Controlling transfer of quantum correlations among bi-partitions of a composite quantum system by combining different noisy environments*

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The correlation dynamics are investigated for various bi-partitions of a composite quantum system consisting of two qubits and two independent and non-identical noisy environments. The two qubits have no direct interaction with each other and locally interact with their environments. Classical and quantum correlations including the entanglement are initially prepared only between the two qubits. We find that contrary to the identical noisy environment case, the quantum correlation transfer direction can be controlled by combining different noisy environments. The amplitude-damping environment determines whether there exists the entanglement transfer among bi-partitions of the system. When one qubit is coupled to an amplitude-damping environment and the other one to a bit-flip one, we find a very interesting result that all the quantum and the classical correlations, and even the entanglement, originally existing between the qubits, can be completely transferred without any loss to the qubit coupled to the bit-flit environment and the amplitude-damping environment. We also notice that it is possible to distinguish the quantum correlation from the classical correlation and the entanglement by combining different noisy environments.

Keywords: quantum correlation, noisy environment

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1. Introduction

Entanglement is an essential resource in many quantum information protocols.[1] It is considered as a unique nonclassical correlation existing in the multi-partite quantum state. However, some investigations have shown that even unentangled states can also possess nonclassical correlations.[2−5] Quantum computers based on those states can be used to solve certain tasks more efficiently than their classical counterparts.[6] Besides, distinguishing classical and quantum correlations of the multi-partite quantum state is vital in quantum information theory. As a consequence, a lot of efforts have been devoted to the definition and the study of correlations in quantum systems.[3,7−12] Among them, quantifying the quantumness of correlations with the quantum discord (QD)[2] proposed firstly by Olliver and Zurek has received great attention. And till now, many aspects concerning with the QD have been discussed, e.g., finding analytical expression of the QD for a certain class of quantum states,[9,13,14] the interplay of the QD with quantum phase transitions,[15−19] studying the QD in continuous variable systems.[20,21]

As all realistic systems are inevitably coupled to their surrounding environments, which leads to the degradation of their quantum correlations. Recently, the investigation of the correlation dynamics under the influence of various kinds of decoherence scenarios has become an active subject in both theory and experiment.[22−31] Werlang *et al.*[22] and Fanchini *et al.*[23] investigated the QD dynamics of open systems. It was found that the QD and the entanglement show very different dynamical behaviours and the QD is more robust to the decoherence. Recently, an interesting dynamical behaviour of correlations, named the sudden transition, has been observed.[25,26] It shows that with certain initial states the correlation undergoes a sudden change from a classical decoherence phase to a quantum one,[24,25] or vice versa. This sudden transition behaviour can be explained in a geometrical way and has a connection with the property

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of environment.[24,25] The dynamical behaviour of the QD is also investigated in a solid state system.[27] Experimental investigations of nonclassical correlations have also received much attention.[5,29−31] Using an optical architecture, Lanyon *et al.*[5] experimentally proved that even fully separable states can be used to construct a quantum computer owing to the quantum correlation that is contained in those states . Xu *et al.*[29,30] investigated both the Markovian and the non-Markovian dynamics of classical and quantum correlations and observed the sudden transition behaviour of the QD. In a recent paper, Soares-Pinto *et al.*[31] reported theoretical and experimental results on the dynamics of quantum and classical correlations in an NMR quadrupolar system.

We notice that existing investigations on the correlation dynamics of quantum states consider a system consisting of two quantum subsystems and two local environments.[22−31] In general, the quantum systems are postulated to locally interact with either a common or two identical environments. However, in practice, the two quantum subsystems may be spatially far away from each other and locally interact with different kinds of environments. For example, subsystem A may be coupled to an amplitude-damping environment, while subsystem B may be coupled to a phase-damping one. In this situation, the question how the correlation dynamics of the subsystems is affected by the different combinations of the environments is raised. In fact, it has been shown that the correlation dynamics under the action of an amplitude-damping reservoir displays much different behaviours in comparison with that under the action of a phase-damping one.[28] On the other hand, it may be easier in experiment to prepare quantum correlations, such as the entanglement, between certain bi-partitions than that between other bi-partitions of a composite system. However, a computation and an information processing may require quantum correlations between bi-partitions that are originally uncorrelated with each other. Therefore, we may need to transfer quantum correlations initially prepared in one bi-partition to another. Motivated by the above considerations, we here study the correlation dynamics for various bi-partitions of a composite system consisting of two qubits A and B and two local environments E_A and E_B. In contrast to the previous investigations,[22−28] we here assume that E_A and E_B are non-identical. We find that contrary to that stated in Refs. [28] and [32], the transfer direction of either entanglement or other quantum correlations can be controlled by combining different environments. We also notice that the amplitude-damping process determines whether there exists the entanglement transfer among bi-partitions of a composite system. For the amplitude-damping plus bit-flip environment case, we find a very interesting result that all the quantum and the classical correlations, and even the entanglement, originally existing between the qubits, can be completely transferred without any loss to the qubit coupled to the bit-flit environment and the amplitude-damping one. We also notice that it is possible to distinguish the quantum correlation from the classical correlation and the entanglement by combining different environments.

The remainder of the present paper is organized as follows. In Section 2, several definitions of quantum and classical correlations are reviewed. In Section 3, the model is introduced, the numerical results are shown, and the detailed discussion is given. Finally, a brief summary is given in Section 4.

2. Classical and quantum correlations

As is well known, the total correlation between two random variables A and B can be described by the mutual information.[1,33,34] In the classical information theory (CIT), there are two equivalent expressions for the classical mutual information as follows:[1−3]

$$I_{\rm C}(A:B) = H(A) + H(B) - H(A,B), \quad (1)$$

$$J_{\rm C}(A:B) = H(A) - H(A\|B), \quad (2)$$

where $H(X) = -\sum_x p_x \log_2 p_x$ ($X = A$, B, and AB) is the Shannon entropy for variable X, with p_x being the probability of X taking value x; and $H(A\|B) = -\sum_{a,b} p_{ab} \log_2 p_{a|b} = H(A,B) - H(B)$ ($p_{a|b} = p_{ab}/p_b$) is the conditional entropy, which represents a weighted average of the entropies of A given the value of B.

