rangle. \tag{17}$$

These conditions are remarkably different from those for the four-wave mixing, where the sidebands are in the stimulated amplification, two Gaussian fields a_1

and a_2 are formed, and the entanglement happens when[9−14]

$$|\langle a_1 a_2\rangle|^2 > \langle a_1^\dagger a_1\rangle\langle a_2^\dagger a_2\rangle. \tag{18}$$

The right-hand side of inequality (18) is the product of the sideband intensities instead of the intensity correlations as in inequalities (16) and (17). The spontaneous parametric process occurs for different dressed state population case and disappears for the equal case ($\tan\phi = 1$). Since the collective resonance fluorescence provides an efficient non-Gaussian entangled source of light, it is no longer necessary to transform the entangled Gaussian light into the non-Gaussian source in various applications in quantum communications and computations. The resonance fluorescence, which acts as a typical source of entangled light, can provide a more convenient alternative for future complicated quantum networks.

4. Discussion and conclusion

The collective phase dephasing can be considered as an independent effect, as shown in Ref. [47]. For the present case, we assume that the dipolar interaction between the atoms[55] is negligibly weak. The phase damping can be described by introducing phenomenologically the following term into the master equation

$$\dot{\rho} = \frac{\gamma_{\rm p}}{2}(2J_3\rho J_3 - J_3^2\rho - \rho J_3^2), \tag{19}$$

where $\gamma_{\rm p}$ is the phase damping rate. By transforming to the dressed state representation, we rewrite the master equation with new parameters $A_0(\gamma_{\rm p}) = \gamma\cos^2\phi\sin^2\phi + \gamma_{\rm p}\cos^2(2\phi)$, $A_1(\gamma_{\rm p}) = \gamma\cos^4\phi + \gamma_{\rm p}\sin^2(2\phi)$, and $A_2(\gamma_{\rm p}) = \gamma\sin^4\phi + \gamma_{\rm p}\sin^2(2\phi)$. The phase damping rate $\gamma_{\rm p}$ is contained in parameter $\xi = \frac{A_2(\gamma_{\rm p})}{A_1(\gamma_{\rm p})}$. Note that we have $\xi(\gamma_{\rm p}\neq 0) < \xi(\gamma_{\rm p} = 0)$ for $\Delta < 0$ and $\xi^{-1}(\gamma_{\rm p}\neq 0) < \xi^{-1}(\gamma_{\rm p} = 0)$ for $\Delta > 0$. It is unexpected that the phase damping does not spoil but enhances the quantum correlation. Essentially, the phase damping in the bare atomic state picture gives the population transfer in the dressed state picture. This is seen by noting that $\gamma_{\rm p}\sin^2(2\phi)$ appears in A_1 and A_2 simultaneously. In fact, a similar effect was once predicted by John and Quang,[47] who showed that the atomic sub-Poissonian distribution in a confined photonic system was improved due to such phase damping. Finally, in order to avoid the Doppler broadening in experiments, an ensemble of sub-Doppler cooled atoms or ultracold atoms can be employed. For such cold atoms, the Doppler broadening is substantially smaller than the natural linewidths and the quantum correlations can be well preserved.

In conclusion, we have examined the sideband quantum correlation between the two well-separated sidebands in the collective resonance fluorescence from strongly driven two-level atoms. It is shown that it is possible to obtain the non-Gaussian entanglement via the spontaneous parametric process when there are more than three atoms in the ensemble. The collective resonance fluorescence is a continuous source for non-Gaussian entangled light and has various applications in quantum information and computation.

References

[1] Braunstein S L and van Loock P 2005 *Rev. Mod. Phys.* **77** 513

[2] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 *Rev. Mod. Phys.* **81** 865

[3] Guhne O and Toth G 2009 *Phys. Rep.* **474** 1

[4] Ou Z Y, Pereira S F, Kimble H J and Peng K C 1992 *Phys. Rev. Lett.* **68** 3663

[5] Aoki T, Takei N, Yonezawa H, Wakui K, Hiraoka T, Furusawa A and van Loock P 2003 *Phys. Rev. Lett.* **91** 080404

[6] Su X L , Tan A H, Jia X J, Zhang J, Xie C D and Peng K C 2007 *Phys. Rev. Lett.* **98** 070502

[7] Josse V, Dantan A, Vernac L, Bramati A, Pinard M and Giacobino E 2003 *Phys. Rev. Lett.* **91** 103601

[8] Josse V, Dantan A, Bramati A, Pinard M and Giacobino E 2004 *Phys. Rev. Lett.* **92** 123601

[9] Wu Y and Yang X X 2004 *Phys. Rev.* A **70** 053818

[10] Ikram M, Li G X and Zubairy M S 2007 *Phys. Rev.* A **76** 042317

[11] Zhou L, Guo H Y and Patnaik A K 2009 *Phys. Rev.* A **79** 062102

[12] Lü X Y, Huang P, Yang W X and Yang X X 2009 *Phys. Rev.* A **80** 032303

[13] Cheng G L, Hu X M, Zhong W X and Li Q 2008 *Phys. Rev.* A **78** 033811

[14] Zhang X and Hu X M 2010 *Phys. Rev.* A **81** 013811

[15] Gomes R M, Salles A, Toscano F, Souto Ribeiro P H and Walborn S P 2009 *Proc. Natl. Acad. Sci. USA* **106** 21517

