Dynamics of a multi-mode maximum entangled coherent state over an amplitude damping channel

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(Received 16 June 2011; revised manuscript received 15 July 2011)

The dynamics of the maximum entangled coherent state traveling through an amplitude damping channel is investigated. For small values of the transmissivity rate, the traveling state is very fragile to this noise channel, which suffers from the phase flip error with high probability. The entanglement decays smoothly for larger values of the transmissivity rate and speedily for smaller values of this rate. As the number of modes increases, the traveling state over this noise channel quickly loses its entanglement. The odd and even states vanish at the same value of field intensity.

Keywords: entanglement, quantum communication, decoherence, coherent states

PACS: 03.67.–a, 03.67.Hk **DOI:** 10.1088/1674-1056/20/11/110303

1. Introduction

Entanglement is one of the fundamental properties of quantum information theory, which has been considered as a nonclassical resource for many applications, such as quantum teleportation[1,2] and super dense coding.[3,4] To achieve these tasks with high efficiency, one needs maximum entangled states and perfect local operations, which are very difficult to establish in the real word. Therefore, investigating the dynamics of entanglement in the presence of imperfect circumstances is very important in the context of quantum information processing. For example, the dynamics of multiparity entanglement under the influence of decoherence is investigated in Refs. [5]–[7]. The dynamics of entangled atoms interacting with a deformed cavity mode is investigated by Metwally.[8]

Coherent states play important roles in many fields of physics, particularly in quantum technologies and quantum optics.[9,10] For example, two entangled coherent states are used to realize an effective quantum computation[11,12] and quantum teleportation.[13] El Allati *et al.*[14] have suggested a system of a three mode coherent state and used it to perform quantum teleportation. Communication via an entangled coherent quantum network is investigated in Ref. [15] and results show that the probability of performing successful teleportation through this network depends on its size.

The entanglement properties of an optical coherent entangled state consisting of two entangled modes under an amplitude damping channel is discussed by Wickert *et al.*[16] The dynamics of the Greenberger–Horne–Zeilinger (GHZ) state through the amplitude damping channel is investigated by Konrad *et al.*[7] This motivates us to investigate the entanglement properties of a class of maximum entangled coherent states consisting of three modes passing through a damping channel. In addition, we study the dynamics of a multi-entangled coherent state passing through this noise channel. The effect of this channel is equivalent to a photon absorption followed by a phase flip operator. The suppression of the traveling state over this channel is discussed and we quantify the bound entanglement of the output state as well as the survival amount of entanglement.

The paper is organized as follows: in Section 2, we review the suggested entangled muti-modes coherent state (MMCS) and the amount of entanglement over a perfect environment is quantified.[14] The entanglement of the MMCS over an amplitude damping channel is investigated in Section 3, where we quantify

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the bound of entanglement for a maximum entangled state consisting of three modes. The dynamics of a multi-mode entangled coherent state passing through the damping channel is discussed. Finally, we summarize our results in Section 4.

2. Perfect environment

Entangled coherent states have been proposed as an important resource in quantum information processing, ensuring or teleporting an unknown quantum states. These states can be written as a function of the Fock state[17−19] as

$$|\pm\alpha\rangle = \exp(-2|\alpha|^2)\sum_{n=0}^{\infty}\frac{(\pm\alpha)^n}{\sqrt{n!}}|n\rangle. \quad (1)$$

The coherent state can be generated from the vacuum state $|0\rangle$ by the displacement operator $D(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$, where $\hat{a}^\dagger$ and $\hat{a}$ are bosons creation and annihilation operators, respectively. One of the properties of these states is the non-orthogonality, and the overlap of two coherent states $|\pm\alpha\rangle$ is $\langle\alpha|-\alpha\rangle = \exp(-2|\alpha|^2)$, which becomes orthogonal by increasing the amplitude $|\alpha|$. Two coherent states can be used as basis states of a logical qubit where $|0\rangle_{\rm L} = |\alpha\rangle$ and $|1\rangle_{\rm L} = |-\alpha\rangle$. One form of the entangled coherent states between three modes can be written as