In the quantum information theory (QIT), the total correlation of a bipartite system can be expressed in terms of the quantum mutual information as[1,33,34]

$$I_{\rm q}(\rho_{A:B}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}), \quad (3)$$

where $S(\rho_{A(B)}) = -\operatorname{Tr}(\rho_{A(B)} \log_2 \rho_{A(B)})$ is the von Neumann entropy of subsystem $A(B)$, and $S(\rho_{AB}) = -\operatorname{Tr}(\rho_{AB} \log_2 \rho_{AB})$ is the entropy of the composite system AB. Quantum mutual information (3) is a straightforward extension of Eq. (1).

From Eq. (2), we can see that the value of $H(A\|B)$ is measurement dependent. Since the quantum measurement may fully destroy a quantum state, the extension of Eq. (2) to the quantum realm is no longer straightforward. The counterpart of Eq. (2) in QIT is defined as[26]

$$J_{\rm q}\left(\rho_{A:\,B}\right) = S(\rho_A) - S_{\left\{\Pi_j^B\right\}}\left(\rho_{A|B}\right), \tag{4}$$

where $\left\{\Pi_j^B\right\}$ describes a set of projectors performed locally on B, and

$$S_{\left\{\Pi_j^B\right\}}\left(\rho_{A|B}\right) = \sum_j q_j S(\rho_A^j)$$

with

$$\rho_A^j = {\rm Tr}_B(\Pi_j^B \rho_{AB} \Pi_j^B)/q_j$$

and $q_j = {\rm Tr}_{AB}\left(\Pi_j^B \rho_{AB} \Pi_j^B\right)$. In the present investigation, we choose $\Pi_j^B = |\theta_j\rangle\langle\theta_j|$, in which $|\theta_1\rangle = \cos\theta\,|0\rangle + {\rm e}^{{\rm i}\,\phi}\sin\theta\,|1\rangle$ and $|\theta_2\rangle = -\cos\theta\,|1\rangle + {\rm e}^{-{\rm i}\,\phi}\sin\theta\,|0\rangle$, with $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$. From Eq. (4), we can see that different choices of $\left\{\Pi_j^B\right\}$ may lead to different values of $J_{\rm q}\left(\rho_{A:\,B}\right)$. The minimum difference between $I_{\rm q}\left(\rho_{A:\,B}\right)$ and $J_{\rm q}\left(\rho_{A:\,B}\right)$ called the quantum discord (QD)[2] can be used to describe the quantum correlation of a bipartite quantum system. We have

$$D\left(\rho_{A:\,B}\right) = \min_{\left\{\Pi_j^B\right\}}\left[I_{\rm q}\left(\rho_{A:\,B}\right) - J_{\rm q}\left(\rho_{A:\,B}\right)\right] \tag{5}$$

or equivalently

$$D\left(\rho_{A:\,B}\right) = I_{\rm q}\left(\rho_{A:\,B}\right) - \max_{\left\{\Pi_j^B\right\}}\left[J_{\rm q}\left(\rho_{A:\,B}\right)\right]. \tag{6}$$

Based on Eqs. (3) and (6), the classical correlation contained in a quantum state may be defined as[3]

$$\begin{aligned} C\left(\rho_{AB}\right) &\equiv I_{\rm q}\left(\rho_{A:\,B}\right) - D\left(\rho_{A:\,B}\right) \\ &= \max_{\left\{\Pi_j^B\right\}}\left[S\left(\rho_A\right) - S_{\left\{\Pi_j^B\right\}}\left(\rho_{A|B}\right)\right]. \end{aligned} \tag{7}$$

Here, we must emphasize that definitions (6) and (7) are valid only when $D\left(\rho_{A:\,B}\right)$ and $C\left(\rho_{AB}\right)$ are symmetric under the interchange of $A \leftrightarrow B$. Otherwise, we need to adopt the two-side measurements for those correlations.[35−38] Under that circumstance, the classical correlation equals the maximum classical mutual information obtained by performing local measurements over both of the subsystems,[35−38]

$$K\left(\rho_{AB}\right) \equiv \max_{\left\{\Pi_i^A \otimes \Pi_j^B\right\}}\left[I_{\rm C}\left(A:\,B\right)\right], \tag{8}$$

where $\left\{\Pi_i^A \otimes \Pi_j^B\right\}$ denotes a set of local measures over subsystems A and B, and $I_{\rm C}\left(A:\,B\right)$ is given by Eq. (1).

Accordingly, the quantum correlation can be defined as[33−36]

$$Q\left(\rho_{AB}\right) \equiv I_{\rm q}\left(\rho_{A:\,B}\right) - K\left(\rho_{AB}\right). \tag{9}$$

Since the two subsystems under consideration are coupled to non-identical environments, their states are asymmetric about the interchange between the two subsystems, we will use Eqs. (8) and (9) to quantify the classical and the quantum correlations, respectively.

3. Dynamics of classical and quantum correlations under the action of various combinations of environments

In this section, we investigate the dynamics of correlations between various bi-partitions of a system consisting of two qubits and two independent noisy environments. In contrast to the previously studied cases where two qubits were coupled to the same kind of noise environment,[28] we here consider qubits A and B coupled to independent and non-identical environments. For example, qubit A may be coupled to an amplitude-damping environment while qubit B to a phase-damping one. We consider three kinds of combined noisy environments, i.e., the amplitude-damping plus the phase-damping environments (APE), the amplitude-damping plus the bit-flip environments (ABE), and the phase-damping plus the phase-flip environments (PPE). In the present investigation, it is always assumed that there is no direct interaction between either the qubits or the environments.

