Rapid Communication

Long-distance quantum state transfer through cavity-assisted interaction*

Li Yu-Ning(李宇宁), Mei Feng(梅 锋), Yu Ya-Fei(於亚飞)†, and Zhang Zhi-Ming(张智明)‡

Laboratory of Photonic Information Technology and Laboratory of Quantum Information Technology, South China Normal University, Guangzhou 510006, *China*

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We propose a scheme for long-distance quantum state transfer between different atoms based on cavity-assisted interactions. In our scheme, a coherent optical pulse sequentially interacts with two distant atoms trapped in separated cavities. Through the measurement of the state of the first atom and the homodyne detection of the final output coherent light, the quantum state can be transferred into the second atom with a success probability of unity and a fidelity of unity. In addition, our scheme neither requires the high-Q cavity working in the strong coupling regime nor employs the single-photon quantum channel, which greatly relaxes the experimental requirements.

Keywords: quantum state transfer, coherent optical pulse, homodyne detection

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In quantum information science, quantum networks play an important role, which can accomplish tasks that are impossible in the realm of classical physics. Quantum networks are composed of spatially separated quantum nodes connected by quantum channels,[1] each node is a quantum system that stores quantum information and processes the information locally using quantum gates. The quantum state transfer between different nodes through the quantum channels plays an important role, which can allow the physical implementation of quantum cryptography,[2,3] quantum secret sharing,[4] distributed quantum computation (QC)[5] and even the simulation of quantum many-body physics.[6] The first quantum state transfer scheme was proposed in 1997[5] and is based on the strong coupling between a high-Q optical cavity and an atom. In the proposed scheme, laser beams are first used to transfer the internal state of an atom at the first node to the state of the cavity mode. The generated photons leak out of the cavity, propagate as a wave packet along the transfer line, and enter into the optical cavity at the second node. Finally, the state of the second cavity is transferred to the internal state of another atom. There are many other proposals for such a transfer, including two single atoms in separated cavities,[7,8] electron spins in quantum dots[9] and atomic ensembles in free space.[10] However, those proposals each requires a strong atom–cavity coupling[5,9] and a single-photon quantum channel.[5,7−10] Despite recent experimental developments, those requirements are still a great challenge in the laboratory.

In this paper, with single atoms trapped in separated cavities as the quantum nodes and a coherent light as the quantum channel, we propose a scheme for long-distance quantum state transfer between different atoms through the cavity-assisted interactions. The basic building block of our scheme is the locally controlled phase-shift (CPS) gate between a single atom trapped in a low-Q cavity and an input coherent light, with the atom–cavity coupling being in the intermediate coupling region. By reflecting an input coherent optical pulse from two distant quantum nodes, the input coherent light will experience two CPS gates, and the quantum state will be transferred from the first atom into the second one. After the measurement of the state of the first atom and the homodyne detection of the final output coherent light, the quantum state can be reconstructed according to the outcomes of both measurements. Moreover, we analyse the per-

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†Corresponding author. E-mail: yfyuks@hotmail.com

‡Corresponding author. E-mail: zmzhang@scnu.edu.cn

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formance of our scheme and find that both the success probability and the fidelity can approach unity. Compared with the previous quantum state transfer schemes,[5,7−11] our scheme neither requires the high-Q cavity working in the strong coupling regime nor employs the single-photon quantum channel, which greatly relaxes the experimental requirements.

First, we introduce the basic building block of our scheme. As illustrated in Fig. 1(a), a three-level atom is trapped in a one-sided optical cavity. Here $a_{\rm in}$ and $a_{\rm out}$ are photon annihilation operators for the input and the output fields, respectively. States $|0\rangle$ and $|1\rangle$ are two ground states, state $|e\rangle$ is a low excited state (Fig. 1(b)). The qubit is encoded in ground states $|0\rangle$ and $|1\rangle$. The transition $|1\rangle \rightarrow |e\rangle$ driven by the input field is coupled to cavity mode $a_{\rm c}$, while state $|0\rangle$ is decoupled to the cavity mode due to the large hyperfine splitting. In the rotating frame with respect to the frequency of the input pulse, the interaction between the atom and the cavity mode is described by the Hamiltonian[12]

$$H = \delta_1 |e\rangle\langle e| + \delta_2 a^\dagger a + g(a\sigma_+ + a^\dagger \sigma_-), \qquad (1)$$

where a and a^+ are the photon annihilation and the photon creation operators of the cavity mode, respectively, $\sigma_+ = |e\rangle\langle 1|$, $\sigma_- = |1\rangle\langle e|$, $\delta_1 = \omega_0 - \omega_{\rm p}$, $\delta_2 = \omega_{\rm c} - \omega_{\rm p}$, ω_0 is the resonant frequency between excited state $|e\rangle$ and ground state $|1\rangle$, $\omega_{\rm c}$ is the frequency of the cavity mode, $\omega_{\rm p}$ is the frequency of the input pulse, and g is the atom–cavity coupling rate.

