Kurtosis parameters of super Lorentz–Gauss beams through a paraxial and real *ABCD* optical system*

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Based on the propagation equation of higher-order intensity moments, analytical propagation expressions for the kurtosis parameters of a super Lorentz–Gauss (SLG) SLG_{01} beam through a paraxial and real *ABCD* optical system are derived. By replacing the parameters in the expressions of the kurtosis parameters of the SLG_{01} beam, the kurtosis parameters of the SLG_{10} and SLG_{11} beams through a paraxial and real *ABCD* optical system can be easily obtained. The kurtosis parameters of an SLG_{01} beam through a paraxial and real *ABCD* optical system depend on two ratios. One is the ratio of the transfer matrix element B to the product of the transfer matrix element A and the diffraction-free range of the super-Lorentzian part. The other is the ratio of the width parameter of the super-Lorentzian part to the waist of the Gaussian part. As a numerical example, the properties of the kurtosis parameters of an SLG_{01} beam propagating in free space are illustrated. The influences of different parameters on the kurtosis parameters of an SLG_{01} beam are analysed in detail.

Keywords: super Lorentz–Gauss beam, kurtosis parameter, intensity moment

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1. Introduction

Because the angular spreading of a Lorentz–Gaussian distribution is higher than that of a Gaussian distribution, Lorentz–Gauss beams provide more appropriate models than Gaussian beams to describe the radiation emitted by a diode laser.[1,2] The properties of Lorentz–Gauss beams have been extensively investigated.[3−13] The researches show that Lorentz–Gauss beams are only applicable to the description of the fundamental mode emitted by a single mode diode laser. Therefore, an orthogonal beam family of super Lorentz–Gauss (SLG) modes has been introduced to describe the high-order mode beams emitted by the multi-mode diode lasers.[14] The SLG_{00} beam is just the Lorentz–Gauss beam. Based on the convolution theorem of the Fourier transform, analytical propagation formulae of the SLG_{01} and SLG_{11} beams through a paraxial *ABCD* optical system have been derived.[14] The beam propagation factors of the SLG_{01} and SLG_{11} beams denote that SLG beams are more appropriate than Hermite-Gaussian (HG) beams to describe the higher-order modes emitted by the multi-mode diode lasers.[14] Far-field properties of the SLG_{01} and SLG_{11} beams have also been investigated from the vectorial structure.[15] Based on the extended Huygens-Fresnel integral and the HG expansion of a Lorentzian function, analytical formulae for the average intensities and the effective beam sizes of the SLG_{01} and SLG_{11} beams have been presented in turbulent atmosphere.[16] The properties of the mutual coherence functions and the spatial complex degrees of coherence for the partially coherent SLG_{01} and SLG_{11} beams through a paraxial and real *ABCD* optical system have been examined.[17] Based on the vectorial Rayleigh–Sommerfeld integral formulae, nonparaxial propagation properties of an SLG_{01} beam have been evaluated in free space.[18]

The fourth-order moment has been used to define the so-called kurtosis parameter. As the kurtosis parameter is employed to describe the flatness (or sharpness) degree of the beams, it is an important parameter to valuate the beam propagation. The kurtosis parameters of various laser beams have been widely investigated.[19−27] However, to our best knowledge, research on the kurtosis parameters of SLG beams has not been reported elsewhere. In the rest of this paper, therefore, we investigate the kurtosis parameters of SLG beams through a paraxial and real *ABCD* optical

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system. Here, we mainly pay attention to the simple case, where only the SLG_{01}, SLG_{10} and SLG_{11} beams are considered. As a numerical example, the properties of the kurtosis parameters of an SLG_{01} beam propagating in free space will be illustrated.

2. The kurtosis parameters of an SLG_{01} beam through a paraxial and real *ABCD* optical system

In the Cartesian coordinate system, the z-axis is taken to be the propagation axis. The SLG beams in the source plane $z = 0$ take the form of

$$E(x_0, y_0, 0) = C^{\mathrm{mm}'} E_{\mathrm{m}}(x_0, 0) E_{\mathrm{m}'}(y_0, 0), \quad (1)$$

with $E_{\mathrm{m}}(x_0, 0)$ and $E_{\mathrm{m}'}(y_0, 0)$ given by

$$E_{2p}(j_0, 0) = \exp\Big(\frac{\mathrm{i}\, k j_0^2}{2q_0}\Big)\Big[j_0^{2p} + \sum_{l=1}^{p-1} c_{(2p)(2l)} j_0^{2l}\Big] \times \prod_{s=0}^{M} \frac{1}{j_0^2 + w_{sj}^2}, \quad (2)$$

$$E_{2p+1}(j_0, 0) = \exp\Big(\frac{\mathrm{i}\, k j_0^2}{2q_0}\Big) \times \Big[j_0^{2p+1} + \sum_{l=1}^{p-1} c_{(2p+1)(2l+1)} j_0^{2l+1}\Big] \times \prod_{s=0}^{M} \frac{1}{j_0^2 + w_{sj}^2}, \quad (3)$$

