Transfer of entangled state, entanglement swapping and quantum information processing via the Rydberg blockade*

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We provide a scheme with which the transfer of the entangled state and the entanglement swapping can be realized in a system of neutral atoms via the Rydberg blockade. Our idea can be extended to teleport an unknown atomic state. According to the latest theoretical research of the Rydberg excitation and experimental reports of the Rydberg blockade effect in quantum information processing, we discuss the experimental feasibility of our scheme.

Keywords: Rydberg blockade, entanglement, entanglement swapping

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1. Introduction

In the last decades, the entanglement has attracted the attention of both theorists and experimentalists.[1−20] Entanglement is at the heart of our understanding of quantum physics. The non-locality of entanglement can help us to demonstrate fundamental quantum properties, which are far beyond the conceptual frame work defined by classical physics. In addition, entanglement is useful as a resource for quantum information processes, such as quantum remote state preparation,[3−5] quantum teleportation[6,7] and quantum secret communication.[8,9] In recent years, entanglement has been demonstrated in lots of different physical systems, such as ions,[10] photon,[11,12] atomic system by cavity QED[13] and atomic ensembles.[14,15] Latest studies show that efficient two-particle and multi-particle entanglement can be realized in a system of neutral atoms by using the Rydberg blockade.[16]

In the so-called Rydberg blockade effect, one atom of an atomic ensemble in a confined volume is first excited to the Rydberg state, then it will block any further Rydberg excitation due to the strong Rydberg–Rydberg interaction that can shift the atomic resonance. The Rydberg blockade leads to an effective implementation of the individually addressed neutral atom, which is considered as a very promising approach to the scalable quantum computer.[21] The Rydberg method has a number of attractive features. It does not require the cooling of the atoms to the ground state of the confining potentials and the precise control of the two-atom interaction strength; it can be operated on μs time scales; and it is not limited to having the nearest neighbor interaction, which is advantageous for scaling up to multi-particle systems.

The advantages of the Rydberg method motivate us to devise schemes for its applications in quantum information processing. In this paper, we propose a scheme with which the transfer of entanglement and entanglement swapping are studied via the Rydberg blockade. The principle of our scheme relies on the Rydberg blockade effect demonstrated recently with two single atoms.[22] The two-qubit quantum logic gate realized in Ref. [21] is applied in our scheme. In addition, we also investigate quantum teleportation via the Rydberg blockade effect.

This paper has been organized as follows. In Section 2, we investigate the transfer of an entangled state. Entanglement swapping and the teleportation of an unknown atomic state are studied in Section 3. In the last section, we give some simple conclusions and discuss the experimental feasibility of our scheme.

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2. Transfer of entangled state

For two systems i and j, which have the same basis of space, their states can be expressed as $|\Psi\rangle_i = \sum_{k=1}^{n} \alpha_k |a_k\rangle_i$ and $|\Psi\rangle_j = \sum_{k=1}^{n} \beta_k |a_k\rangle_j$, respectively, and unitary operations are performed on state $|\Psi\rangle_i |\Psi\rangle_j$. If $|\Psi\rangle_i$ evolves as $|\Psi\rangle_i = \sum_{k=1}^{n} \beta_k |a_k\rangle_i$ and $|\Psi\rangle_j$ evolves as $|\Psi\rangle_j = \sum_{k=1}^{n} \alpha_k |a_k\rangle_j$, the process is called as the transfer of quantum state. Taking two qubits as an example, the initial state for the two qubits is $(\alpha|0\rangle_1 + \beta|1\rangle_1)(\gamma|0\rangle_2 + \theta|1\rangle_2)$. After unitary operations are performed, the final state is $(\gamma|0\rangle_1 + \theta|1\rangle_1)(\alpha|0\rangle_2 + \beta|1\rangle_2)$, which is the transfer of the qubit state. In order to obtain the transfer of the entangled state, we first consider the transfer of the qubit state.

