Electric and magnetic dipole couplings in split ring resonator metamaterials*

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In this paper, the electric and the magnetic dipole couplings between the outer and the inner rings of a single split ring resonator (SRR) are investigated. We numerically demonstrate that the magnetic resonance frequency can be substantially modified by changing the couplings of the electric and magnetic dipoles, and give a theoretical expression of the magnetic resonance frequency. The results in this work are expected to be conducive to a deeper understanding of the SRR and other similar metamaterials, and provide new guidance for complex metamaterials design with a tailored electromagnetic response.

Keywords: metamaterials, split ring resonator, coupling

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1. Introduction

In recent years, a group of artificial microscopic structures named metamaterials[1−5] have received much attention in the scientific community due to their excellent physical properties and novel potential applications. The dielectric permittivity ε and the magnetic permeability μ in this kind of medium could be controlled arbitrarily, which allows arbitrary bending of electromagnetic fields in the theoretical frame of transformation optics.[6,7] In 1968, Veselago[8] presented the original idea of left-handed material with negative ε and negative μ simultaneously. Lately in 2001, Shelby[2] fabricated the left-handed material with combination of periodical metallic arrays of split ring resonators (SRRs) and wires, and demonstrated the negative refraction experimentally. Since that, various artificial structures such as S-shape resonators,[9,10] paired nano rods,[11] dendritic cells,[12,13] and double fishnet structures[14,15] have been proposed in pursuit of negative refraction or negative magnetic response. Many applications such as perfect lens,[16] invisibility cloaks[7,17,18] and electromagnetic absorber[19,20] are also developed based on the design of metamaterials.

The original and most basic magnetic atom of metamaterials is the SRR. Recently, a great many of studies investigating the electromagnetic coupling effects especially, have been conducted on this atom. Liu *et al.*[21] fabricated a magnetic dimmer composed of two single SRRs, and investigated the magnetic hybridization and the optical activity at optical frequencies. Liu *et al.*[22] theoretically and experimentally demonstrated that the optical properties of the SRR array can be substantially modified by changing the twist angles between the two SRR atoms of different layers, arising from the variation of electric and magnetic interactions between them. Sersic *et al.*[23] fabricated near-infrared planar metamaterials with different inter-SRR spaces along different directions, and showed strong electric and magnetic interactions between adjacent SRRs in metamaterials. Singh *et al.*[24] designed a metamaterial that consists of two closely spaced SRRs that have their splits in nonidentical positions within the ring, and demonstrated that the coupling between a dark mode and a bright mode can result in the quasi-electromagnetically induced transparency. Powell *et al.*[25] analysed the near-field inter-

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action between neighbouring SRRs, and showed that manipulating the near-field interaction could tune the response of metamaterials. All these studies are conducted on the condition that the electromagnetic wave is normally incident upon the SRR surface, where the negative magnetic resonance should be created by multi-layers of an SRR rather than a single SRR. In fact, an SRR of parallel incidence acts as a key magnetic atom for metamaterial design. The magnetic resonance is induced by a single SRR (induced by the outer and/or inner ring of the SRR) and there indeed exists strong interaction between the outer and the inner rings of a single SRR. However, to our knowledge, neither the coupling between the outer and the inner rings of single SRR, nor the coupling effect of a parallel incidence SRR has been reported.

In this paper, we study a group of SRRs, each composed of two split rings, and the splits in the outer and the inner rings are different in the sense of the difference between SRRs (Fig. 1). The different directions of the ring splits will result in different resonant electric and magnetic dipoles. It is found that the coupling between electric and magnetic dipoles may greatly affect the magnetic resonance of the SRR. We numerically demonstrate that the magnetic resonance frequency can be substantially modified by different orientations of the splits, and give a theoretical analysis for this electromagnetic coupling. The results in this work would be conducible to a deeper understanding of the SRR and other similar metamaterials, and provide new guidance for complex metamaterial design with a tailored electromagnetic response.