where $C^{\mathrm{mm}'}$ is a constant that makes the expressions of the field unitless; $j_0 = x_0$ or y_0 and $j = x$ or y (hereafter); $k = 2\pi/\lambda$ with λ being the optical wavelength; $q_0 = -\mathrm{i}\, k w_0^2/2$ is the q parameter in the source plane; w_0 is the waist of the Gaussian part; M is an integer; and $p = 0, 1, \ldots, M-1$. The coefficients $c_{(2p)(2l)}$ and $c_{(2p+1)(2l+1)}$ are calculated by[4]

$$c_{(2p)(2l)} = -\frac{R(2p)}{R(2l)},\ c_{(2p+1)(2l+1)} = -\frac{R(2p+1)}{R(2l+1)}, \quad (4)$$

where $R(2p)$, $R(2l)$, $R(2p+1)$ and $R(2l+1)$ are given by

$$R(2u+t) = 2\pi(-1)^{u+1} \times \sum_{s=0}^{M-1} \lim_{j_0 \to w_{sj}} \frac{\partial}{\partial j_0}\Big[\frac{j_0^{2u+t}}{(j_0^+ w_{sj})^2} \prod_{h=0, h\neq s}^{M-1} \frac{j_0^t}{j_0^2 + w_{hj}^2}\Big], \quad (u = p, \text{ or } l, t = 0 \text{ or } 1). \quad (5)$$

Here, w_{sj} is the width parameter of the s-th Lorentzian part in the j-direction. In the simple case of M=1, there are four mutually orthogonal SLG beams

$$E^{00}(x_0, y_0, 0) = E^0(x_0, 0) E^0(y_0, 0), \quad (6)$$

$$E^{01}(x_0, y_0, 0) = C_{y_0} E^0(x_0, 0) E^1(y_0, 0), \quad (7)$$

$$E^{10}(x_0, y_0, 0) = C_{x_0} E^1(x_0, 0) E^0(y_0, 0), \quad (8)$$

$$E^{11}(x_0, y_0, 0) = C_{x_0} C_{y_0} E^1(x_0, 0) E^1(y_0, 0), \quad (9)$$

where $E^0(j_0, 0)$, $E^1(j_0, 0)$ and C_{j_0} are given by

$$E^0(j_0, 0) = \frac{1}{1 + \dfrac{j_0^2}{w_{0j}^2}} \exp\Big(-\frac{j_0^2}{w_0^2}\Big),$$

$$E^1(j_0, 0) = \frac{j_0}{1 + \dfrac{j_0^2}{w_{0j}^2}} \exp\Big(-\frac{j_0^2}{w_0^2}\Big),$$

$$C_{j_0} = \Big(\frac{1}{w_0^2} + \frac{1}{w_{0j}^2}\Big)^{1/2}. \quad (10)$$

They are referred to as SLG_{00}, SLG_{01}, SLG_{10} and SLG_{11} beams, respectively. The SLG_{00}, SLG_{01}, SLG_{10} and SLG_{11} beams correspond to the HG_{00}, HG_{01}, HG_{10} and HG_{11} beams, respectively. The SLG_{00} beam is just the Lorentz–Gauss beam and the kurtosis parameter of the Lorentz–Gauss beam has been investigated.[28] Accordingly, the kurtosis parameters of the SLG_{01}, SLG_{10} and SLG_{11} beams will be discussed in the rest of the paper.

The optical system of concern is orthogonal and is described by the same $ABCD$ matrix in each of the mutually orthogonal planes, $x - z$ and $y - z$. The A, B, C, and D are the transfer matrix elements of the paraxial optical system. Moreover, there is no inherent aperture between the source and the output planes. Therefore, A, B, C and D are all realvalued. By using the Collins integral formula, we can obtain the transformation formulae of the SLG_{01}, SLG_{10} and SLG_{11} beams passing through a paraxial and real $ABCD$ optical system

$$E^{01}(x, y, z) = \frac{\pi k C_{x_0} w_{0x} w_{0y}^2}{8B} \exp(\mathrm{i}\, kz) \exp\Big[\mathrm{i}\, \frac{k(x^2 + y^2)}{2q}\Big][E_x^+(x, z) + E_x^-(x, z)][E_y^+(y, z) - E_y^-(y, z)], \quad (11)$$

$$E^{10}(x, y, z) = \frac{\pi k C_{y_0} w_{0x}^2 w_{0y}}{8B} \exp(\mathrm{i}\, kz) \exp\Big[\mathrm{i}\, \frac{k(x^2 + y^2)}{2q}\Big][E_x^+(x, z) - E_x^-(x, z)][E_y^+(y, z) + E_y^-(y, z)], \quad (12)$$

$$E^{11}(x, y, z) = \frac{\mathrm{i}\, \pi k C_{x_0} C_{y_0} w_{0x}^2 w_{0y}^2}{8B} \exp(\mathrm{i}\, kz) \exp\Big[\mathrm{i}\, \frac{k(x^2 + y^2)}{2q}\Big][E_x^+(x, z) - E_x^-(x, z)][E_y^+(y, z) - E_y^-(y, z)], \quad (13)$$

where $E_j^{\pm}(j,z)$ and q are given by

$$E_j^{\pm}(j,z) = \exp\Big[\frac{kA'}{2\,\mathrm{i}\,B}\Big(w_{0j} \pm \mathrm{i}\,\frac{j}{A'}\Big)^2\Big] \times \mathrm{erfc}\Big[\sqrt{\frac{kA'}{2\,\mathrm{i}\,B}}\Big(w_{0j} \pm \mathrm{i}\,\frac{j}{A'}\Big)\Big], \quad (14)$$

$$q = \frac{A'}{C'} = \frac{Aq_0 + B}{Cq_0 + D}, \quad (15)$$

where $\mathrm{erfc}(x)$ is the complementary error function. $A' = A + B/q_0$ and $C' = C + D/q_0$.