$$\begin{aligned}\rho_\alpha = \frac{1}{N_\theta^2}\Big\{&|\sqrt{2}\alpha,\alpha,\alpha\rangle_{345}\langle\sqrt{2}\alpha,\alpha,\alpha| \\ &+ \mathrm{e}^{-\mathrm{i}\,\theta}|\sqrt{2}\alpha,\alpha,\alpha\rangle_{345}\langle-\sqrt{2}\alpha,-\alpha,-\alpha| \\ &+ \mathrm{e}^{\mathrm{i}\,\theta}|-\sqrt{2}\alpha,-\alpha,-\alpha\rangle_{345}\langle\sqrt{2}\alpha,\alpha,\alpha| \\ &+ |-\sqrt{2}\alpha,-\alpha,-\alpha\rangle_{345}\langle-\sqrt{2}\alpha,-\alpha,-\alpha|\Big\}, \quad (2)\end{aligned}$$

where $N_\theta = \sqrt{2(1+\mathrm{e}^{-16|\alpha|^2}\cos\theta)}$ is the normalization factor. Setting $\theta = \pi$ in Eq. (2), we can obtain a maximum entangled state defined by

$$\begin{aligned}&\rho_\alpha^- = |\psi_\alpha^-\rangle\langle\psi_\alpha^-|, \\ &|\psi_\alpha^-\rangle = \frac{1}{\sqrt{N_\alpha}}(|\sqrt{2}\alpha,\alpha,\alpha\rangle - |-\sqrt{2}\alpha,-\alpha,-\alpha\rangle), \quad (3)\end{aligned}$$

where $N_\alpha = 2(1-\mathrm{e}^{-8|\alpha|^2})$ is the normalization factor. We use the concurrence to quantify entanglement between two qubits, which is denoted by $\mathcal{C}(|\psi_\alpha\rangle)$ as[20,21]

$$\mathcal{C}^{1/23}(|\psi_\alpha\rangle) = \frac{1-\exp(-8|\alpha|^2)}{1+\exp(-8|\alpha|^2)\cos\theta}. \quad (4)$$

Figure 1 describes the dynamics of entanglement contained in the state $|\psi_\alpha\rangle$ as a function of θ and $|\alpha|$. It is clear that, at $\theta = \pi$ the concurrence $\mathcal{C} = 1$, namely, the entanglement is maximum and is independent of $|\alpha|$. However, the concurrence is less than 1 ebit for the small amplitude, but it increases to one ebit for larger amplitudes. Therefore, this state represents two classes of entangled coherent states: the first is partial entangled states and the second is the maximum entangled one (see Ref. [14] for more details).

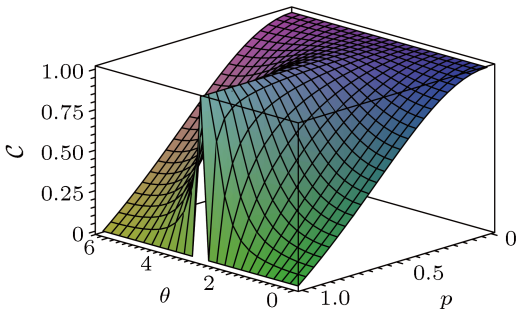


Fig. 1. (colour online) Concurrence for the coherent states for a range of θ and p, with $p = \langle\alpha|-\alpha\rangle^2$.

3. Entanglement through noise environment

3.1. Amplitude damping: description

In this section, we investigate the dynamics of the maximum entangled state (3) when it passes through an amplitude damping channel, which is defined by a photon loss and phase flip with probability $p_{\rm f}$. The photon loss is due to the interaction of traveling state (3) with an optical fibre prepared in a vacuum state. This interaction transfer the state $|\pm\alpha\rangle|0\rangle_E$ to $|\pm\sqrt{\eta}\alpha\rangle|\pm\sqrt{1-\eta}|\alpha\rangle\rangle_E$, where η is called the the transmissivity rate.[16] Tracing out the environment mode, one obtains a new state where the amplitude is reduced from α to $\alpha\sqrt{\eta}$.[16] Therefore, the state vector $|\psi_\alpha^-\rangle$ changes to $|\psi_\eta^-\rangle$, where

$$\begin{aligned}|\psi_\eta^-\rangle = \frac{1}{\sqrt{N_\eta}}&(|\sqrt{2}\alpha\sqrt{\eta},\alpha\sqrt{\eta},\alpha\sqrt{\eta}\rangle \\ &- |-\sqrt{2}\alpha\sqrt{\eta},-\alpha\sqrt{\eta},-\alpha\sqrt{\eta}\rangle). \quad (5)\end{aligned}$$