We consider two ^{87}Rb atoms confined in a potential.[22] Each atom has three states $|0\rangle$, $|1\rangle$, and $|r\rangle$, where $|0\rangle$ and $|1\rangle$ stand for qubit states, and $|r\rangle$ is the Rydberg state (see Fig. 1). We can choose Rydberg state $|r\rangle = |79\mathrm{d}_{5/2},\ M_j = 1/2\rangle$, qubit states $|0\rangle = |5\mathrm{s}_{1/2}, F = 2\rangle$ and $|1\rangle = |5\mathrm{p}_{3/2}, F = 2\rangle$. We assume that the initial state of the two atoms is

$$(\alpha|0\rangle_1 + \beta|1\rangle_1)(\gamma|0\rangle_2 + \theta|1\rangle_2). \tag{1}$$

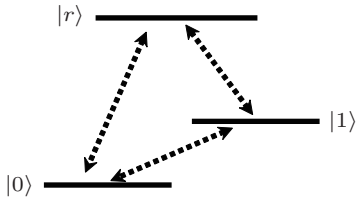


Fig. 1. Level scheme diagram and transitions between levels.

In order to realize the transfer of quantum state between the two atoms, a third atom with initial state $|0\rangle_3$ is introduced. The state of the three atoms is written as

$$(\alpha|0\rangle_1 + \beta|1\rangle_1)(\gamma|0\rangle_2 + \theta|1\rangle_2)|0\rangle_3. \tag{2}$$

For atoms 2 and 3, sequential rotations U_{mn} are performed, where $m = 2$ and $n = 3$. The sequence of operations of U_{mn} is shown in Fig. 2, which includes five steps. In step 1, we use a π pulse between states $|1\rangle_2$ and $|r\rangle_2$ and obtain

$$(\alpha|0\rangle_1 + \beta|1\rangle_1)(\gamma|0\rangle_2 + \theta|r\rangle_2)|0\rangle_3. \tag{3}$$

Then we use a second π pulse to drive the transition between states $|0\rangle_3$ and $|r\rangle_3$ of atom 3 (step 2). Due to the Rydberg blockade effect, two atoms cannot be simultaneously excited to a Rydberg state. After the operation of the second π pulse, the system is in

$$(\alpha|0\rangle_1 + \beta|1\rangle_1)(\gamma|0\rangle_2|r\rangle_3 + \theta|r\rangle_2|0\rangle_3). \tag{4}$$

In step 3, we use a π pulse that is the same as the second one to drive the state rotations of qubit state $|0\rangle_2$ and Rydberg state $|r\rangle_2$, and obtain

$$(\alpha|0\rangle_1 + \beta|1\rangle_1)|0\rangle_2(\gamma|r\rangle_3 + \theta|0\rangle_3). \tag{5}$$

Next, states $|0\rangle_3$ and $|r\rangle_3$ are coupled by a π pulse, and the state of the three atoms is evolved to

$$(\alpha|0\rangle_1 + \beta|1\rangle_1)|0\rangle_2(\gamma|0\rangle_3 + \theta|r\rangle_3). \tag{6}$$

Finally, a π pulse is used between states $|1\rangle_3$ and $|r\rangle_3$, and the state of the three atoms becomes

$$(\alpha|0\rangle_1 + \beta|1\rangle_1)|0\rangle_2(\gamma|0\rangle_3 + \theta|1\rangle_3). \tag{7}$$

For atoms 1 and 2, the sequential operations shown in Fig. 2 are performed, where $m = 1$ and $n = 2$. We obtain

$$|0\rangle_1(\alpha|0\rangle_2 + \beta|1\rangle_2)(\gamma|0\rangle_3 + \theta|1\rangle_3). \tag{8}$$

For atoms 1 and 3, the sequential rotations shown in Fig. 2 are applied, where $m = 3$ and $n = 1$, and the final state of the three atoms is

$$(\gamma|0\rangle_1 + \theta|1\rangle_1)(\alpha|0\rangle_2 + \beta|1\rangle_2)|0\rangle_3. \tag{9}$$

By using the above steps, we realize the transfer of the qubit state between qubits 1 and 2, and we have discarded the whole phase factor.

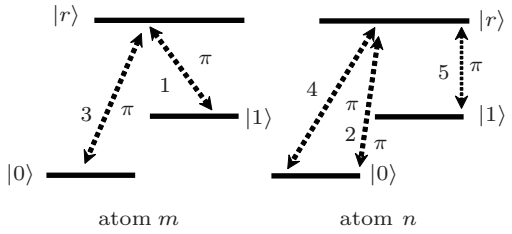


Fig. 2. Level scheme and sequence of operations for U_{mn}.