The initial state of the two qubits A and B is given by

$$\rho_{AB}(0) = \frac{1}{4}\sum_{i=0}^{3} c_i \sigma_A^i \otimes \sigma_B^i, \tag{10}$$

where $\sigma_{A,B}^i$ is the Pauli matrix ($i = x, y, z$), $\sigma_{A,B}^0$ is the 2×2 identity matrix, and coefficients c_i ($c_0 \equiv 1$) take real values with the restriction that ρ_{AB} must be positive and normalized. State (10) represents a general class of quantum states including both the

Werner ($|c_1| = |c_2| = |c_3| = a$, $0 < a < 1$) and the Bell ($|c_1| = |c_2| = |c_3| = 1$) states. In the following discussion, we will set $c_1 = c_2 = c_3 = -a$.

3.1. Combining amplitude-damping and phase-damping environments

In this subsection, we will investigate the dynamics of correlations among qubits A, B, and amplitude-damping and phase-damping noisy environments.

The amplitude damping describes a quantum operation, in which the quantum system exchanges energy with the environment. Many physical processes, such as the spontaneously emitting of photon and the relaxation of spin system at high temperature to equilibrium with its environment, can be described by it.[1] The state map corresponding to the action of the amplitude-damping environment over qubit A is given by[28,39,40]

$$|0\rangle_A |0\rangle_{E_A} \to |0\rangle_A |0\rangle_{E_A}, \tag{11}$$

$$|1\rangle_A |0\rangle_{E_A} \to \sqrt{q}\,|1\rangle_A |0\rangle_{E_A} + \sqrt{p}\,|0\rangle_A |1\rangle_{E_A}, \tag{12}$$

where $|0\rangle_A$ and $|1\rangle_A$ are the ground and the excited states of qubit A, respectively, and $|0\rangle_{E_A}$ and $|1\rangle_{E_A}$ are the states of environment A without and with one excitation distributed over all its modes, respectively. Meanwhile, we set $p \in [0, 1]$, which corresponds to the reduced parameter of time, and $q = 1 - p$. The advantage of using p instead of an explicit function of time is the possibility to describe a wide range of physical systems in the same model.[1,28] The Kraus operators corresponding to Eqs. (11) and (12) are[1]

$$\Gamma_1^A = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{q} \end{pmatrix}, \quad \Gamma_2^A = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}. \tag{13}$$

Contrary to the amplitude-damping process, the phasing-damping one describes the loss of quantum information without the loss of energy.[1] The phase-damping action over qubit B can be described by the state map[28,39,40]

$$|0\rangle_B |0\rangle_{E_B} \to |0\rangle_B |0\rangle_{E_B}, \tag{14}$$

$$|1\rangle_B |0\rangle_{E_B} \to \sqrt{q}\,|1\rangle_B |0\rangle_{E_B} + \sqrt{p}\,|1\rangle_B |1\rangle_{E_B}, \tag{15}$$

where the definitions of $|0\rangle_B, |1\rangle_B, |0\rangle_{EB}$, and $|1\rangle_{EB}$ are similar to those in Eqs. (11) and (12). The Kraus operators describing the phase damping are given by[1]

$$\Gamma_1^B = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{q} \end{pmatrix}, \quad \Gamma_2^B = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{pmatrix}. \tag{16}$$

The initial state of the two environments is assumed to be in the vacuum state $|00\rangle_{E_A E_B}$. Combined with Eq. (10), the initial state of the whole system (qubits plus environments) can be written as

$$\rho_{ABE_AE_B}(0) = \frac{1}{4}\left(\sum_{i=0}^{3} c_i \sigma_i^A \otimes \sigma_i^B\right) \otimes |00\rangle_{E_A E_B} \langle 00|. \tag{17}$$

By using Eqs. (13), (16), and (17), the density matrix of the whole system after the initial moment can be written as

$$\rho_{ABE_AE_B} = \sum_{i,j=1}^{2} \Gamma_i^{(A)} \Gamma_j^{(B)} \rho_{ABE_AE_B}(0) \Gamma_i^{(A)\dagger} \Gamma_j^{(B)\dagger}. \tag{18}$$

Since we are interested in the correlation dynamics of various bi-partitions of the whole system, the reduced density matrices are sufficient for our purpose.

Taking the partial trace of $\rho_{ABE_AE_B}$ over all irrelative variables, we can obtain the reduced density matrices for various bi-partitions of the system. In the representation expanded by the basis $\{|00\rangle_{AB}, |01\rangle_{AB}, |10\rangle_{AB}, |11\rangle_{AB}\}$, we have the following results. The reduced density matrix for qubits A and B is given by

$$\rho_{AB} = \frac{1}{4}\begin{pmatrix} a_+ + pa_- & 0 & 0 & qb_- \\ 0 & a_- + pa_+ & qb_+ & 0 \\ 0 & qb_+ & qa_- & 0 \\ qb_- & 0 & 0 & qa_+ \end{pmatrix}, \tag{19}$$

where $a_\pm = (c_0 \pm c_3)$, and $b_\pm = (c_1 \pm c_2)$. The reduced density matrices for bi-partitions AE_A, BE_B, AE_B, and BE_A are given as

$$\rho_{AE_A} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & p & \sqrt{pq} & 0 \\ 0 & \sqrt{pq} & q & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{20}$$

$$\rho_{BE_B} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & q & \sqrt{pq} \\ 0 & 0 & \sqrt{pq} & p \end{pmatrix}, \tag{21}$$

$$\rho_{AE_B} = \frac{1}{4}\begin{pmatrix} 1+pqa_+ & \sqrt{pq}(pa_+ + a_-) & 0 & 0 \\ \sqrt{pq}(pa_+ + a_-) & p(pa_+ + a_-) & 0 & 0 \\ 0 & 0 & q(qa_+ + a_-) & q\sqrt{pq}a_+ \\ 0 & 0 & q\sqrt{pq}a_+ & pqa_+ \end{pmatrix}, \tag{22}$$

$$\rho_{BE_A} = \frac{1}{4}\begin{pmatrix} a_+ + qa_- & 0 & 0 & \sqrt{pq}b_- \\ 0 & pa_- & \sqrt{pq}b_+ & 0 \\ 0 & \sqrt{pq}b_+ & a_- + qa_+ & 0 \\ \sqrt{pq}b_- & 0 & 0 & pa_+ \end{pmatrix}. \tag{23}$$