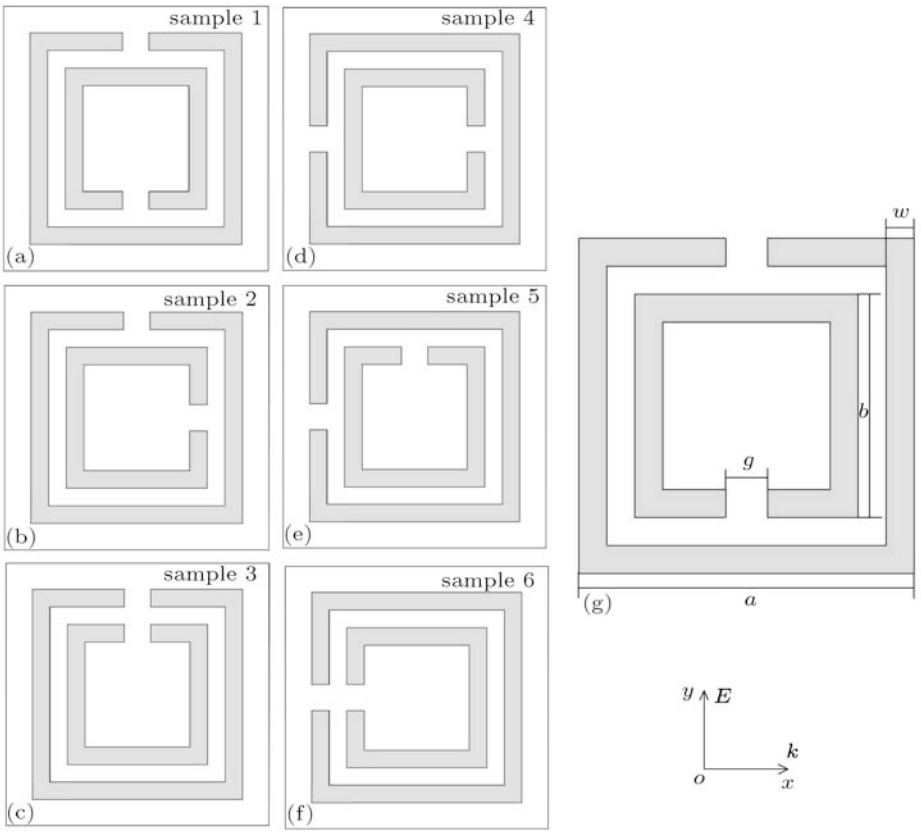


Fig. 1. Schematic representation of the SRR metamaterials, showing the structures of the six SRR samples [(a)–(f)] and dimension parameters of the SRR (g).

2. Simulation and analysis

Figure 1 shows the unit cells of six different SRRs. The geometrical dimensions of these SRRs are kept the same; the only difference is the split orientations of the outer and inner sprit rings. We assume that the copper patterns of 0.03 mm are fabricated on one side of the 0.8-mm thick FR4 printed circuit board. The geometrical dimensions of these SRRs are

all $a = 2.5$ mm, $b = 1.7$ mm, $g = 0.3$ mm, and $w = 0.3$ mm. The lattice lengths are both 3 mm along the $\boldsymbol{x}$ and the $\boldsymbol{y}$ directions. In this work, the interactive couplings between the outer and inner sprit rings are investigated, and we will show that the transmission and the effective magnetic permeability μ are affected intensively by the coupling.

The physical features of our model are examined with a commercial software package, CST Microwave Studio, which is based on a finite integration method. In our simulations, the copper is used as a lossy metal and its electric conductivity is 5.8×10^7 S/m. The permittivity of the epoxy glass substrate is 4.6 with a loss tangent of 0.025.

For electromagnetic wave parallelly incident upon the SRR plane, the magnetic component is perpendicular to the SRR. Plenty of references have shown that the magnetic resonances will be excited in this case. In general, there occur three resonances, of which two resonances are both corresponding to LC resonance of ring and one resonance arises from the mutual coupling. In this paper, we are mainly concerned with the lowest magnetic resonance in the spectrum, which corresponds to the mutual coupling. This is because in this resonance, the lattice length is small compared with the resonant wavelength and thus the effective medium theory is well satisfied.