In addition, if we assume that this traveling state is subject to a phase noise with probability $p_{\rm f}$, then the final resulting effect is equivalent to the effect of the amplitude damping channel. Thus, the final output state $\rho_{\rm adc}^-$ which is obtained from the traveling state (3) through the amplitude damping channel is given by

$$\rho_{\rm adc}^- = (1-p_{\rm f})\rho_\eta^- + p_{\rm f}Z\rho_\eta^- Z, \quad (6)$$

where $\rho_\eta^- = |\psi_\eta^-\rangle\langle\psi_\eta^-|$ and Z is the phase flip error, which is defined as $Z(\lambda_1|0\rangle_{\rm L} + \lambda_2|1\rangle_{\rm L}) = \lambda_1|0\rangle_{\rm L} - \lambda_2|1\rangle_{\rm L}$. This operator affects the traveling state with probability $p_{\rm f}$ and with $(1-p_{\rm f})$ the state passes safely. In terms of α and η, the probability $p_{\rm f}$ is given by

$$p_{\rm f} = \frac{1 - {\rm e}^{-8|\alpha|^2} - {\rm e}^{-4(1-\eta)|\alpha|^2} + {\rm e}^{-4(1+\eta)|\alpha|^2}}{2(1 - {\rm e}^{-8|\alpha|^2})}. \quad (7)$$

The behaviour of the probability is shown in Fig. 2 for different values of the transmissivity rate, η. It displays that for small values of the field's intensity ($\alpha \simeq 0$), the minimum values of $p_{\rm f}$ increases as the noise strength η decreases. In a small range of field intensity $\alpha \in [0, 4]$, $p_{\rm f}$ increases faster and reaches its maximum value (1/2) as the noise strength increases. However, for larger values of the field intensity the dynamics of $p_{\rm f}$ is independent of the noise strength, where $p_{\rm f} = 1/2$. This means that for larger values of the transmissivity rate $\eta \simeq 1$, the traveling state is almost the maximum and its resistance to phase flip error is stronger.

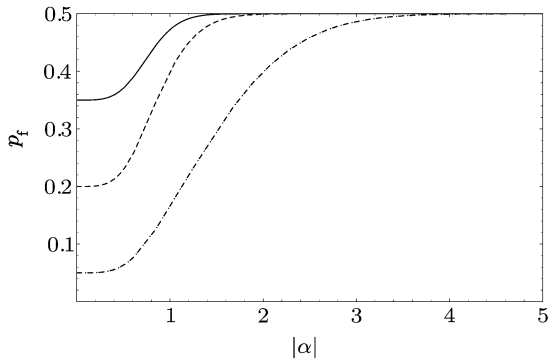


Fig. 2. Phase flip probability $p_{\rm f}$ as a function of the field intensity $|\alpha|$. The dash-dot, dot, and solid curves are for $\eta = 0.9$, 0.6, and 0.3 respectively.

3.2. Dynamics of entanglement: three qubit

To investigate the entanglement of a maximum entangled tripartite state (which is defined by Eq. (3)), passing through amplitude damping channel, we consider the following situation: assume that we have a source supplying three users, Alice, Bob and Charlie with a maximum entangled state of type (3). For simplicity, it is assumed that during the transition from the source to the users, Bob's and Charlie's qubits are forced to pass through the amplitude damping channel. According to this suggested scenario, the dynamics of the traveling state is given by

$$\rho_{\rm adc} = (1 \otimes \mathcal{S}_1 \otimes \mathcal{S}_2)\rho_\alpha^-, \quad (8)$$

where $\mathcal{S}_1$ and $\mathcal{S}_2$ represent the damping channels which effect on Bob's and Charlie's qubits respectively. For simplicity, we set $\mathcal{S}_1 = \mathcal{S}_2 = \mathcal{S}$ and rewrite the state vector $|\psi_\alpha^-\rangle$ by using the orthogonal bases u and v defined as

$$|\alpha\rangle = \lambda_\alpha|u\rangle + \mu_\alpha|v\rangle, \quad |-\alpha\rangle = \lambda_\alpha|u\rangle - \mu_\alpha|v\rangle, \quad (9)$$

and $\lambda_\alpha = (\frac{1+{\rm e}^{-2|\alpha|^2}}{2})^{1/2}$ and $\mu_\alpha = (\frac{1-{\rm e}^{-2|\alpha|^2}}{2})^{1/2}$. Then the output state vector can be written as