The kurtosis parameter in one transversal dimension, e.g., the x-direction, is defined as[29]

$$K_x = \frac{\langle x^4\rangle}{\langle x^2\rangle^2}, \quad (16)$$

where $\langle x^4\rangle$ and $\langle x^2\rangle$ are the second- and fourth-order intensity moments in the spatial domain and are given by

$$\langle x^n\rangle = \frac{\int_{-\infty}^{\infty} x^n |E(x,y,z)|^2\,\mathrm{d}x}{\int_{-\infty}^{\infty} |E(x,y,z)|^2\,\mathrm{d}x}, \quad n = 2, 4. \quad (17)$$

Due to the existence of the complementary error function, it is difficult to obtain the analytical expression of the kurtosis parameter by using Eq. (17). Therefore, another method is used to obtain the kurtosis parameter. The transformation of higher-order intensity moments in the quadratic-index media, which is characterized by its $ABCD$ matrix, has been derived up to order 8.[30] For $n \leq 4$, the transformation formula is[30]

$$\langle j^n\rangle = \langle (Aj_0 + B\Theta_j)^n\rangle + 3\delta_{n4}A^2B^2/k^2, \quad (18)$$

where δ_{n4} is the Kronecker function, Θ_j is the parameter in the spatial-frequency domain of the source plane $z = 0$. Equation (18) is not applicable to the complex optical system. Therefore, the optical system we consider is real, which means that the inherent aperture is not included in the optical system. As a result, the kurtosis parameter of an optical beam through a paraxial and real $ABCD$ optical system can be expressed as

$$K_j = \frac{\langle (Aj_0 + B\Theta_j)^4\rangle + 3A^2B^2/k^2}{\langle (Aj_0 + B\Theta_j)^2\rangle^2}. \quad (19)$$

The generalized intensity moment of order $n_1 + n_2$ is calculated by

$$\langle j_0^{n_1}\Theta_j^{n_2}\rangle = \frac{1}{2(\mathrm{i}\,k)^{n_2}} \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} j_0^{n_1} E(x_0,y_0,0)\frac{\partial^{n_2}}{\partial j_0^{n_2}}E^*(x_0,y_0,0)\,\mathrm{d}x_0\,\mathrm{d}y_0}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |E(x_0,y_0,0)|^2\,\mathrm{d}x_0\,\mathrm{d}y_0} + \mathrm{c.c.}, \quad (20)$$

where the asterisk denotes the complex conjugation and c.c. means taking the complex conjugation of the preceding item. Therefore, we have

$$\langle j_0\Theta_j\rangle = 0, \quad \langle j_0^3\Theta_j\rangle = 0, \quad \langle j_0\Theta_j^3\rangle = 0. \quad (21)$$

Inserting Eq. (21) into Eq. (19), we find that the kurtosis parameter of an optical beam through a paraxial and real $ABCD$ optical system yields

$$K_j = \frac{\langle j_0^4\rangle + 3(B/A)^2(2\langle j_0^2\Theta_j^2\rangle + 1/k^2) + (B/A)^4\langle\Theta_j^4\rangle}{[\langle j_0^2\rangle + (B/A)^2\langle\Theta_j^2\rangle]^2}. \quad (22)$$

To obtain the kurtosis parameter of an optical beam through a paraxial and real $ABCD$ optical system, one should first obtain $\langle j_0^2\rangle$, $\langle\Theta_j^2\rangle$, $\langle j_0^2\Theta_j^2\rangle$, $\langle j_0^4\rangle$ and $\langle\Theta_j^4\rangle$.

We note that the field expressions of the SLG_{01}, SLG_{10} and SLG_{11} beams can be separable in the x- and y-directions. If we obtain K_x and K_y of the SLG_{01} beam, we can obtain K_x and K_y of the SLG_{10} and SLG_{11} beams by replacing the parameters. Therefore, K_x and K_y of the SLG_{01} beam are derived hereafter. Substituting Eq. (7) into Eq. (20), we find that the second-order moments of the SLG_{01} beam $\langle x_0^2\rangle$ and $\langle y_0^2\rangle$ turn out to be

$$\langle x_0^2\rangle = w_{0x}^2 \frac{\int_{-\infty}^{\infty}\exp(-a^2\tau^2)\frac{\tau^2}{(1+\tau^2)^2}\,\mathrm{d}\tau}{\int_{-\infty}^{\infty}\exp(-a^2\tau^2)\frac{1}{(1+\tau^2)^2}\,\mathrm{d}\tau} = w_{0x}^2 \frac{-2a + \sqrt{\pi}(1+2a^2)\exp(a^2)\mathrm{erfc}(a)}{2a + \sqrt{\pi}(1-2a^2)\exp(a^2)\mathrm{erfc}(a)}, \quad (23)$$