First, we study the microwave transmissions of these SRR metamaterials. For the exciting of these samples, we use a parallelly incident wave with the wave vector $\boldsymbol{k}$ along the $\boldsymbol{x}$ direction and the electric polarization along the $\boldsymbol{y}$ direction. Figure 2(a) shows the transmissions for samples 1, 2 and 3 presented in Fig. 1. For each system there is an apparently observable resonance. With the methods given in Refs. [26] and [27], the negative permeabilities can be obtained as shown in Fig. 2(b), which confirm that these resonances are negative magnetic resonances. However, it can be obviously found that the resonant frequencies of these resonances are much different from each other. Similarly, Fig. 2(c) shows the transmissions for samples 4, 5 and 6, which have the corresponding negative permeabilities as indicated in Fig. 2(d). Still, the resonant frequencies of these resonances are much different from each other. We will show that these differences are mainly due to the mutual electromagnetic coupling between the outer and the inner rings.

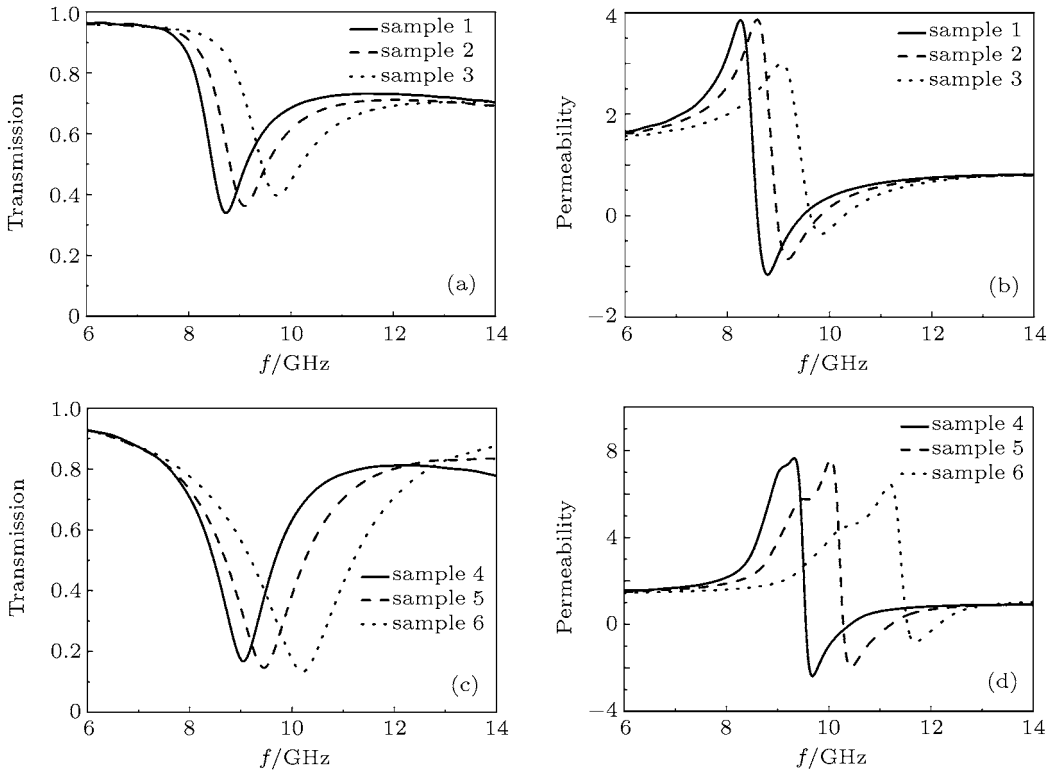


Fig. 2. Transmissions (a) and magnetic permeabilities (b) of SRR samples 1, 2 and 3. The transmissions (c) and magnetic permeabilities (d) of SRR samples 4, 5 and 6.