$$\begin{aligned}|\psi_{\rm out}\rangle = {} & (1 + {\rm e}^{{\rm i}\,\theta})\Big\{\lambda_{\sqrt{2}\alpha}\lambda_\alpha^2|uuu\rangle \\ & + \lambda_{\sqrt{2}\alpha}\lambda_\alpha\mu_\alpha\big(|uuv\rangle + |uvu\rangle\big) \\ & + \lambda_{\sqrt{2}\alpha}\mu_\alpha^2|uvv\rangle + \mu_{\sqrt{2}\alpha}\lambda_\alpha^2|vuu\rangle \\ & + \mu_{\sqrt{2}\alpha}\lambda_\alpha\mu_\alpha\big(|vuv\rangle + |vuu\rangle\big) \\ & + \mu_{\sqrt{2}\alpha}\mu_\alpha^2|vvv\rangle\Big\} \\ & + (1 - {\rm e}^{{\rm i}\,\theta}\Big\{\lambda_{\sqrt{2}\alpha}\lambda_\alpha^2|uuu\rangle \\ & - \lambda_{\sqrt{2}\alpha}\lambda_\alpha\mu_\alpha\big(|uuv\rangle + |uvu\rangle\big) \\ & + \lambda_{\sqrt{2}\alpha}\mu_\alpha^2|uvv\rangle - \mu_{\sqrt{2}\alpha}\lambda_\alpha^2|vuu\rangle \\ & + \mu_{\sqrt{2}\alpha}\lambda_\alpha\mu_\alpha\big(|vuv\rangle + |vuu\rangle\big) \\ & - \mu_{\sqrt{2}\alpha}\mu_\alpha^2|vvv\rangle\Big\}. \end{aligned} \quad (10)$$

The lower bound of entanglement of state $\rho_{\rm out}$ can be quantified by using a procedure described in Ref. [7]. This procedure state that the concurrence for any two qubits state $|\zeta\rangle\langle\zeta|$ passing either in one or two sides of channels $\mathcal{S}_1$ and $\mathcal{S}_2$ is bounded from above in terms of the evolution of the concurrence of the maximally entangled state under either one of the one-sided channels as

$$\begin{aligned} & \mathcal{C}\big[(\mathcal{S}_1 \otimes \mathcal{S}_2)|\zeta\rangle\langle\zeta|\big] \\ & = \mathcal{C}\big[(\mathcal{S}_1 \otimes \mathcal{S}_2)|\phi\rangle\langle\phi|\big]\mathcal{C}\big[|\zeta\rangle\langle\zeta|\big], \end{aligned} \quad (11)$$

where $|\phi\rangle\langle\phi|$ is a maximum entangled two qubits state. For a three qubits state, we use the same procedure, where we consider GHZ state represent the maximum entangled state. Therefore, the concurrence of the maximum entangled state (3) is bounded from above as[22]

$$\begin{aligned} & \mathcal{C}^{23/1}[(1 \otimes S \otimes S)\rho_\alpha^-] \\ & \leq \mathcal{C}^{23/1}[(1 \otimes S \otimes S)|GHZ\rangle\langle GHZ|]\mathcal{C}^{23/1}[\rho_\alpha^-]. \end{aligned} \quad (12)$$

To quantify the degree of entanglement of the output state $\rho_{\rm out} = |\psi_{\rm out}\rangle\langle\psi_{\rm out}|$, we have to reexpress the GHZ in the new bases u and v as

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|uuu\rangle + \langle vvv|). \quad (13)$$

As a first step, we consider one side effect of the amplitude damping channel on the GHZ state. This evolution is defined as

$$(1\otimes 1\otimes S)|GHZ_{uv}\rangle\langle GHZ_{uv}| = \begin{pmatrix} a & 0 & 0 & 0 & 0 & 0 & 0 & f \\ 0 & b & 0 & 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathrm{e}^{*} & 0 & 0 & 0 & 0 & c & 0 \\ f^{*} & 0 & 0 & 0 & 0 & 0 & 0 & d, \end{pmatrix}, \quad (14)$$