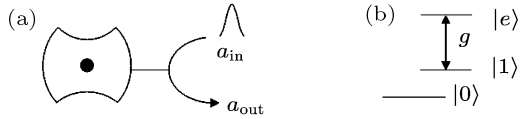


Fig. 1. (a) Setup for the basic building block, where an input coherent optical pulse is reflected by a cavity with a single trapped atom, and (b) level structure of the atom trapped in the cavity.

Through directly solving the motion equation of the atomic and the field operators and using the input–output relation $a_{\rm out} = a_{\rm in} + \sqrt{\kappa}a$, where κ is the decay rate of the cavity field, we can obtain the relationship between the input and the output optical pulses as[12−14]

$$a_{\rm out} = r a_{\rm in}, \qquad (2)$$

where r is the reflection coefficient and is expressed as

$$r = \frac{{\rm i}\delta_1({\rm i}\delta_2 - \kappa/2) + p_1 g^2}{{\rm i}\delta_1({\rm i}\delta_2 + \kappa/2) + p_1 g^2}, \qquad (3)$$

where population $p_1 = 0$ ($p_1 = 1$) when the atom is in the state $|0\rangle$ ($|1\rangle$). We define r_0 and r_1 as the reflection coefficients when the atom is in states $|0\rangle$ and $|1\rangle$, respectively. The initial input optical pulse is initialized in a coherent state $|\alpha\rangle_{\rm in}$, and the atom is in a superposition state, then the initial state of the system will be

$$\begin{aligned} |\varphi\rangle_i &= \frac{|0\rangle + |1\rangle}{\sqrt{2}}|\alpha\rangle_{\rm in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}} D(\alpha)|0\rangle_{\rm in} \\ &= \frac{|0\rangle + |1\rangle}{\sqrt{2}}\, {\rm e}^{\alpha a_{\rm in}^\dagger - \alpha^* a_{\rm in}}|0\rangle_{\rm in}, \end{aligned} \qquad (4)$$

where $D(\alpha) = {\rm e}^{\alpha a^+ - \alpha^* a}$ is the displacement operator, and α is the amplitude of the coherent state. After interacting with the cavity–atom system, according to the reflection coefficient, the final state becomes

$$\begin{aligned} |\varphi\rangle_{\rm f} &= \frac{1}{\sqrt{2}}(|0\rangle D(r_0\alpha) + |1\rangle D(r_1\alpha))|0\rangle_{\rm out} \\ &= \frac{1}{\sqrt{2}}(|0\rangle|\alpha\, {\rm e}^{{\rm i}\varphi_0}\rangle_{\rm out} + |1\rangle|\alpha\, {\rm e}^{{\rm i}\varphi_1}\rangle_{\rm out}), \end{aligned} \qquad (5)$$

where $|\alpha\rangle_{\rm out}$ represents the output optical pulse, and $\varphi_s = \arg(r_s)$, $s = 0, 1$.[12−14]

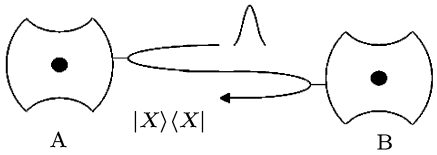


Fig. 2. Setup for the transfer of a state between separated cavities, where $|X\rangle\langle X|$ represents the homodyne detection.