In order to clarify the underlying physics of the electromagnetic couplings, we introduce a Lagrangian formalism.[21] A single split ring can be described as an equivalent LC circuit with a resonance frequency $\omega = 1/(LC)^{1/2}$. It consists of a magnetic coil with inductance L and capacitance C. If we define the total charge Q accumulated in the split as a generalized coordinate, the Lagrangian expression of a split ring can be written as follows:

$$\Gamma = (L\dot{Q}^2/2) - (Q^2/2C), \tag{1}$$

where the first term on the right-hand side refers to the kinetic energy of the oscillations ($\dot{Q}$ is the induced current) and the second term is the electrostatic energy stored in the slit. Considering the electric and the magnetic interaction terms between the outer and inner rings, the Lagrangian expression of the coupled SRR systems can be written as

$$\begin{aligned}\Gamma = {} & \frac{1}{2}(L_1\dot{Q}_1^2 - Q_1^2/C_1) + \frac{1}{2}(L_2\dot{Q}_2^2 - Q_2^2/C_2) \\ & + M_{\rm H}\dot{Q}_1\dot{Q}_2 + M_{\rm E}\omega_1\omega_2 Q_1 Q_2.\end{aligned} \tag{2}$$

Here, the subscripts 1 and 2 indicate the relevant physical parameters of the outer and inner rings, respectively. $M_{\rm H}$ and $M_{\rm E}$ are the mutual inductances for the magnetic and the electric interactions.

In this work, the splits of the outer and inner rings are exactly the same, which means the capacitance C_1 is equal to C_2. If we define $l = L_2/L_1$ $(0 < l < 1)$, and $q = Q_2/Q_1$ $(0 < q < 1)$, we can simplify the Lagrangian expression into the following expression:

$$\begin{aligned}\Gamma = {} & \frac{1}{2}\dot{Q}_1^2(L_1 + L_1 l q^2 + 2M_{\rm H}q) \\ & - \frac{1}{2}Q_1^2\omega_1^2(L_1 + L_1 q^2 - 2M_{\rm E}l^{-1/2}q).\end{aligned} \tag{3}$$

By substituting Γ into the Euler-Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \Gamma}{\partial \dot{Q}_i}\right) - \frac{\partial \Gamma}{\partial Q_i} = 0, \quad i = 1, 2, \tag{4}$$

we have

$$\begin{aligned}\ddot{Q}_1(1 + lq^2 + 2M_{\rm H}q/L_1) + Q_1\omega_1^2(1 + q^2 \\ + 2M_{\rm E}l^{-1/2}q/L_1) = 0.\end{aligned} \tag{5}$$

Considering that the excitation source is a time harmonic electromagnetic wave, the charge $Q_1(\boldsymbol{r}, t) = Q_1(\boldsymbol{r})\cos\omega t$, and then $\ddot{Q}_1(\boldsymbol{r}, t) = -\omega^2 Q_1(\boldsymbol{r}, t)$.

Finally, the resonance frequency of the coupled SRR could be obtained as

$$\omega = \omega_1\sqrt{\frac{1 + q^2 + 2\kappa_{\rm E}l^{-1/2}q}{1 + lq^2 + 2\kappa_{\rm H}q}}, \tag{6}$$

where $\kappa_{\rm H} = M_{\rm H}/L_1$ and $\kappa_{\rm E} = M_{\rm E}/L_1$ are the coefficients of the overall magnetic and electric interactions, respectively.

Here, we can find in Eq. (6) that interaction coefficients $\kappa_{\rm H}$ and $\kappa_{\rm E}$ have great effects on the magnetic resonant frequency. In the case of magnetic interaction, if the magnetic dipoles of the outer and inner rings are aligned in the same direction, the magnetic interaction coefficient will be larger, so the magnetic resonant frequency turns much lower, and in contrast, the opposite magnetic dipoles will result in higher magnetic resonant frequency. In the case of electric interaction, the larger total electric dipole means the larger electric interaction coefficient, and the magnetic resonant frequency will be higher. If there is no electric dipole in the system, the electric interaction coefficient will be zero, which means no electric interaction on this magnetic resonance. We should indicate that the magnetic interaction coefficient $\kappa_{\rm H}$ is in the denominator while the electric interaction coefficient $\kappa_{\rm E}$ is in the numerator, therefore, the magnetic interactions always have more effects on the resonance.