From Eqs. (20) and (23), we can see that the reduced density matrices for bi-partitions AE_A and BE_B are independent of c_i, so the correlation dynamical behaviours of the partitions are not affected by the initial states. The reduced density matrix for partition E_AE_B is given by

$$\rho_{E_AE_B} = \frac{1}{4}\begin{pmatrix} (1+q^2)a_+ + 2qa_- & \sqrt{pq}(a_- + qa_+) & 0 & 0 \\ \sqrt{pq}(a_- + qa_+) & p(a_- + qa_+) & 0 & 0 \\ 0 & 0 & p(a_- + qa_+) & p\sqrt{pq}a_+ \\ 0 & 0 & p\sqrt{pq}a_+ & p^2a_+ \end{pmatrix}. \tag{24}$$

Explicitly knowing the reduced density matrices for all the bi-partitions, we are in a position to investigate the dynamical behaviours of various correlations, i.e., total correlation (3), classical correlation (8), quantum correlation (9), and the entanglement. To quantify the entanglement, we use the Wootters[41] concurrence, which is defined as $C = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$, where λ_i are the eigenvalues of matrix $R = \rho_{AB}(\sigma_A^y \otimes \sigma_B^y)\rho_{AB}^*(\sigma_A^y \otimes \sigma_B^y)$ arranged in the decreasing order of magnitude, ρ_{AB}^* is the complex conjugate of ρ_{AB}, and $\sigma_{A,B}^y$ are the Pauli matrices for subsystems A and B. The concurrence attains its maximum value 1 for maximally entangled states and 0 for separable states.

In Fig. 1, various correlations are plotted each as a function of parameter p for bi-partitions AB and E_AE_B, where the initial state is the Werner state with $a = 0.4$. We can see that entanglement shows the sudden death (ESD) behaviour.[42–44] Contrary to what happens to the entanglement, the quantum correlation of AB has no sudden death but monotonically decreases to zero in the asymptotic limit ($p = 1$). It indicates the robustness of the quantum correlation to the decoherence.[22,23]

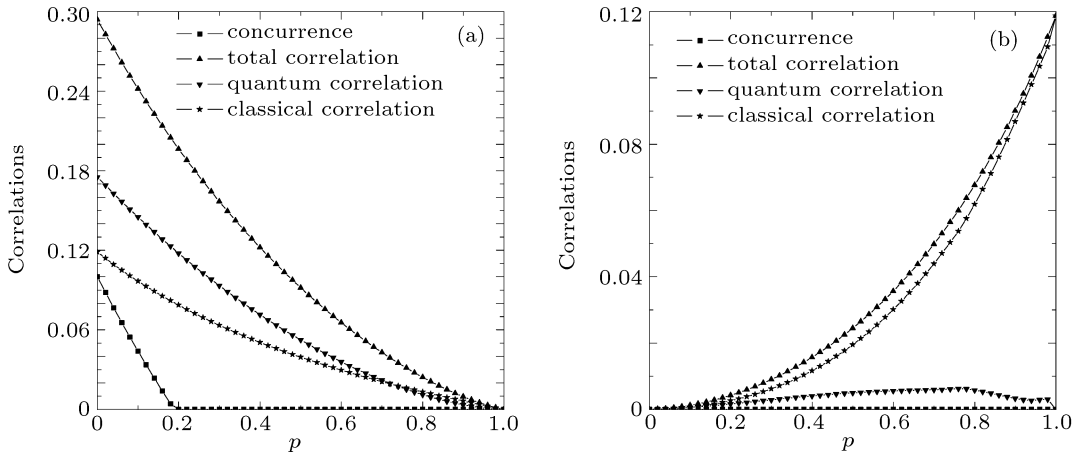


Fig. 1. Correlation dynamics for partitions AB (a) and E_AE_B (b) under the action of the amplitude-damping and phase-damping reservoirs. The initial state is the Werner state ($a = 0.4$).

Although the classical correlation between qubits A and B decreases monotonically as p increases, the classical correlation between E_A and E_B increases monotonically and arrives at the maximal value in the asymptotic limit ($p = 1$). In this sense, we may say that the classical correlation lost from partition AB partially transfers to partition E_AE_B.

However, much different from the quantum correlation between A and B, the quantum correlation of partition E_AE_B first increases and finally decreases to zero in the asymptotic limit, and shows the sudden change behaviour[13] during the evolution. From Fig. 1, we can see that there is no entanglement between E_A and E_B in the whole p range. Therefore, in contrast to the classical correlation, the entanglement and the quantum correlation of partition AB are not transferred to the two environments. This result is much different from that obtained only with the amplitude-damping channels[28] presented in Ref. [32].

The correlation dynamics for other bi-partitions, i.e., AE_A, AE_B, BE_A, and BE_B, are shown in Fig. 2. In these figures, we can see that the correlation dynamics between A and the environments differ from that between B and the environments. All the correlations (classical and quantum correlations) between A and the environments completely vanish in the asymptotic limit. However, the classical correlation can be created between B and E_A (E_B). This result differs from the situation with only the amplitude-damping environments, where the qubits can only have the nonclassical correlation with its own environment.[28] Another interesting result shown in Fig. 2 is that both the classical and the quantum correlations of BE_A exhibit the sudden change behaviour.[13] Figure 2 also shows that during the evolution, the quantum correlation can be constructed in all the bi-partitions, while the entanglement can be created only in AE_A.

The correlation dynamics with the Bell initial state ($a = 1$) are plotted in Fig. 3. It is very interesting that in the asymptotic limit the quantum correlation of AB always remains and is equal to its initial value 1, while the entanglement and the classical correlation decay to zero. This result completely differs from that of the Werner state shown in Fig. 2. It shows that the nonclassical correlation can exist even in a separable state.[2] We also observe that the classical correlation transfers to partitions E_AE_B and BE_A.