$$\langle y_0^2\rangle = w_{0y}^2 \frac{\int_{-\infty}^{\infty}\exp(-b^2\tau^2)\frac{\tau^4}{(1+\tau^2)^2}\,\mathrm{d}\tau}{\int_{-\infty}^{\infty}\exp(-b^2\tau^2)\frac{\tau^2}{(1+\tau^2)^2}\,\mathrm{d}\tau} = w_{0y}^2 \frac{2 + 2b^2 - \sqrt{\pi}(3b+2b^3)\exp(b^2)\mathrm{erfc}(b)}{-2b^2 + \sqrt{\pi}(b+2b^3)\exp(b^2)\mathrm{erfc}(b)}, \quad (24)$$

where $a=\sqrt{2}w_{0x}/w_0$ and $b=\sqrt{2}w_{0y}/w_0$. Similarly, the second-order moments in the spatial-frequency domain of the SLG_{01} beam $\langle\Theta_x^2\rangle$ and $\langle\Theta_y^2\rangle$ are

$$\begin{aligned}\langle\Theta_x^2\rangle &= \frac{1}{k^2w_{0x}^2}\frac{\int_{-\infty}^{\infty}\exp(-a^2\tau^2)\dfrac{2+a^2-(6+2a^2+a^4)\tau^2-(3a^2+2a^4)\tau^4-a^4\tau^6}{(1+\tau^2)^4}d\tau}{\int_{-\infty}^{\infty}\exp(-a^2\tau^2)\dfrac{1}{(1+\tau^2)^2}\,\mathrm{d}\tau}\\ &= \frac{1}{6k^2w_{0x}^2}\frac{6a+4a^3+4a^5+\sqrt{\pi}(3-6a^4-4a^6)\exp(a^2)\mathrm{erfc}(a)}{2a+\sqrt{\pi}(1-2a^2)\exp(a^2)\mathrm{erfc}(a)},\end{aligned}\tag{25}$$

$$\begin{aligned}\langle\Theta_y^2\rangle &= \frac{1}{k^2w_{0y}^2}\frac{\int_{-\infty}^{\infty}\exp(-b^2\tau^2)\dfrac{(6+3b^2)\tau^2-(2-2b^2+b^4)\tau^4-(b^2+2b^4)\tau^6-b^4\tau^8}{(1+\tau^2)^4}\,\mathrm{d}\tau}{\int_{-\infty}^{\infty}\exp(-b^2\tau^2)\dfrac{\tau^2}{(1+\tau^2)^2}\,\mathrm{d}\tau}\\ &= \frac{1}{6k^2w_{0y}^2}\frac{6b-16b^3-4b^5+\sqrt{\pi}(3+18b^4+4b^6)\exp(b^2)\mathrm{erfc}(b)}{-2b+\sqrt{\pi}(1+2b^2)\exp(b^2)\mathrm{erfc}(b)}.\end{aligned}\tag{26}$$

The cross moments of the SLG_{01} beam $\langle x_0^2\Theta_x^2\rangle$ and $\langle y_0^2\Theta_y^2\rangle$ are

$$\begin{aligned}\langle x_0^2\Theta_x^2\rangle &= \frac{1}{k^2}\frac{\int_{-\infty}^{\infty}\exp(-a^2\tau^2)\dfrac{(2+a^2)\tau^2-(6+2a^2+a^4)\tau^4-(3a^2+2a^4)\tau^6-a^4\tau^8}{(1+\tau^2)^4}\,\mathrm{d}\tau}{\int_{-\infty}^{\infty}\exp(-a^2\tau^2)\dfrac{1}{(1+\tau^2)^2}\,\mathrm{d}\tau}\\ &= \frac{1}{6k^2}\frac{-6a-16a^3-4a^5+\sqrt{\pi}(-3+12a^2+18a^4+4a^6)\exp(a^2)\mathrm{erfc}(a)}{2a+\sqrt{\pi}(1-2a^2)\exp(a^2)\mathrm{erfc}(a)},\end{aligned}\tag{27}$$

$$\begin{aligned}\langle y_0^2\Theta_y^2\rangle &= \frac{1}{k^2}\frac{\int_{-\infty}^{\infty}\exp(-b^2\tau^2)\dfrac{(6+3b^2)\tau^4-(2-2b^2+b^4)\tau^6-(b^2+2b^4)\tau^8-b^4\tau^{10}}{(1+\tau^2)^4}\,\mathrm{d}\tau}{\int_{-\infty}^{\infty}\exp(-b^2\tau^2)\dfrac{\tau^2}{(1+\tau^2)^2}\,\mathrm{d}\tau}\\ &= \frac{1}{6k^2}\frac{24b+28b^3+4b^5-\sqrt{\pi}(3+36b^2+30b^4+4b^6)\exp(b^2)\mathrm{erfc}(b)}{-2b+\sqrt{\pi}(1+2b^2)\exp(b^2)\mathrm{erfc}(b)}.\end{aligned}\tag{28}$$