Now, we investigate the transfer of the entanglement. We assume the initial state of four atoms can be expressed as

$$(\alpha|0\rangle_1|0\rangle_2 + \beta|1\rangle_1|1\rangle_2)(\gamma|0\rangle_3|0\rangle_4 + \theta|1\rangle_3|1\rangle_4), \tag{10}$$

where atoms 1 and 2 and atoms 3 and 4 are in two-particle entangled states. If the four-particle system

undergoes unitary operations, and the state of the system evolves to

$$(\gamma\,|0\rangle_1\,|0\rangle_2+\theta\,|1\rangle_1\,|1\rangle_2)(\alpha\,|0\rangle_3\,|0\rangle_4+\beta\,|1\rangle_3\,|1\rangle_4), \quad (11)$$

it is called the transfer of the entanglement.

In order to realize the transformation from state (10) to state (11), a controlled-NOT gate is required. The controlled-NOT gate protocol[21] is shown in Fig. 3. We now show how to complete the transfer of the entanglement.

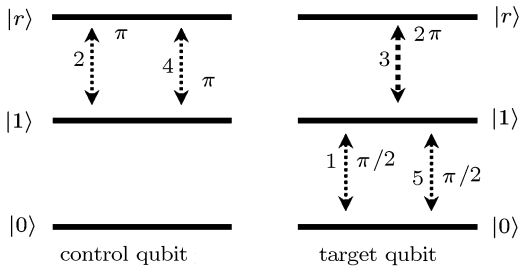


Fig. 3. Level scheme diagram and sequence of operations for the controlled-NOT gate.

First, taking atom 1 (3) as the control qubit and atom 2 (4) as the target qubit, the controlled-NOT gate is performed on atoms 1 (3) and 2 (4), and we obtain

$$(\alpha\,|0\rangle_1+\beta\,|1\rangle_1)\,|0\rangle_2\,(\gamma\,|0\rangle_3+\theta\,|1\rangle_3)\,|0\rangle_4\,. \quad (12)$$

For atoms 2 and 3, we perform sequential operations shown in Fig. 2, where $m=3$ and $n=2$. The sequence results in the state

$$(\alpha\,|0\rangle_1+\beta\,|1\rangle_1)(\gamma\,|0\rangle_2+\theta\,|1\rangle_2)\,|0\rangle_3\,|0\rangle_4\,. \quad (13)$$

The sequential operations shown in Fig. 2 are again applied between atoms 1 and 4, where $m=1$ and $n=4$, which lead to

$$|0\rangle_1\,(\gamma\,|0\rangle_2+\theta\,|1\rangle_2)\,|0\rangle_3\,(\alpha\,|0\rangle_4+\beta\,|1\rangle_4). \quad (14)$$

Finally, the controlled-NOT gate is performed on atoms 2 and 1 and atoms 4 and 3 respectively. The final state of the four atoms is

$$(\gamma\,|0\rangle_1\,|0\rangle_2+\theta\,|1\rangle_1\,|1\rangle_2)(\alpha\,|0\rangle_3\,|0\rangle_4+\beta\,|1\rangle_3\,|1\rangle_4). \quad (15)$$

Comparing the above state with state (11), we realize the transfer of the entanglement.

3. Entanglement swapping and quantum information processing

We consider four atoms 1, 2, 3 and 4. Atom 1 belongs to Alice, atoms 2 and 3 belong to Bob, and Carol has atom 4. Alice's atom 1 and Bob's atom 2 are entangled, and so are Bob's atom 3 and Carol's atom 4. If Bob performs a projective measurement on his two particles in the Bell state basis and communicates the results of the measurement to Alice or Carol. Although atom 1 of Alice and atom 4 of Carol never interact with each other, they are now entangled, which is called as the entanglement swapping. The entanglement swapping is first introduced in Ref. [23]. It has already been demonstrated experimentally for single photon polarization states.[24]

Now we consider the realization of the entanglement swapping via the Rydberg blockade. Assuming the initial state of four atoms 1, 2, 3 and 4 can be expressed as

$$\frac{1}{\sqrt{2}}(|0\rangle_1\,|0\rangle_2+|1\rangle_1\,|1\rangle_2)\frac{1}{\sqrt{2}}(|0\rangle_3\,|0\rangle_4+|1\rangle_3\,|1\rangle_4), \quad (16)$$

that is, atoms 1 (for Alice) and 2 (for Bob) are in a Bell state, and atoms 3 (for Bob) and 4 (for Carol) are mutually entangled (also in a Bell state).