where

$$a = P_{\mathrm{s}}\frac{\lambda^2_{\sqrt{\eta}u}}{4\lambda_u^2}, \quad b = (1-P_{\mathrm{s}})\frac{\mu^2_{\sqrt{\eta}u}}{4\mu_u^2},$$
$$f = P_{\mathrm{s}}\frac{\mu_{\sqrt{\eta}u}\nu_{\sqrt{\eta}u}}{4\mu_u\nu_u}, \quad e = -(1-P_{\mathrm{s}})\frac{\lambda_{\sqrt{\eta}u}\mu_{\sqrt{\eta}u}}{4\lambda_u\mu_\alpha},$$
$$c = (1-P_{\mathrm{s}})\frac{\lambda^2_{\sqrt{\eta}\alpha}}{4\lambda_u^2}, \quad d = P_{\mathrm{s}}\frac{\mu^2_{\sqrt{\eta}u}}{4\lambda_u^2}.$$
$$P_{\mathrm{s}} = \frac{1}{2} + \frac{\mathrm{e}^{-4(1-\eta)|u|^2} - \mathrm{e}^{-4(1+\eta)|u|^2}}{2(1-\mathrm{e}^{-8|u|^2})}. \quad (15)$$

It is clear that the outer and the inner elements of the state represent the "unflipped" and "flipped" GHZ states of reduced, $\sqrt{\eta}u$ amplitude respectively. Then the dynamics of GHZ state through two-sides amplitude damping channel is given by

$$\begin{aligned}&(1\otimes S\otimes S)|GHZ_{u,u,u}\rangle\langle GHZ_{u,u,u}| \\ &= P_{\mathrm{s}}|GHZ_{u,\sqrt{\eta}u,\sqrt{\eta}u}\rangle\langle GHZ_{u,\sqrt{\eta}u,\sqrt{\eta}u}| \\ &\quad + (1-P_{\mathrm{s}})Z|GHZ_{u,\sqrt{\eta}u,\sqrt{\eta}u}\rangle\langle GHZ_{u,\sqrt{\eta}u,\sqrt{\eta}u}|Z. \quad (16)\end{aligned}$$

The concurrence of the traveling state (10) through the amplitude damping channel is given by

$$\mathcal{C}(\rho) = 2\max[0, |e| - \sqrt{ad}, |f| - \sqrt{bc}]. \quad (17)$$

Figure 3 shows the dynamics of the concurrence $\mathcal{C}(\rho)$ for different values of the transmissivity rate η. If the traveling state through the amplitude damping channel is partially entangled state, i.e., η is small, then the entanglement, which is represented by the concurrence, is very small and vanishes for small values of the field intensity. However, for larger values of η, the initial entanglement is large and decreases smoothly as the field's intensity increases. Therefore, to keep the entanglement of the MMECS over the amplitude damping channel survival for a long time, one has to decrease the field's intensity. It is clear that for larger values of the transmissivity rate η, the traveling state is more robust.

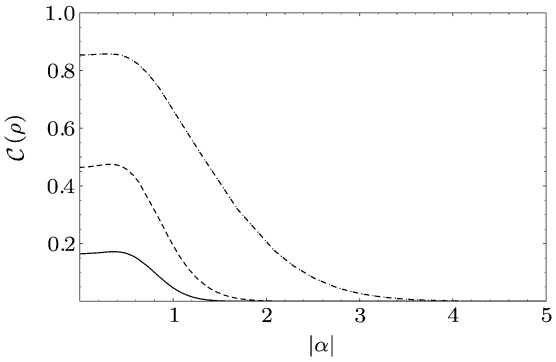


Fig. 3. The concurrence $\mathcal{C}(\rho)$ as a function of η and $|\alpha|$. The dash-dot, dot, and solid curves are evaluated at $\eta = 0.9$, 0.6, and 0.3 respectively.

3.3. Dynamics of entanglement: *m* modes

In this section, we assume that the users share a coherent state of m modes given by

$$\begin{aligned}|\Psi^{\pm}_{0,\ldots,m}\rangle \;=\; & A^{\pm}_{m+1}\Big(|2^{\frac{m-1}{2}}\alpha\rangle_0,\ldots,|2^{\frac{1}{2}}\alpha\rangle_{m-2}|\alpha\rangle_{m-1}|\alpha\rangle_m \\ & \pm|-2^{\frac{m-1}{2}}\alpha\rangle_0,\ldots,|-2^{\frac{1}{2}}\alpha\rangle_{m-2}|-\alpha\rangle_{m-1}|-\alpha\rangle_m|\Big), \quad (18)\end{aligned}$$

where $A^{\pm}_{m+1} = [2(1\pm\mathrm{e}^{-2^{m+1}|\alpha|^2})]^{-\frac{1}{2}}$ is the normalized factor. This state can be generated from Schrödinger state and optics devices. In Ref. [9], we have shown that this state represents a quantum network shared between multiusers that one user called the emitter posses the mode 0 and the other users share the remaining m modes. Moreover, we have employed this state to teleport multipartite states of m modes. The degree of entanglement of the network which is defined by the state $\rho_{\mathrm{gen}} = |\psi^{\pm}_{0,\ldots,m}\rangle\langle\psi^{\pm}_{0,\ldots,m}|$ is given by[15]

$$\mathcal{C}^{0/1,2,\ldots,m} = 1. \quad (19)$$

for $\theta = \pi$ or for $\theta = 0$.