The fourth-order moments of the SLG_{01} beam $\langle x_0^4\rangle$ and $\langle y_0^4\rangle$ turn out to be

$$\langle x_0^4\rangle = w_{0x}^4\frac{\int_{-\infty}^{\infty}\exp(-a^2\tau^2)\dfrac{\tau^4}{(1+\tau^2)^2}\,\mathrm{d}\tau}{\int_{-\infty}^{\infty}\exp(-a^2\tau^2)\dfrac{1}{(1+\tau^2)^2}\,\mathrm{d}\tau} = w_{0x}^4\frac{2+2a^2-\sqrt{\pi}(3a+2a^3)\exp(a^2)\mathrm{erfc}(a)}{2a^2+\sqrt{\pi}(a-2a^3)\exp(a^2)\mathrm{erfc}(a)},\tag{29}$$

$$\langle y_0^4\rangle = w_{0y}^4\frac{\int_{-\infty}^{\infty}\exp(-b^2\tau^2)\dfrac{\tau^6}{(1+\tau^2)^2}\,\mathrm{d}\tau}{\int_{-\infty}^{\infty}\exp(-b^2\tau^2)\dfrac{\tau^2}{(1+\tau^2)^2}\,\mathrm{d}\tau} = w_{0y}^4\frac{1-4b^2-2b^4+\sqrt{\pi}(5b^3+2b^5)\exp(b^2)\mathrm{erfc}(b)}{-2b^4+\sqrt{\pi}(b^3+2b^5)\exp(b^2)\mathrm{erfc}(b)}.\tag{30}$$

The fourth-order moments in the spatial-frequency domain of the SLG_{01} beam $\langle\Theta_x^4\rangle$ and $\langle\Theta_y^4\rangle$ are given by

$$\begin{aligned}\langle\Theta_x^4\rangle &= \frac{1}{k^4w_{0x}^4\int_{-\infty}^{\infty}\exp(-a^2\tau^2)\frac{1}{(1+\tau^2)^2}\,\mathrm{d}\tau}\int_{-\infty}^{\infty}\frac{\exp(-a^2\tau^2)}{(1+\tau^2)^6}\Big[24(1-10\tau^2+5\tau^4)+12a^2(1-9\tau^2-5\tau^4+5\tau^6)\\ &\quad+3a^4(1-8\tau^2-14\tau^4+5\tau^8)+2a^6\tau^2(\tau^2-3)(1+\tau^2)^3+a^8(\tau+\tau^3)^4\Big]\,\mathrm{d}\tau\\ &= \frac{1}{10k^4w_{0x}^4}\frac{30a+20a^3+8a^5+28a^7+4a^9+\sqrt{\pi}(15-20a^6-30a^8-4a^{10})\exp(a^2)\mathrm{erfc}(a)}{2a+\sqrt{\pi}(1-2a^2)\exp(a^2)\mathrm{erfc}(a)},\end{aligned}\tag{31}$$

$$\begin{aligned}\langle\Theta_y^4\rangle &= \frac{1}{k^4w_{0y}^4\int_{-\infty}^{\infty}\exp(-b^2\tau^2)\frac{\tau^2}{(1+\tau^2)^2}\,\mathrm{d}\tau}\int_{-\infty}^{\infty}\frac{\tau^2\exp(-b^2\tau^2)}{(1+\tau^2)^6}\Big[24(5-10\tau^2+\tau^4)+12b^2(5-5\tau^2-9\tau^4+\tau^6)\\ &\quad+3b^4(5-14\tau^4-8\tau^6+\tau^8)-2b^6\tau^2(5+\tau^2)(1+\tau^2)^3+b^8(\tau+\tau^3)^4\Big]\,\mathrm{d}\tau\end{aligned}$$

$$= \frac{1}{10k^4 w_{0y}^4} \frac{30b + 20b^3 - 78b^5 - 48b^7 - 4b^9 + \sqrt{\pi}(15 + 100b^6 + 50b^8 + 4b^{10}) \exp(b^2)\mathrm{erfc}(b)}{-2b + \sqrt{\pi}(1 + 2b^2) \exp(b^2)\mathrm{erfc}(b)}. \quad (32)$$

By using Eqs. (23)–(32), we can calculate K_x and K_y of the SLG_{01} beam through a paraxial and real $ABCD$ optical system as

$$K_x = \frac{2a + (1 - 2a^2)\varepsilon(a)}{\left\{-2a + (1 + 2a^2)\varepsilon(a) + \frac{\delta_x^2}{6}[6a + 4a^3 + 4a^5 + (3 - 6a^4 - 4a^6)\varepsilon(a)]\right\}^2} \times \left\{2a^{-1} + 2a - (3 + 2a^2)\varepsilon(a) + 2a^2\delta_x^2[-8a - 2a^3 + (3 + 9a^2 + 2a^4)\varepsilon(a)] + \frac{\delta_x^4}{10}[30a + 20a^3 + 8a^5 + 28a^7 + 4a^9 + (15 - 20a^6 - 30a^8 - 4a^{10})\varepsilon(a)]\right\}, \quad (33)$$