To verify the above discussion, the current distribution of each sample at the relevant resonance is calculated and discussed. For sample 1, the surface currents of the two metal rings, which are driven by the magnetic field, rotate around the same direction. The resulting magnetic dipoles are aligned parallelly and reinforced. Considering the symmetry of this structure along the $\boldsymbol{x}$ direction, the electric field has little effect on this resonance. Therefore, this magnetic resonance is due to the overlapped magnetic dipoles of inner and outer sprit rings, and so the resonance frequency (8.74 GHz) is much lower, as shown in Fig. 2(a).

For sample 2, the surface currents of the two metal rings rotate around the same direction. Hence, magnetic dipoles are aligned parallelly and reinforced. Compared with the case in sample 1, the inner ring is not symmetric along $\boldsymbol{x}$ direction, and thus the electric field drives an electric LC resonance during this excitation. The extra electric resonance in the inner

ring will have influence on the magnetic resonance. Therefore, we find that the frequency of this magnetic resonance (9.10 GHz) is slightly higher than that in sample 1.

For sample 3, the surface currents of the two metal rings rotate around the opposite directions. Thus, the resulting magnetic dipoles are aligned anti-parallelly and weakened. Similar to the case in sample 1, due to the symmetry of this structure along $\boldsymbol{x}$ direction, the electric field has little effect on this resonance. This magnetic resonance is due mainly to the weakened magnetic dipoles of inner and outer sprit rings, and the resonance frequency is much higher than those in samples 1 and 2.

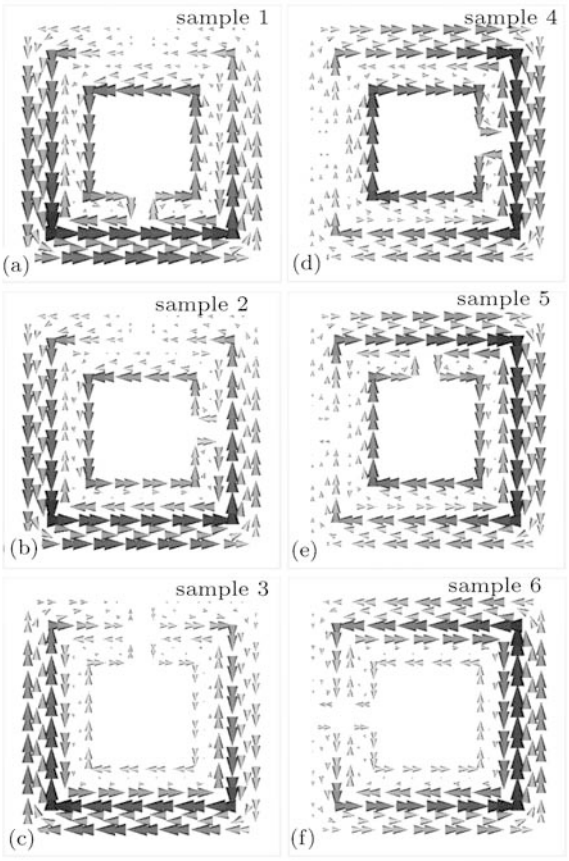


Fig. 3. Surface currents of the SRR samples at frequencies just above their corresponding resonance frequencies.

For sample 4, the surface currents of the two metal rings rotate around in the same direction. Thus, the two magnetic dipoles are aligned parallelly and reinforced. We can also find that the inner and the outer rings are both not symmetric along $\boldsymbol{x}$ direction, and the electric field would drive electric LC resonances to both split rings during this excitation. The two electric dipoles are anti-parallelly and weakened. Therefore, this electric resonance may have less influence on the magnetic resonance than that in sample 2, and the resonance frequency will be between those in samples 1 and 2. In fact, our simulation shows this resonance frequency is 9.05 GHz, which is indeed between those in samples 1 and 2.

For sample 5, the surface currents of the two metal rings rotate around the same direction. The resulting magnetic dipoles are aligned parallelly and reinforced. Compared with the case in sample 2, the outer ring instead of the inner ring is unsymmetric along the $\boldsymbol{x}$ direction. Thus, the electric field would drive an electric LC resonance on the outer ring, and the electric dipole of sample 5 is stronger than the one of sample 2. As a consequence, the electric resonance here may have more effects on the magnetic resonance, and the resonance frequency will be between those of sample 2 and sample 3. We can find that this resonance frequency is 9.46 GHz in Fig. 2(c), which is indeed between those of samples 2 and 3.