Now, we begin to describe the process of transferring a state from one cavity to the other one. As illustrated in Fig. 2, there are two cavities, cavity A and cavity B, each of which traps an atom. Cavity A and atom 1 belong to Alice, cavity B and atom 2 belong to Bob. Atom 1 is prepared in a state $|\phi_{\rm a}\rangle = a|0\rangle + b|1\rangle$ with $|a|^2 + |b|^2 = 1$, atom 2 is prepared in the superposition state $|\phi_{\rm b}\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. Now, Alice wants to transfer state $|\phi_{\rm a}\rangle$ to Bob. Let coherent optical pulse $|\alpha\rangle$ be reflected by the two cavities sequentially. When the coherent pulse passes through the two cavities in turn, an operation R_2R_1 is applied to the two atoms and the input coherent optical pulse $|\alpha\rangle$, where R_1 and R_2 denote the effects of coherent optical pulse $|\alpha\rangle$ being reflected by the two cavities, respectively. Then the final state of the system becomes

$$\begin{aligned} |\psi_1\rangle &= R_2R_1\left[\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes (a|0\rangle + b|1\rangle) \otimes |\alpha\rangle\right] \\ &= R_2\left[a|0\rangle|\alpha\, {\rm e}^{{\rm i}\varphi_0}\rangle\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)\right. \end{aligned}$$

$$
\begin{aligned}
&+ b|1\rangle|\alpha\,\mathrm{e}^{\mathrm{i}\varphi_1}\rangle\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\Big] \\
&= \frac{a|00\rangle|\alpha\,\mathrm{e}^{2\mathrm{i}\varphi_0}\rangle + a|01\rangle|\alpha\,\mathrm{e}^{\mathrm{i}(\varphi_0+\varphi_1)}\rangle}{\sqrt{2}} \\
&+ \frac{b|10\rangle|\alpha\,\mathrm{e}^{\mathrm{i}(\varphi_0+\varphi_1)}\rangle + b|11\rangle|\alpha\,\mathrm{e}^{2\mathrm{i}\varphi_1}\rangle}{\sqrt{2}}. \qquad (6)
\end{aligned}
$$

In the case of a low-Q cavity ($\kappa \gg \gamma$), where γ is the spontaneous emission rate, we can have phase shifts $\phi_0 = \pi/2$ and $\phi_1 = -\pi/2$ by adjusting $\delta_1 = \delta_2 = \kappa/2$ and $g = \kappa/\sqrt{2}$. Note that here we do not require the cavity to possess a high-Q factor and work in the strong coupling regime, which is within the current experimental technology. Then the final state evolves into

$$
\begin{aligned}
|\psi_2\rangle = \frac{1}{\sqrt{2}}&(a|00\rangle|-\alpha\rangle + a|01\rangle|\alpha\rangle \\
&+ b|10\rangle|\alpha\rangle + b|11\rangle|-\alpha\rangle). \qquad (7)
\end{aligned}
$$

The operation is performed on the first qubit at the basis of $|+\rangle$ and $|-\rangle$, with $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$, then the state in Eq. (7) becomes

$$
\begin{aligned}
|\psi_3\rangle &= \frac{1}{2}a(|+\rangle + |-\rangle)|0\rangle|-\alpha\rangle \\
&+ \frac{1}{2}a(|+\rangle + |-\rangle)|1\rangle|\alpha\rangle \\
&+ \frac{1}{2}b(|+\rangle - |-\rangle)|0\rangle|\alpha\rangle \\
&+ \frac{1}{2}b(|+\rangle - |-\rangle)|1\rangle|-\alpha\rangle \\
&= \frac{1}{2}[|+\rangle|-\alpha\rangle(a|0\rangle + b|1\rangle) \\
&+ |-\rangle|-\alpha\rangle(a|0\rangle - b|1\rangle)] \\
&+ \frac{1}{2}[|+\rangle|\alpha\rangle(a|1\rangle + b|0\rangle) \\
&+ |-\rangle|\alpha\rangle(a|1\rangle - b|0\rangle)]. \qquad (8)
\end{aligned}
$$

Now, Alice performs a measurement on the state of the first atom, and Bob makes a homodyne detection on the final output coherent light. Assuming that the amplitude of the coherent light is large enough, Bob can distinguish unambiguously coherent light $|\alpha\rangle$ from $|-\alpha\rangle$ by using the homodyne detection.[15–17] If the measurement result is $\{|-\alpha\rangle|+\rangle, |-\alpha\rangle|-\rangle, |\alpha\rangle|+\rangle, |\alpha\rangle|-\rangle\}$, Bob can use the corresponding local operations $\{I, \sigma_z, \sigma_x, \mathrm{i}\sigma_y\}$ (here I is the identical operator, σ_z, σ_x, and σ_y are the Pauli operators) to reconstruct the unknown quantum state, then the quantum state transfer between the two distant atoms is completed.