$$K_y = \frac{-2b + (1 + 2b^2)\varepsilon(b)}{\left\{2b^{-1} + 2b - (3 + 2b^2)\varepsilon(b) + \frac{\delta_y^2}{6}[6b - 16b^3 - 4b^5 + (3 + 18b^4 + 4b^6)\varepsilon(b)]\right\}^2} \times \left\{b^{-3} - 4b^{-1} - 2b + (5 + 2b^2)\varepsilon(b) + 2b\delta_y^2[9 + 14b^2 + 2b^4 - (15b + 15b^3 + 2b^5)\varepsilon(b)] + \frac{\delta_y^4}{10}[30b + 20b^3 - 78b^5 - 48b^7 - 4b^9 + (15 + 100b^6 + 50b^8 + 4b^{10})\varepsilon(b)]\right\}, \quad (34)$$

where

$$\varepsilon(a) = \sqrt{\pi}\exp(a^2)\mathrm{erfc}(a), \quad \varepsilon(b) = \sqrt{\pi}\exp(b^2)\mathrm{erfc}(b),$$

$\delta_j = B/Az_{rj}$ and $z_{rj} = kw_{0j}^2$ is the diffraction-free range in the j-direction for the super-Lorentzian part. The K_x of the SLG_{01} beam through a paraxial and real $ABCD$ optical system depends on the parameters a and δ_x. The K_y of the SLG_{01} beam through a paraxial and real $ABCD$ optical system depends on the parameters b and δ_y. As to the SLG_{10} beam, its kurtosis parameter in the x-direction can be obtained by replacing δ_y and b with δ_x and a in the expression of K_y of the SLG_{01} beam and the kurtosis parameter in the y-direction can be obtained by replacing δ_x and a with δ_y and b in the expression of K_x of the SLG_{01} beam. As to the SLG_{11} beam, its kurtosis parameter in the y-direction is just K_y of the SLG_{01} beam. By replacing δ_y and b with δ_x and a in the expression of K_y of the SLG_{01} beam, we can obtain the kurtosis parameter in the x-direction of the SLG_{11} beam.

3. Numerical results and analyses

As a numerical example, we consider the propagation of an SLG_{01} beam in free space. The matrix elements for free space of the distance z are $A = 1$, $B = z$, $C = 0$ and $D = 1$. In free space, K_x of the SLG_{01} beam is determined by the ratio z/z_{rx} and the parameter a and K_y of the SLG_{01} beam is determined by the ratio z/z_{ry} and the parameter b. The kurtosis parameter K_x of an SLG_{01} beam as a function of z/z_{rx} and a is plotted in Fig. 1(a), and K_y of the

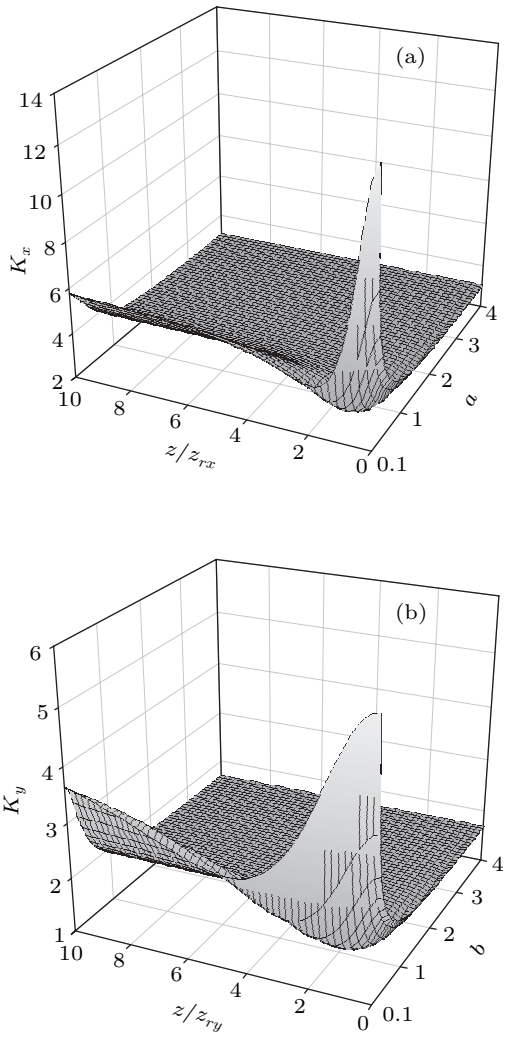


Fig. 1. (a) The K_x of the SLG_{01} beam as a function of z/z_{rx} and a. (b) The K_y of the SLG_{01} beam as a function of z/z_{ry} and b.

SLG_{01} beam as a function of z/z_{ry} and b is depicted in Fig. 1(b). When a and b tend to zero, K_x and K_y drastically increase and take a large value. Therefore, the starting values of a and b in Fig. 1 are both 0.1. When a is large enough, the kurtosis parameter K_x approaches the value of 3 approximately at any value of z/z_{rx}. When b is large enough, the kurtosis parameter K_y approaches the value of 1.67 approximately at any value of z/z_{ry}.