The main aim of this section is to investigate the entangled and separable properties of this multipartite state. Let us assume that there are m modes of

the state (18) passing through an amplitude damping channel. In this case the output state can be written as

$$\rho_{\rm out}^{\pm} = (1 - p_{{\rm f},m})\rho_{\eta,0,\ldots,m}^{\pm} + p_{{\rm f},m} Z \rho_{\eta,0,\ldots,m}^{\pm} Z, \quad (20)$$

where $\rho_{\rm out}^{\pm} = |\Psi_{0,\ldots,m}^{\pm}\rangle\langle\Psi_{0,\ldots,m}^{\pm}|$ and $p_{{\rm f},m}$ is the probability that the phase flip affects the traveling state through the amplitude damping channel. This probability is given by

$$p_{{\rm f},m} = \frac{1 - {\rm e}^{-2^m|\alpha|^2} - {\rm e}^{-2^{m-1}(1-\eta)|\alpha|^2} + {\rm e}^{-2^{m-1}(1+\eta)|\alpha|^2}}{2(1 - {\rm e}^{-2^m|\alpha|^2})}, \quad m \geq 1. \quad (21)$$

Figure 4(a) displays the dynamics of the probability $p_{\rm f}$ of the phase bit flip error which affects the traveling state through the amplitude damping channel for different values of the modes while transmissivity rate is large ($\eta = 0.99$), i.e., the traveling state is almost the maximum. It is clear that, for small vales of modes, the probability $p_{{\rm f},m}$ increases gradually to its maximum value ($= 0.5$) for lager values of the field intensity $|\alpha|$. However, for larger values of m, $p_{{\rm f},m}$ increases abruptly and reaches the maximum bound for small values of the field intensity. In Fig. 4(b), we assume that the traveling state (18) through the amplitude damping channel is the partially entangled state, where we set the transmissivity rate $\eta = 0.1$. In this case, the resistance of the input state (18) for the phase bit flip error is very fragile, where $p_{{\rm f},m}$ reaches its maximum values for smaller values of the field intensity.

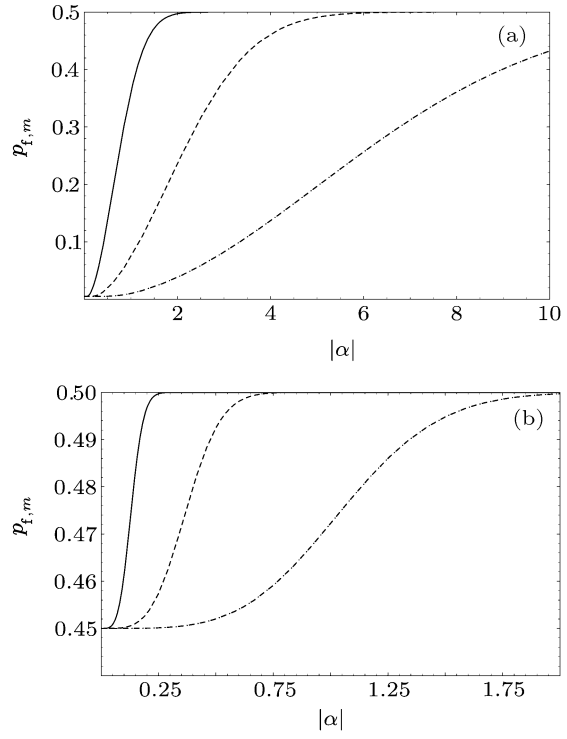


Fig. 4. The probability of the phase flip error $p_{{\rm f},m}$ which affect the traveling state (18). The dash-dot, dot and solid curves are for $m = 2$, 5, 8, respectively, and the transmissivity rate (a) $\eta = 0.99$ and (b) $\eta = 0.1$.