In the following, we will show how to use the homodyne detection to distinguish the final output coherent states and analyse its influence on the quantum state transfer. Here we choose a homodyne measurement on the position quadrature of the coherent state $X = (a + a^\dagger)/\sqrt{2}$,[15] the wave function of the coherent state can be written in the position space. If the atomic measurement result is $|+\rangle$, we can obtain the final state of the system as

$$
\begin{aligned}
|\psi\rangle = \frac{1}{\sqrt{2}}&[f(x,-\alpha)(a|0\rangle + b|1\rangle) \\
&+ f(x,\alpha)(a|1\rangle + b|0\rangle)], \qquad (9)
\end{aligned}
$$

where $f(x,\pm\alpha) = (1/\pi)^{1/4}\exp[-(x \mp \sqrt{2}\alpha)^2/2]$ are two Gaussian curves corresponding to coherent states $|\pm\alpha\rangle$, the midpoint of the two peaks of the curves is located at $x_\mathrm{m} = 0$, and the distance between the two peaks is $x_\mathrm{d} = 2\sqrt{2}\alpha$. If the measurement result is $x \geq x_\mathrm{m}$ or $x < x_\mathrm{m}$, the state of the atom will turn into $a|0\rangle + b|1\rangle$ or $a|1\rangle + b|0\rangle$. In fact, there are some photon losses in practice, which include contributions from the channel attenuation and the atomic spontaneous emission. For the long-distance quantum state transfer distribution in our scheme, the external loss is the dominant photon loss and can be quantified with parameter $1 - \eta^2$, where η^2 is the efficient channel transmission.[16,17] Suppose that the measurement on the atomic state is perfect for a particular case, the success probability and the fidelity for measure result $|-\alpha\rangle|+\rangle$ are calculated as[12]

$$
\begin{aligned}
P_\mathrm{s} &= \mathrm{Tr}\int_{x_\mathrm{m}}^{+\infty}\mathrm{d}_x\rho(x), \\
F &= \frac{1}{P_\mathrm{s}}\int_{x_\mathrm{m}}^{+\infty}\mathrm{d}_x\langle\varphi_\mathrm{a}|\rho(x)|\varphi_\mathrm{a}\rangle, \qquad (10)
\end{aligned}
$$

where $\rho(x)$ is the density matrix of the practical final state of the system.

As shown in Fig. 3, even in the presence of the channel loss and the atomic spontaneous emission, we can obtain an ideal fidelity of the quantum state transfer by increasing α. The spontaneous emission rate used in Fig. 3 is very large compared with that of the current experimental technology.[18,19] However, even with a large spontaneous emission rate, for the initial input coherent optical pulse with a mean photon number $\langle n\rangle = 3$, a fidelity of unity and an ideal success probability of 1/4 could still be achieved. It also holds for the other three measurement results. So the fidelity of our scheme is perfect and the total success probability is unity.

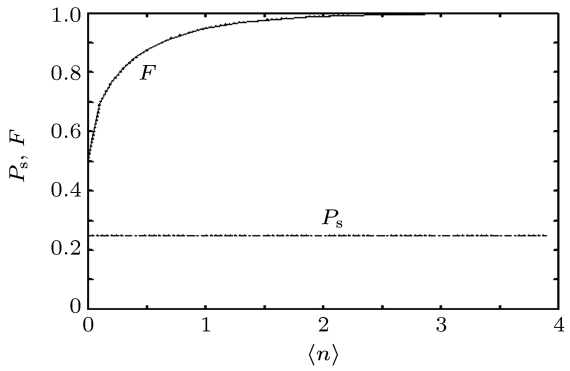


Fig. 3. Calculated success probability and fidelity for measurement result $|-\alpha\rangle|+\rangle$. The atomic spontaneous emission rates $\gamma = 0.2\kappa$ (dashed curve) and $\gamma = 0.4\kappa$ (dash-dotted curve) are used. The channel transmission is $\eta^2 = 2/3$.

In conclusion, with single atoms trapped in separated low-Q cavities working in an intermediate coupling region as the quantum nodes and a coherent optical pulse as the quantum channel, we have proposed a scheme for the long-distance quantum state transfer between different atoms through two CPS gates. In our scheme, through the input coherent optical pulse reflected from a quantum node, a CPS gate between the atom and the coherent pulse is generated. With the CPS gate, single-atom state measurement, high efficiency homodyne detection and long-distance quantum state transfer can be achieved with a high success probability and a high fidelity even in the presence of practical photon losses. In addition, our scheme is within the current experiment technology, as it needs neither a strong coupling cavity nor a single photon quantum channel.

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