To further reveal the variation laws of K_x and K_y versus the ratios z/z_{rx} and z/z_{ry}, the parameters a and b are fixed, which are shown in Fig. 2. When the parameter a is small such as $a = 0.5$, K_x first quickly decreases, then increases and finally tends to a saturated value with increasing the ratio z/z_{rx}. A minimum value of K_x exists with the varying ratio z/z_{rx}. When the parameter a increases, the minimum value of K_x slowly increases. However, the saturated value of K_x decreases with the increasing parameter a. Moreover, the location where the minimum value of K_x emerges is different for the different value of a. With the increasing parameter a, the location where the minimum value of K_x emerges moves towards the source plane. When $a = 3.0$, the minimum value and the saturated value of K_x nearly overlap. In this case, K_x is approximately equal to 3 and is nearly kept unchanged with the varying ratio z/z_{rx}. When the parameter a is large enough, the field distribution in the x-direction of the SLG_{01} beam nearly reduces to a Gaussian distribution. The kurtosis parameter of the Gaussian distribution is just 3 and is invariant upon propagation. The K_y versus the ratio z/z_{ry} has the similar law of K_x versus the ratio z/z_{rx}. As the SLG_{01} beam is the zeroth-order and the first-order modes in the x- and y-directions, the corresponding K_y has the smaller value. When the parameter b is large enough, the field distribution in the y-direction of the SLG_{01} beam nearly reduces to the field distribution in the y-direction of HG_{01} beam. The kurtosis parameter in the y-direction of HG_{01} beam is just 5/3 and keeps invariant upon propagation.

Figure 3 represents the kurtosis parameters K_x and K_y as a function of the parameters a and b, where the ratios z/z_{rx} and z/z_{ry} are fixed. When the ratio z/z_{rx} is small such as $z/z_{rx} = 0.5$, K_x first quickly decreases, then increases, and finally tends to 3 with the increasing parameter a. When the ratio z/z_{rx} increases, the minimum value of K_x increases, and

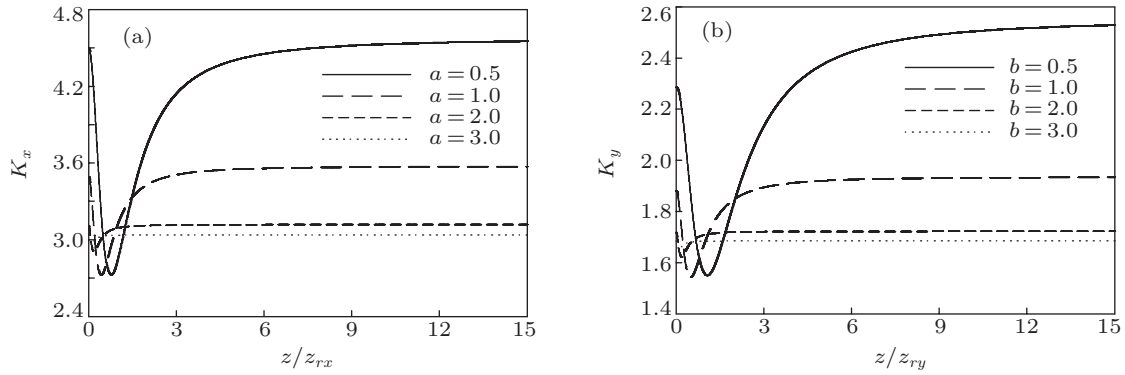


Fig. 2. (a) The K_x of the SLG_{01} beam with a given value of the parameter a as a function of z/z_{rx}. (b) The K_y of the SLG_{01} beam with a given value of the parameter b as a function of z/z_{ry}.

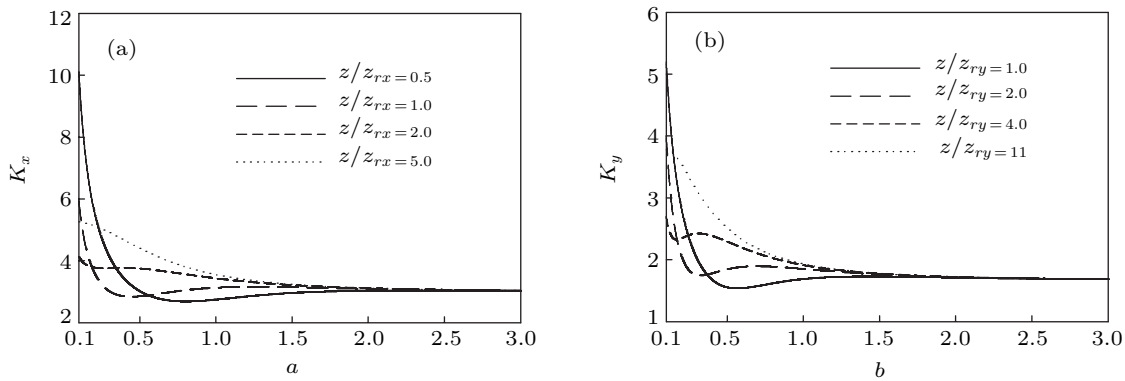


Fig. 3. (a) The K_x of the SLG_{01} beam with a given value of the ratio z/z_{rx} as a function of a. (b) The K_y of the SLG_{01} beam with a given value of the ratio z/z_{ry} as a function of b.