In order to realize the entanglement swapping, Bob first performs a controlled-NOT gate (shown in Fig. 3) on his atoms 2 and 3, where atom 2 represents the control qubit, and atom 3 is the target qubit. The operation leads to

$$\begin{aligned}&\frac{1}{2}(|0\rangle_1\,|0\rangle_2\,|0\rangle_3\,|0\rangle_4+|0\rangle_1\,|0\rangle_2\,|1\rangle_3\,|1\rangle_4\\&+|1\rangle_1\,|1\rangle_2\,|0\rangle_3\,|0\rangle_4+|1\rangle_1\,|1\rangle_2\,|0\rangle_3\,|1\rangle_4). \quad (17)\end{aligned}$$

Next, a $\pi/2$ pulse is applied on atom 2, which results in $|0\rangle_2\rightarrow\frac{1}{\sqrt{2}}(|0\rangle_2+|1\rangle_2)$ and $|1\rangle_2\rightarrow\frac{1}{\sqrt{2}}(|0\rangle_2-|1\rangle_2)$, then state (17) becomes

$$\begin{aligned}&\frac{1}{2}\,|0\rangle_2\,|0\rangle_3\,\frac{1}{\sqrt{2}}(|0\rangle_1\,|0\rangle_4-|1\rangle_1\,|1\rangle_4)\\&+\frac{1}{2}\,|0\rangle_2\,|1\rangle_3\,\frac{1}{\sqrt{2}}(|0\rangle_1\,|1\rangle_4-|1\rangle_1\,|0\rangle_4)\\&+\frac{1}{2}\,|1\rangle_2\,|0\rangle_3\,\frac{1}{\sqrt{2}}(|0\rangle_1\,|0\rangle_4+|1\rangle_1\,|1\rangle_4)\\&+\frac{1}{2}\,|1\rangle_2\,|1\rangle_3\,\frac{1}{\sqrt{2}}(|0\rangle_1\,|1\rangle_4+|1\rangle_1\,|0\rangle_4). \quad (18)\end{aligned}$$

Based on state (18), Bob can perform the joint measurement by detecting atoms 2 and 3 separately. Then, Bob transmits the outcomes of the joint measurement to receiver Alice or Coral. Alice (Carol) then knows that her atom has been entangled with Carol's (Alice's) atom. In the above process, only Bob performs the sequential operations, and Alice's and

Carol's atoms never interact with each other, however Alice's and Carol's atoms are now entangled. The entanglement swapping is successfully realized by using the Rydberg blockade.

As an example of the application of the Rydberg blockade effect, we consider the teleportation of an unknown atomic state. Assuming atom 1 (belonged to the sender) is initially prepared in a superposition state $|\phi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1$, where α and β are unknown coefficients, and $|\alpha|^2 + |\beta|^2 = 1$, and atoms 2 (belonged to the sender) and 3 (belonged to the receiver) are prepared in a two-particle entangled state $(|0\rangle_2|0\rangle_3 + |1\rangle_2|1\rangle_3)/\sqrt{2}$, then the state of the three-particle system can be expressed as

$$(\alpha|0\rangle_1 + \beta|1\rangle_1)\frac{1}{\sqrt{2}}(|0\rangle_2|0\rangle_3 + |1\rangle_2|1\rangle_3). \quad (19)$$

Taking atom 1 as the control qubit and atom 2 as the target qubit, we perform a controlled-NOT gate on atoms 1 and 2 and obtain

$$\begin{aligned}&\frac{\alpha}{\sqrt{2}}|0\rangle_1|0\rangle_2|0\rangle_3 + \frac{\alpha}{\sqrt{2}}|0\rangle_1|1\rangle_2|1\rangle_3\\ &+\frac{\beta}{\sqrt{2}}|1\rangle_1|1\rangle_2|0\rangle_3 + \frac{\beta}{\sqrt{2}}|1\rangle_1|0\rangle_2|1\rangle_3. \quad (20)\end{aligned}$$

We then use a $\pi/2$ pulse on atom 1 and let atom 1 undergo the transitions $|0\rangle_1 \to (|0\rangle_1 + |1\rangle_1)/\sqrt{2}$ and $|1\rangle_1 \to (|1\rangle_1 - |0\rangle_1)/\sqrt{2}$. Then the state of the whole system evolves to

$$\begin{aligned}&\frac{1}{\sqrt{2}}|0\rangle_1|0\rangle_2(\alpha|0\rangle_3 - \beta|1\rangle_3)\\ &+\frac{1}{\sqrt{2}}|0\rangle_1|1\rangle_2(\alpha|1\rangle_3 - \beta|0\rangle_3)\\ &+\frac{1}{\sqrt{2}}|1\rangle_1|0\rangle_2(\alpha|0\rangle_3 + \beta|1\rangle_3)\\ &+\frac{1}{\sqrt{2}}|1\rangle_1|1\rangle_2(\alpha|1\rangle_3 + \beta|0\rangle_3). \quad (21)\end{aligned}$$