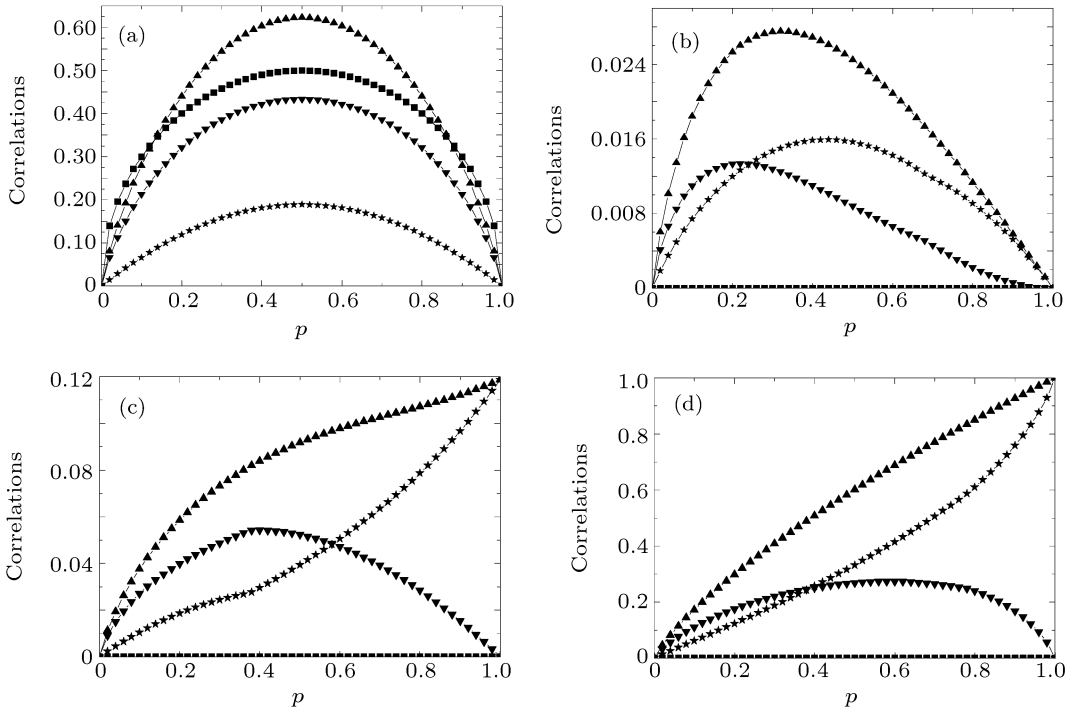


Fig. 2. Correlation dynamics for partitions AE_A (a), AE_B (b), BE_A (c), and BE_B (d) under the action of amplitude-damping and phase-damping environments. The initial state is the Werner state ($a = 0.4$). The lines marked with triangle, flipped triangle, star, and square represent total, quantum, classical correlations and the entanglement, respectively.

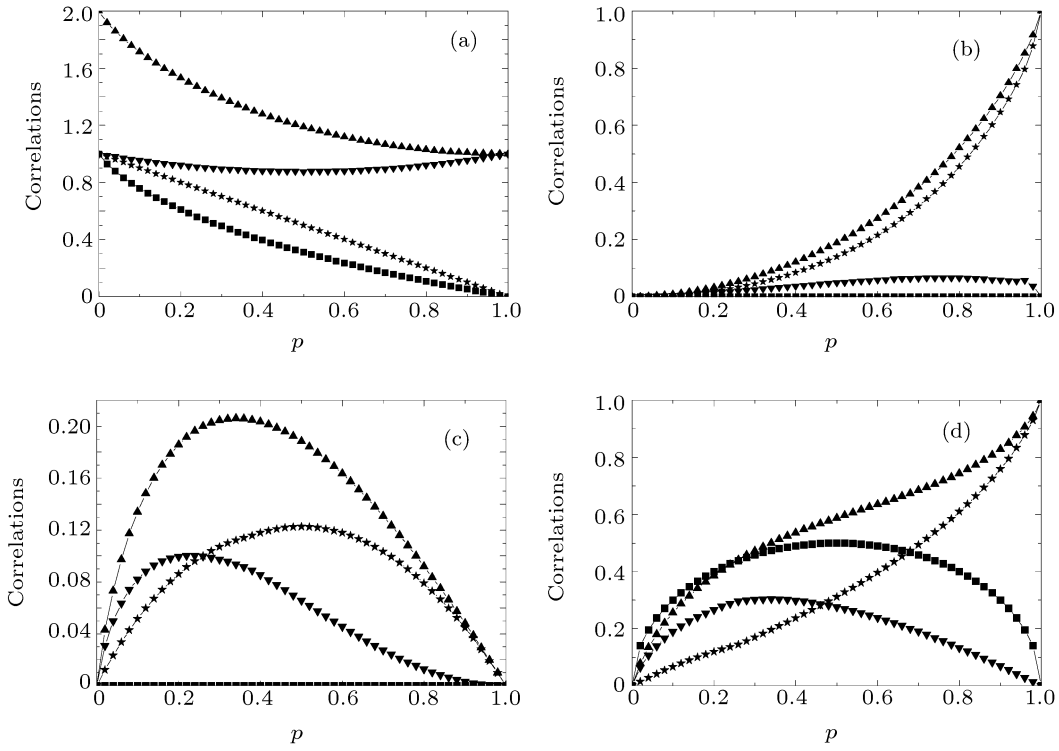


Fig. 3. Correlation dynamics for partitions AB (a), E_AE_B (b), AE_B (c), and BE_A (d) with the amplitude-damping and phase-damping environments. The initial state is the Bell state ($a = 1$). The lines marked with triangle, flipped triangle, star, and square represent total, quantum, classical correlations and the entanglement, respectively.