the location where the minimum value of K_x emerges moves towards $a = 0.1$. When the ratio $z/z_{rx} = 5.0$, the minimum value disappears. In this case, K_x first decreases and then tends to 3 with the increasing parameter a. When $a \geq 2.5$, K_x is approximately equal to 3 no matter what value the ratio z/z_{rx} is. The K_y versus the parameter b has the similar law of K_x versus the parameter a. When K_y versus the parameter b has the same law of K_x versus the parameter a, the corresponding value of z/z_{ry} is larger than that of z/z_{rx}. When b and z/z_{ry} are both smaller than and equal to 1, K_y of the SLG_{01} beam is smaller than 5/3, which can be found in Figs. 2 and 3. This phenomenon can be interpreted as follows: When b is smaller than and equal to 1, the divergence of the SLG_{01} beam is very large.[14] Moreover, the spreading of the SLG_{01} beam is also relatively large in the region of $z/z_{ry} \leq 1$. The above two reasons result in the flat intensity distribution of the SLG_{01} beam under the conditions of $b \leq 1$ and $z/z_{ry} \leq 1$. As a result, the corresponding K_y is smaller than 5/3.

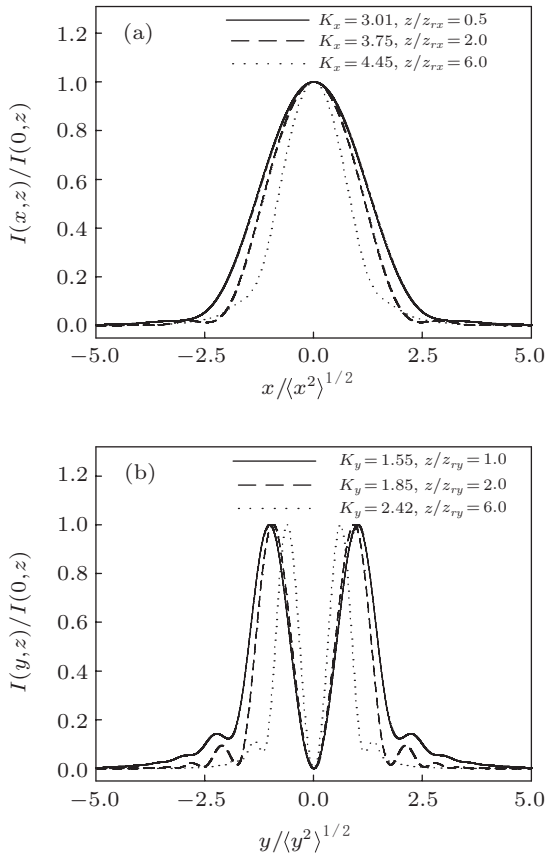


Fig. 4. (a) Relative intensity distributions in the x-direction of the SLG_{01} beam with $a = 0.5$ for different values of z/z_{rx}. (b) Relative intensity distributions in the y-direction of the SLG_{01} beam with $b = 0.5$ for different values of z/z_{ry}. The intensity I is defined as the square of the amplitude of the optical field.

For the sake of intuition, the relative intensity distributions of the SLG_{01} beam in the different reference planes are shown in Fig. 4. In the different reference planes, the SLG_{01} beam has different beam widths in the x- and y-directions, respectively. Therefore, the transversal coordinates in Fig. 4 are divided by the corresponding beam widths for convenient comparison. Upon propagation, the kurtosis parameters of the SLG_{01} beam are changed, which are marked in Fig. 4. With the increasing of the kurtosis parameters, the sections of the light intensity distributions of the SLG_{01} beam are steepened. Therefore, the kurtosis parameters represent the degree of flatness (or sharpness) of the beam intensity distribution. Figure 4 can interpret the variation laws of the solid curves in Fig. 2.

4. Conclusions

Based on the transformation of higher-order intensity moments by a paraxial and real *ABCD* optical system, analytical propagation expressions for the kurtosis parameters of the SLG_{01} beam through a paraxial and real *ABCD* optical system are derived. By using the replacement of the parameters in the expressions of the kurtosis parameters of the SLG_{01} beam, the kurtosis parameters of the SLG_{10} and SLG_{11} beams through a paraxial and real *ABCD* optical system can be conveniently obtained. The kurtosis parameter in the x-direction of the SLG_{01} beam through a paraxial *ABCD* optical system depends on δ_x and a. The kurtosis parameter in the y-direction of the SLG_{01} beam through a paraxial *ABCD* optical system depends on δ_y and b. The kurtosis parameters in the x- and y-directions of the SLG_{01} beam are independent of each other. As a numerical example, the properties of the kurtosis parameters of an SLG_{01} beam propagating in free space are illustrated. The influences of the different parameters on the kurtosis parameters of an SLG_{01} beam are discussed in detail. The properties of the kurtosis parameters of other more complicated SLG beam through a paraxial and real *ABCD* optical system can be analysed by the same procedure. When an SLG beam propagates in the atmospheric turbulence, the method used here is not applicable and one can use the approach suggested in Refs. [31] and [32]. This research is beneficial to the practical applications of an SLG beam and is useful for optical designs involving the multi-mode diode lasers.

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