Based on state (21) and the principle of teleportation,[6] the sender makes a joint measurement (with basis $\{|0\rangle_1|0\rangle_2, |0\rangle_1|1\rangle_2, |1\rangle_1|0\rangle_2, |1\rangle_1|1\rangle_2\}$) on atoms 1 and 2 (his component of the shared entangled state) and sends his results of the measurements to the receiver via their classical communication channels. The receiver uses those classical information to choose an appropriate unitary transformation, which is then performed on atom 3 (his component of the shared entangled state). Finally, the receiver reconstructs the unknown atomic state from atom 3, that is, the unknown atomic state is faithfully teleported from the sender to the receiver.

4. Discussion and conclusion

In this paper, we have proposed a scheme in which the transfer of the entangled state and the entanglement swapping are realized by using the Rydberg blockade effect. Our ideas originate from the latest theoretical and experimental studies about the Rydberg blockade. In a system of neutral atoms, experiments show a single Rydberg-excited radium atom blocks the excitation of a second atom located more than 10 μm away and the probability of double excitation is small.[22] Theoretical studies[25,26] about the dynamic of the Rydberg excitation are consistent with the experimental data. In our scheme, the sequential unitary operations made up of single qubit and two-qubit gates are based on the latest experiments of the Rydberg blockade. According to the theory of excitation dynamics, in a two-atom system (atom i and j) driven by an external laser field, the interaction Hamiltonian is expressed as $H = \hbar\Delta_{rr}|r\rangle_i|r\rangle_j\,{}_j\langle r|\,{}_i\langle r| + \frac{\hbar\Omega_R}{2}(|r\rangle_j\,{}_j\langle 1| + |r\rangle_j\,{}_j\langle 0| + |r\rangle_i\,{}_i\langle 1| + |r\rangle_i\,{}_i\langle 0| + \text{h.c.})$, where Ω_R is the Rabi frequency, $\hbar$ is the reduced Plank constant and $\Delta_{rr}|r\rangle_i|r\rangle_j\,{}_j\langle r|\,{}_i\langle r|$ is the strong Rydberg interaction energy. Through solving the equation of evolution of the system,[27,28] we can calculate the change of the probability P_{rr} of the double excitation (i.e. state $|r\rangle$ are simultaneously populated by two atoms.) with time t. When the Rydberg–Rydberg interaction is weak (dashed curve in Fig. 4), the population of double Rydberg level $|r\rangle_i|r\rangle_j$ is large. The population of doubly excited state $|r\rangle_i|r\rangle_j$ decreases with the increase of the Rydberg–Rydberg interaction, that is, the double excitation is blocked (solid curve in Fig. 4).

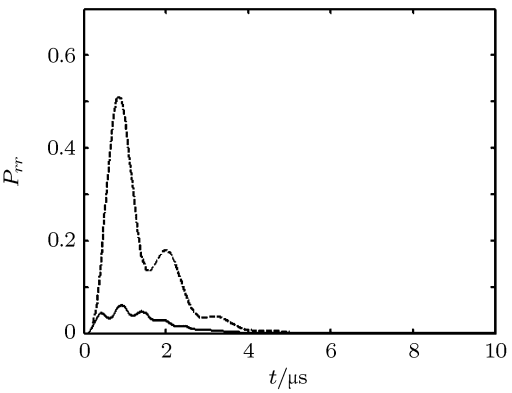


Fig. 4. Time evolution of double-excitation probability P_{rr}, where the Rabi frequency is $\Omega_R = 2 \times 10^6$ s^{-1}, the dissipation rate is $\gamma = 1.5 \times 10^6$ s^{-1} and the Rydberg-Rydberg interaction energy is $\Delta_{rr} = 5.0 \times 10^6$ s^{-1} (for dashed curve) and $\Delta_{rr} = 14 \times 10^6$ s^{-1} (for solid curve).

We extend the Rydberg blockade to teleport an unknown atomic state. The system with atoms interacting with laser fields is extensively applied in quantum information processing.[29−32] The latest experimental and theoretical studies might provide support for the experimental realization of our scheme in the future.

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