For sample 6, the surface currents of the two metal rings rotate around the opposite directions. The resulting magnetic dipoles are aligned anti-parallelly and weakened. Compared with the case in sample 3, the inner and out rings are both not symmetric along $\boldsymbol{x}$ direction. Thus, the electric field would drive two electric LC resonances during this excitation, and these two dipoles are parallel and reinforced. The magnetic resonance is well influenced by the superposed electric resonances, and resonance would be much higher than that in sample 3. We find in Fig. 2(c) that the resonance frequency is 10.22 GHz, which is the highest in resonant frequencies of all the 6 samples.

Table 1. Comparison between electric dipoles, magnetic dipoles, and the resonance frequencies of SRR metamaterials.

Sample	1	2	3	4	5	6
$\boldsymbol{m}$	$\boldsymbol{m}_{\rm o}+\boldsymbol{m}_i$	$\boldsymbol{m}_{\rm o}+\boldsymbol{m}_i$	$\boldsymbol{m}_{\rm o}-\boldsymbol{m}_i$	$\boldsymbol{m}_{\rm o}+\boldsymbol{m}_i$	$\boldsymbol{m}_{\rm o}+\boldsymbol{m}_i$	$\boldsymbol{m}_{\rm o}-\boldsymbol{m}_i$
$\boldsymbol{p}$	0	$\boldsymbol{p}_i$	0	$\boldsymbol{p}_{\rm o}-\boldsymbol{p}_i$	$\boldsymbol{p}_{\rm o}$	$\boldsymbol{p}_{\rm o}+\boldsymbol{p}_i$
f/GHz	8.736	9.096	9.736	9.048	9.456	10.224

Here, we list the electric and the magnetic dipoles of all the six SRR metamaterials in Table 1 for a brief comparison. For samples 1, 2, 4, and 5, the magnetic dipoles are all aligned parallelly and reinforced, but the total electric dipoles are different and can be ordered as $\boldsymbol{p}_1 < \boldsymbol{p}_4 < \boldsymbol{p}_2 < \boldsymbol{p}_5$, and therefore, we can find the magnetic resonant frequencies to be ordered as $f_1 < f_4 < f_2 < f_5$. For samples 3 and 6, the magnetic dipoles are aligned anti-parallelly and weakened, sample 3 has no electric dipole while sample 6 has a superimposed electric dipole, then we can find their magnetic resonant frequencies are ordered as $f_3 < f_6$. As mentioned before, the magnetic interactions always have more effects on resonant frequency, and we can finally order the resonant frequencies as follows: $f_1 < f_4 < f_2 < f_5 < f_3 < f_6$. The result is in good accordance with the numerical simulation result. Another appearance we can find in Fig. 2 is that the transmission dips of samples 4, 5, and 6 are much deeper than those of samples 1, 2, and 3. That phenomenon may be caused mainly by the additional electric dipoles of the outer rings in samples 4, 5, and 6. We will study the effects of the electric and the magnetic dipole coupling on resonant intensity and try to explain the internal mechanism in our next work.

3. Conclusion

The frequency shifts of the magnetic resonance of SRR metamaterials are studied systematically in this paper. By introducing the magnetic and the electric interaction terms into the Lagrangian formalism, the electromagnetic coupling effect on the magnetic resonant frequency is investigated. It is found that a larger total magnetic dipole will cause a much lower resonant frequency, while a larger total electric dipole will induce a higher resonant frequency. The magnetic resonant frequency is the coupling effect of both magnetic and the electric interactions. Numerical full wave simulation results are in good accordance with the theoretical analyses, which confirms the validity of the theoretical analyses. The results in this work are expected to help deeply understand the SRR metamaterials and other similar metamaterials, and well design complex metamaterials design through the interplay of electric and magnetic interactions. Although this work takes the microwave frequency for example, the same effects could be applied to the design of THz or optical frequency metamaterials.

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