To quantify the degree of entanglement contained in the traveling state (18) through the amplitude damping channel, we rewrite the lower bound of entanglement to include m modes. Therefore, Eq. (12) can be generalized as

$$\begin{aligned} &\mathcal{C}[(1\otimes,\ldots,\otimes\mathcal{S}\otimes\mathcal{S})\rho_{\rm gen}^{\pm}] \\ &\leq \mathcal{C}[(1\otimes,\ldots,\otimes\mathcal{S}\otimes\mathcal{S})|GHZ_{\alpha,\ldots,\alpha}\rangle\langle GHZ_{\alpha,\ldots,\alpha}|] \\ &\quad \times \mathcal{C}^{1/2,\ldots,m}[\rho_{\rm gen}^{\pm}]. \end{aligned} \quad (22)$$

To evaluate this bound of entanglement, one has to investigate the effect of the amplitude noise channel on the $(m+1)$ modes of GHZ state which in the orthogonal basis takes the form

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|u,\ldots,u\rangle + |v,\ldots,v\rangle). \quad (23)$$

The dynamics of $|GHZ\rangle$ state through the amplitude damping channel is given by

$$\begin{aligned} &(1\otimes,\ldots,\otimes\mathcal{S}\otimes\mathcal{S})|GHZ_{\alpha,\alpha,\ldots,\alpha}\rangle\langle GHZ_{\alpha,\alpha,\ldots,\alpha}| \\ &= (1 - P_{{\rm f},m})|GHZ_{\alpha,\sqrt{\eta}\alpha,\ldots,\sqrt{\eta}\alpha}\rangle\langle GHZ_{\alpha,\sqrt{\eta}\alpha,\ldots,\sqrt{\eta}\alpha}| \\ &\quad + P_{{\rm f},m} Z|GHZ_{\alpha,\sqrt{\eta}\alpha,\ldots,\sqrt{\eta}\alpha}\rangle\langle GHZ_{\alpha,\sqrt{\eta}\alpha,\ldots,\ldots,\sqrt{\eta}\alpha}|Z. \end{aligned} \quad (24)$$

The amount of entanglement is quantified by means of the concurrence as

$$\begin{aligned} \mathcal{C}_{\pm} = &\frac{1 - 2p_{{\rm f},m}}{1 \pm \exp\left\{-2^{m-1}(1+\eta)|\alpha|^2\right\}} \\ &\times \sqrt{1 - \exp(-2^m|\alpha|^2)} \\ &\times \sqrt{1 - \exp(-2^m\eta|\alpha|^2)}, \end{aligned} \quad (25)$$

where $\mathcal{C}_+$ and $\mathcal{C}_-$ are for $\theta = 0, \pi$ respectively, and the concurrence $\mathcal{C}^{1/2,\ldots,m}[\rho_{\rm gen}^{\pm}] = 1$ (see Eq. (19)).

The dynamics of entanglement which is represented by concurrence for different values of the phase θ is described in Fig. 5, where the transmissivity rate η is assumed to be fixed. It is clear that for an odd state,

i.e., $\theta = \pi$, the concurrence decreases as the field intensity increases, as as shown in Fig. 5(a). The decay of entanglement depends on the number of modes of the traveling state. For small values of modes, the entanglement decays smoothly and gradually to vanish completely at larger values of the field intensity. However, as the number of modes increases, the entanglement decays fast and abruptly vanishes at small values of the field intensity. The dynamics of the concurrence for an even class of MMECS is shown in Fig. 5(b), where $\theta = 0$. It is clear that for $|\alpha| = 0$, the traveling state is almost separable. However, as $|\alpha|$ increases, the entanglement increases sharply to its maximum value in a very small range of the $|\alpha|$, depending on the number of traveling modes. However, as $|\alpha|$ increases more, the entanglement decays gradually for small values of m and quicly for larger values of m. From Figs. 5(a) and 5(b), the entanglement vanishes for the same value of $|\alpha|$. Therefore, the amount of entanglement contained in the odd and even traveling states over the amplitude damping channel vanishes for the same value of the field intensity.