3.2. Combining amplitude-damping and bit-flip environments

Now we consider the correlation dynamics for various bi-partitions of the system under the action of the amplitude-damping and the flip reservoirs. Suppose that qubits A and B are coupled to the amplitude damping and the bit-flip environments, respectively. The effect of the flip environment on qubit B can be described by the state map[39,40]

$$\begin{aligned} &|0\rangle_B\,|0\rangle_{E_B} \\ \rightarrow\ &\sqrt{p}\,|0\rangle_B\,|0\rangle_{E_B} + (\mathrm{i})^k\sqrt{q}\,|1\rangle_B\,|1\rangle_{E_B}\,, \qquad (25) \\ &|1\rangle_B\,|0\rangle_{E_B} \\ \rightarrow\ &\sqrt{p}\,|1\rangle_B\,|0\rangle_{E_B} + (-\mathrm{i})^k\sqrt{q}\,|0\rangle_B\,|1\rangle_{E_B}\,, \qquad (26) \end{aligned}$$

where $k = 0$ for the bit-flip environment, $k = 1$ for the bit-phase-flip environment. Since the correlation dynamics do not depend on k, in what follows, we consider only the bit flip environment ($k = 0$). The Kraus operators of the bit-flip are[1]

$$\Gamma_1^B = \begin{pmatrix} \sqrt{p} & 0 \\ 0 & \sqrt{p} \end{pmatrix}, \quad \Gamma_2^B = \begin{pmatrix} 0 & \sqrt{q} \\ \sqrt{q} & 0 \end{pmatrix}. \qquad (27)$$

The initial state of the whole system is still supposed to be that given by Eq. (17). From Eqs. (11), (12), (25), and (26), we can obtain the density matrix of the whole system. The reduced density matrices for various bi-partitions of the system can then be analytically obtained, which are given in appendix A.

In Fig. 4, the correlation dynamics for bipartitions AB and BE_A are shown. We observe that the total, the classical, and the quantum correlations of AB vanish at the asymptotic limit $p = 1$, while the entanglement of AB suddenly and completely disappears at a finite point. It is also noticed that in the asymptotic limit, all the correlations including the entanglement are fully transferred to partition BE_A rather than that of the two environments E_A and E_B.[28,32]

In Fig. 5, the correlation dynamics for the other four bi-partitions are shown. From these figures, we can see that although there exist bipartite correlations during the evolution process, they all vanish in the asymptotic limit. The four bi-partitions AE_A, AE_B, BE_B, and BE_A can then be treated as transfer bridges for the correlations.

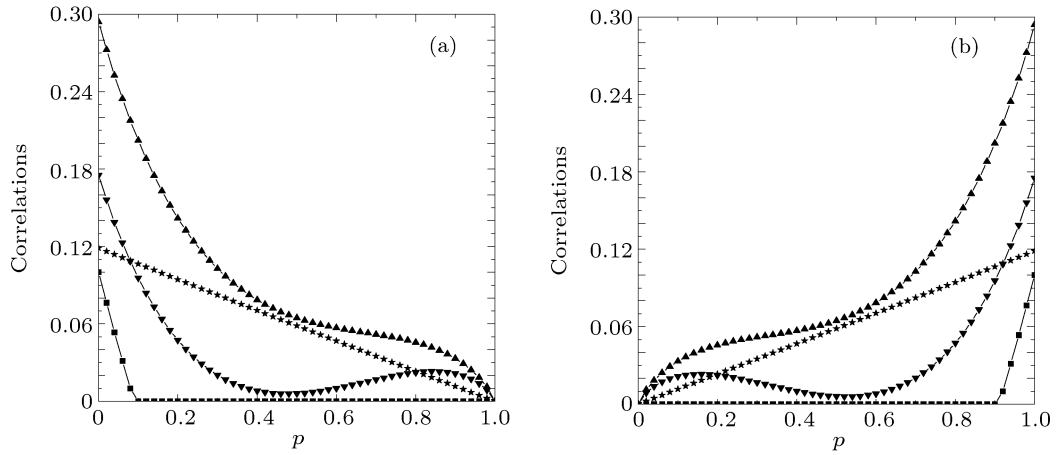


Fig. 4. Correlation dynamics for partitions AB (a) and BE_A (b) with the combined amplitude damping and bit-flip environments. The initial state is the Werner state ($a = 0.4$). The lines marked with triangle, flipped triangle, star, and square represent total, quantum, classical correlations and the entanglement, respectively.

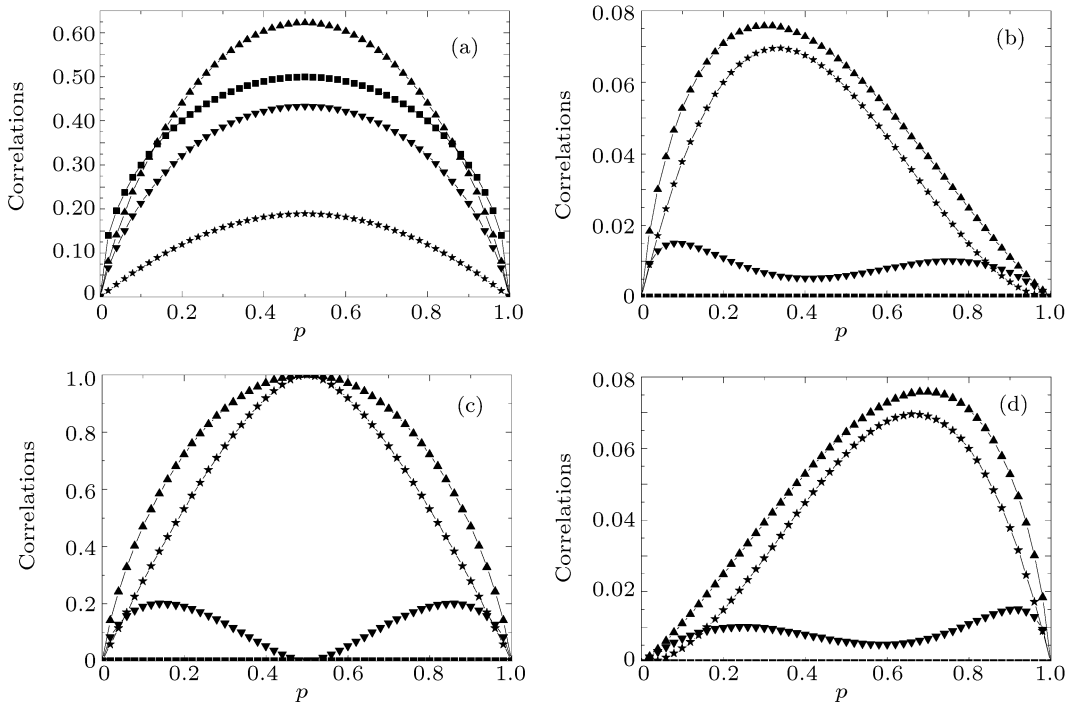


Fig. 5. Correlation dynamics for partitions AE_A (a), AE_B (b), BE_B (c), and E_AE_B (d) with the combined amplitude-damping and bit-fip environments. The initial state is the Werner state ($a = 0.4$). The lines marked with triangle, flipped triangle, star, and square represent total, quantum, classical correlations and the entanglement, respectively.