[16] Short R and Mandel L 1983 *Phys. Rev. Lett.* **51** 384

[17] Lvovsky A I, Hansen H, Aichele T, Benson O, Mlynek J and Schiller S 2001 *Phys. Rev. Lett.* **87** 050402

[18] Ourjoumtsev A, Tualle-Brouri R and Grangier P 2006 *Phys. Rev. Lett.* **96** 213601

[19] Lvovsky A I and Mlynek J 2002 *Phys. Rev. Lett.* **88** 250401

[20] Ourjoumtsev A, Tualle-Brouri R, Laurat J and Grangier P 2006 *Science* **312** 83

[21] Neergaard-Nielsen J S, Melholt Nielsen M, Hettich C, Molmer K and Polzik E S 2006 *Phys. Rev. Lett.* **97** 083604

[22] Ourjoumtsev A, Jeong H, Tualle-Brouri R and Grangier P 2007 *Nature* **448** 784

[23] Dong R, Lassen M, Heersink J, Marquardt C, Filip R, Leuchs G and Andersen U L 2008 *Nat. Phys.* **4** 919
[24] Eisert J, Scheel S and Plenio M B 2002 *Phys. Rev. Lett.* **89** 137903
[25] Opatrny T, Kurizki G and Welsch D G 2000 *Phys. Rev.* A **61** 032302
[26] Dell'Anno F, de Siena S, Albano L and Illuminati F 2007 *Phys. Rev.* A **76** 022301
[27] Cerf N J, Krüger O, Navez P, Werner R F and Wolf M M 2005 *Phys. Rev. Lett.* **95** 070501
[28] Bartlett S D, Sanders B C, Braunstein S L and Nemoto K 2002 *Phys. Rev. Lett.* **88** 097904
[29] Yarnall T, Abouraddy A F, Saleh B E A and Teich M C 2007 *Phys. Rev. Lett.* **99** 170408
[30] Ourjoumtsev A, Tualle-Brouri R, Laurat R and Grangier P 2006 *Science* **312** 83
[31] Ourjoumtsev A, Ferreyrol F, Tualle-Brouri R and Grangier P 2009 *Nat. Phys.* **5** 189
[32] Walls D F and Milburn G J 1994 *Quantum Optics* (Berlin: Springer)
[33] Scully M O and Zubairy M S 1997 *Quantum Optics* (Cambridge: Cambridge University Press)
[34] Carmichael H J 1985 *Phys. Rev. Lett.* **55** 2790
[35] Vogel W 1991 *Phys. Rev. Lett.* **67** 2450
[36] Gerber S, Rotter D, Slodièka L, Eschner J, Carmichael H J and Blatt R 2009 *Phys. Rev. Lett.* **102** 183601
[37] Castro-Beltrán H M 2010 *Opt. Commun.* **283** 4680
[38] Shchukin E and Vogel W 2005 *Phys. Rev. Lett.* **95** 230502
[39] Grünwald P and Vogel W 2010 *Phys. Rev. Lett.* **104** 233602
[40] Mollow B R 1969 *Phys. Rev.* **188** 1969
[41] Wu F Y, Grove R E and Ezekiel S 1975 *Phys. Rev. Lett.* **35** 1426
[42] Aspect A, Roger G, Reynaud S, Dalibard J and Cohen-Tannoudji C 1980 *Phys. Rev. Lett.* **45** 617
[43] Zhou P and Swain S 1999 *Phys. Rev.* A **5**9 841
[44] Hu X M and Zhang X H 2008 *Phys. Rev.* A **77** 063809
[45] Lu Z H, Bali S and Thomas J E 1998 *Phys. Rev. Lett.* **81** 3635
[46] Peng J S and Li G X 1998 *Introduction to Modern Quantum Optics* (Singapore: World Scientific)
[47] John S and Quang T 1997 *Phys. Rev. Lett.* **78** 1888
[48] Schwinger J 1965 *Quantum Theory of Angular Momentum* (New York: Academic Press) p. 229
[49] Cohen-Tannoudji C, Dupont-Roc J and Grynberg G 1992 *Atom-Photon Interactions* (New York: Wiley)
[50] Duan L M, Giedke G, Cirac J I and Zoller P 2000 *Phys. Rev. Lett.* **84** 2722
[51] Simon R 2000 *Phys. Rev. Lett.* **84** 2726
[52] Jayarao A S, Lawande S V and D'Souza R 1990 *Phys. Rev.* A **42** 3044
[53] Joshi A and Puri P R 1991 *Phys. Rev.* A **43** 6428
[54] Boyd R W 1992 *Nonlinear Optics* (Boston: Academic)
[55] Gross M and Haroche S 1982 *Phys. Rep.* **93** 301