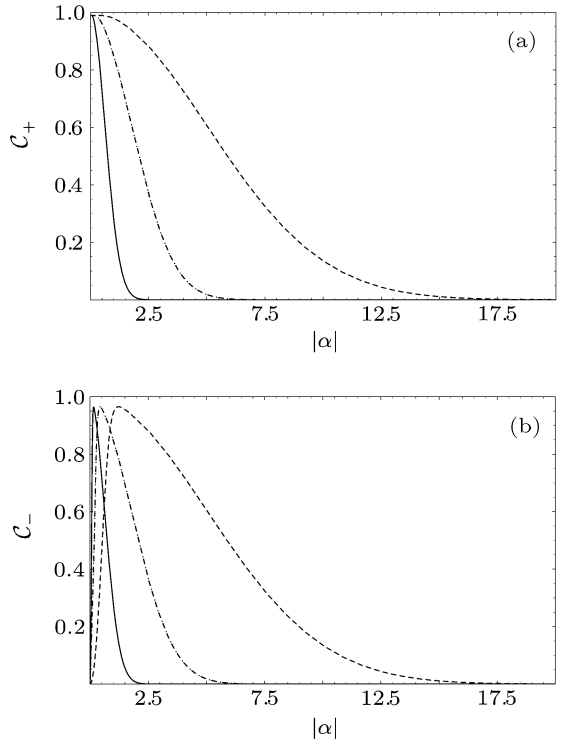


Fig. 5. Dynamics of the concurrence for different values of modes. The dot, dash-dot and solid curves are for $m = 2$, 5, and 8 respectively, and the transmissivity rate $\eta = 0.9$ for (a) $\theta = \pi$, (b) $\theta = 0$.

Figure 6 shows the dynamics of entanglement for small value of $\eta(= 0.1)$, i.e., the traveling state over the amplitude damping channel has an initial small value of entanglement. The general behaviour is the same as that depicted in Fig. 5. However, the initial amount of entanglement is very small and vanishes very quickly at small values of the field intensity.

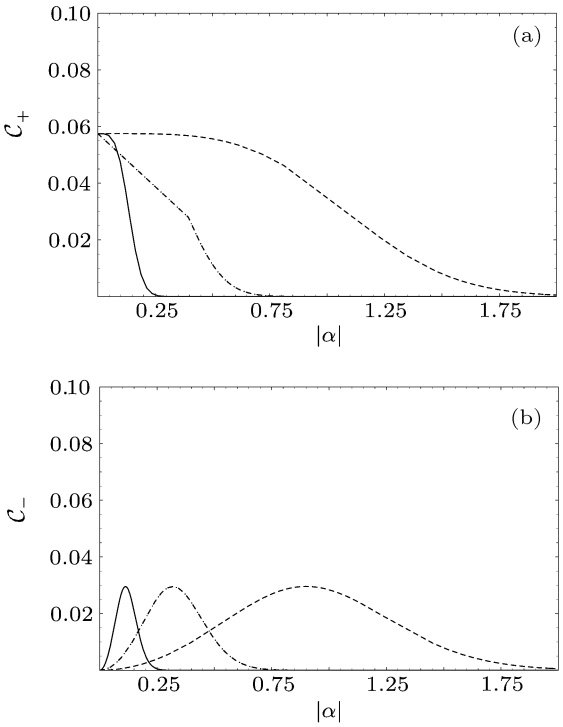


Fig. 6. The same as Fig. 5, but $\eta = 0.1$.

4. Conclusion

The dynamics of a maximum entangled state passing through an amplitude damping channel has been discussed. We find that the entanglement decays gradually for larger values of the field intensity and small values of the transmissivity rate. However, for small values of the transmissivity rate, the entanglement vanishes at small values of the field intensity. Therefore, to increase the resistance of the MMECS to entanglement degradation, one has to increase the field's intensity when the transmissivity rate is large.

The dynamics of a multi-mode entangled state passing through an amplitude damping channel has been investigated. This type of study displays the effects of the noise strength, the phase flip operator and the field intensity. We find that the traveling state suffering from the phase flip effect with high probability for small values of the noise strength absorption parameter. However, the robustness of this multimode entangled state for the phase flip operator decreases as the photon absorption decreases, in which case the traveling state is partially entangled state.

Moreover, this resistance decreases as the field's intensity increases. In addition, the probability of the phase error effects depends on the number of photons for each mode, where the probability is maximized as the number of photons increases.

The entanglement of MMECS for different modes is investigated, where the entanglement decreases gradually for small values of modes. However, as the number of modes increases, the entanglement decays very fast for small values of the field's intensity. It is shown that the entanglement for both the odd and even MMECS states completely vanishes at the same value of the field intensity. The decay rate of the traveling entanglement depends on the field's intensity and the transmissivity rate.

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