In Fig. 6, the correlation dynamics for various partitions of the system with the Bell initial state are shown. It is noticed that all the correlations and the entanglement of AB can be fully transferred to partition BE_A. We also find that there is the death and revival of the entanglement under the action of the combined amplitude-damping and bit-flip environments for the Bell state with $a = 1$.

From the above discussion, we obtain a very interesting result that all the quantum and the classical correlations, and even the entanglement, originally prepared between qubits A and B, can be completely transferred to partition BE_A under the action of the amplitude-damping and the bit-flip environments.

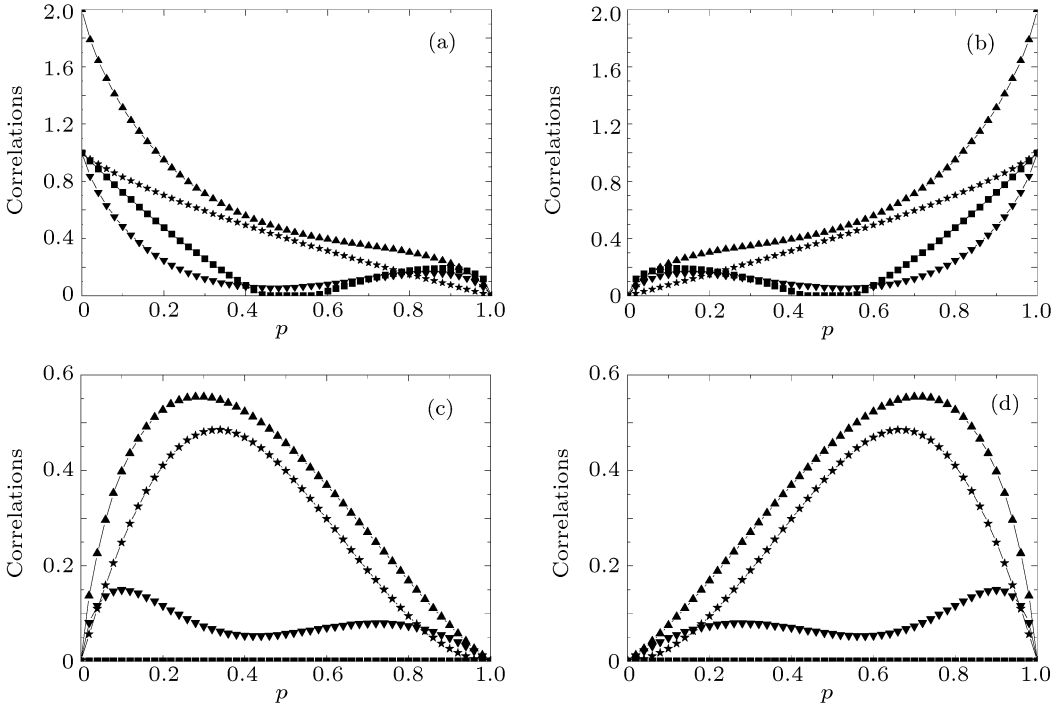


Fig. 6. Correlation dynamics for partitions AB (a), E_AE_B (b), AE_B (c), and BE_A (d) with the combined amplitude damping and bit-flip environments. The initial state is the Bell state ($a = 1$). The lines marked with triangle, flipped triangle, star, and square represent total, quantum, classical correlations and the entanglement, respectively.

3.3. Combining phase-damping and phase-flip environments

The correlation dynamics when two quits A and B interact independently and locally with the phase-damping and the phase-flip environments are investigated in this subsection. The corresponding state map induced by the phase-flip environment over qubit B is given by[39,40]

$$|0\rangle_B|0\rangle_{E_B} \to \sqrt{p}\,|0\rangle_B|0\rangle_{E_B} + \sqrt{q}\,|0\rangle_B|1\rangle_{E_B}\,, \quad (28)$$

$$|1\rangle_B|0\rangle_{E_B} \to \sqrt{p}\,|1\rangle_B|0\rangle_{E_B} - \sqrt{q}\,|1\rangle_B|1\rangle_{E_B}\,. \quad (29)$$

The Kraus operators of the phase-flip environment are[1]

$$I_1^B = \begin{pmatrix} \sqrt{p_1} & 0 \\ 0 & \sqrt{p_1} \end{pmatrix}, \quad I_2^B = \begin{pmatrix} \sqrt{q_1} & 0 \\ 0 & -\sqrt{q_1} \end{pmatrix}. \quad (30)$$

It is known that a unitary recombination of the elements in Eq. (16) can give the expression of Eq. (30).[1] In particular, the phase-damping process is exactly the same as the phase-flip one when $p_1 = (1 + \sqrt{1-q})/2$. In the present investigation, we set $p_1 = p$ and $q_1 = 1 - p_1$, which guarantees that A and B undergo non-identical noisy environments.

Starting from the initial state given in Eq. (17) and using the same method mentioned in the preceding subsections, we can obtain the reduced density matrices for various bi-partitions of the system, which are shown in appendix B.

In Fig. 7, the correlation dynamics for partitions AB and E_AE_B are shown. It can be seen that the total and the quantum correlations display the decay and revival behaviours, while the entanglement exhibits the ESD. This result differs from that of the pure phase-damping environment where the quantum and the total correlations monotonically decrease with the increasing p.[28] In particular, the classical correlation of AB remains a constant in the whole range. This indicates that the classical correlation is immune to the decoherence. This result may lead to an operational way of measuring both the classical and the quantum correlations in a composite system.[13] All the correlations of E_AE_B first increase and reach the maximal values with the increasing p, and then decrease to zero in the asymptotic limit ($p = 1$). This phenomenon also differs from that of the pure phase-damping environment where the classical and the quantum correlations between the environments reach the same finite value at $p = 1$.[28]

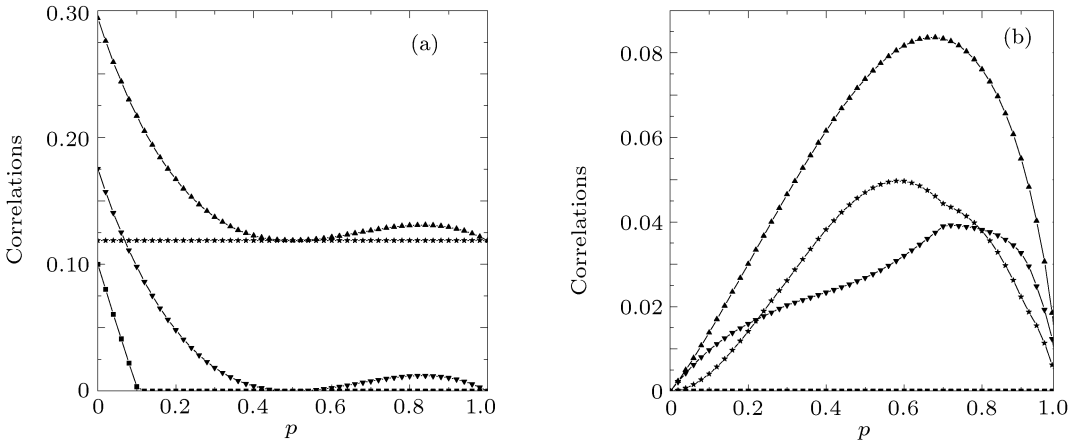


Fig. 7. Correlation dynamics for partitions AB (a) and E_AE_B (b) with the combined phase-damping and phase flip environments. The initial state is the Werner state ($a = 0.4$). The lines marked with triangle, flipped triangle, star, and square represent total, quantum, classical correlations and the entanglement, respectively.

Figure 8 shows the correlation evolution for other bi-partitions of the system. It is observed that only the classical correlation can be created between bi-partitions AE_A and BE_A in the asymptotic limit ($p = 1$). The quantum correlation can exist among the various bi-partitions only during the evolution. However, there exists no entanglement between any partitions during the whole evolution, and the entanglement exiting in the initial state is fully evaporated, which is very similar to that of the pure phase-damping noisy environment.[28]

The correlation dynamics are shown in Fig. 9 when the two qubits are initially in the Bell state. In these figures, it is seen that the entanglement displays the death and revival phenomenon and the classical correlation keeps a constant during the evolution. The quantum correlation and the entanglement existing in the initial state are fully evaporated in the asymptotic limit ($p = 1$). As shown in Fig. 8, only the classical correlation between qubit A (B) and the phase-damping environment can be created.

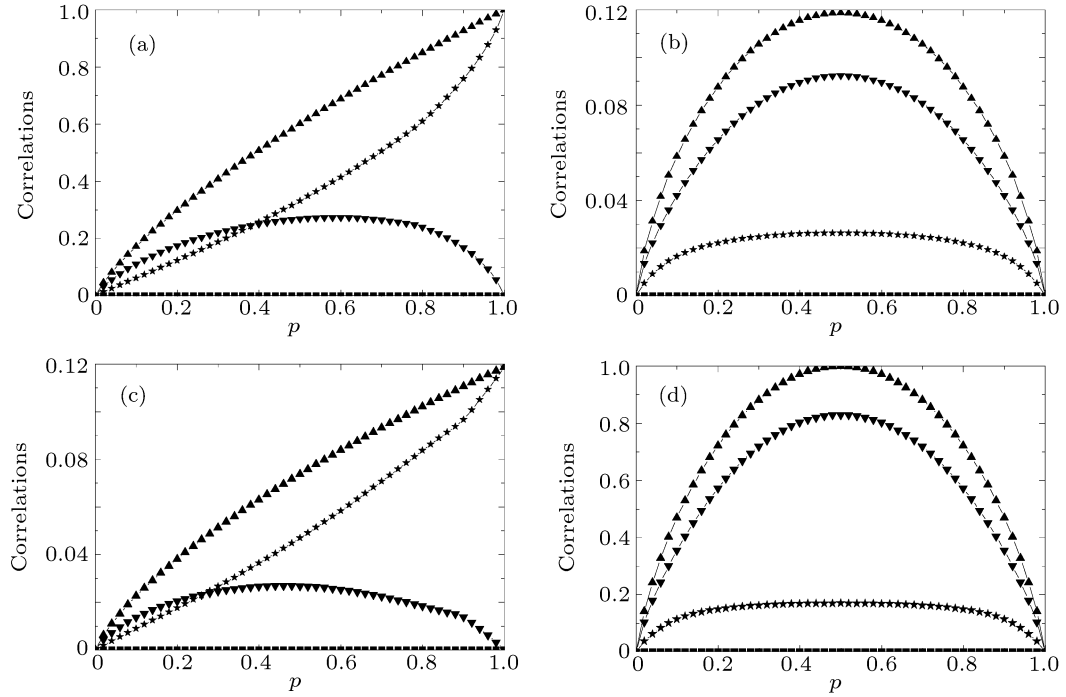


Fig. 8. Correlation dynamics for partitions AE_A (a), AE_B (b), BE_A (c), and BE_B (d) with the combined phase-damping and phase-flip environments for the Werner state ($a = 0.4$). The lines marked with triangle, flipped triangle, star, and square represent total, quantum, classical correlations and the entanglement, respectively.

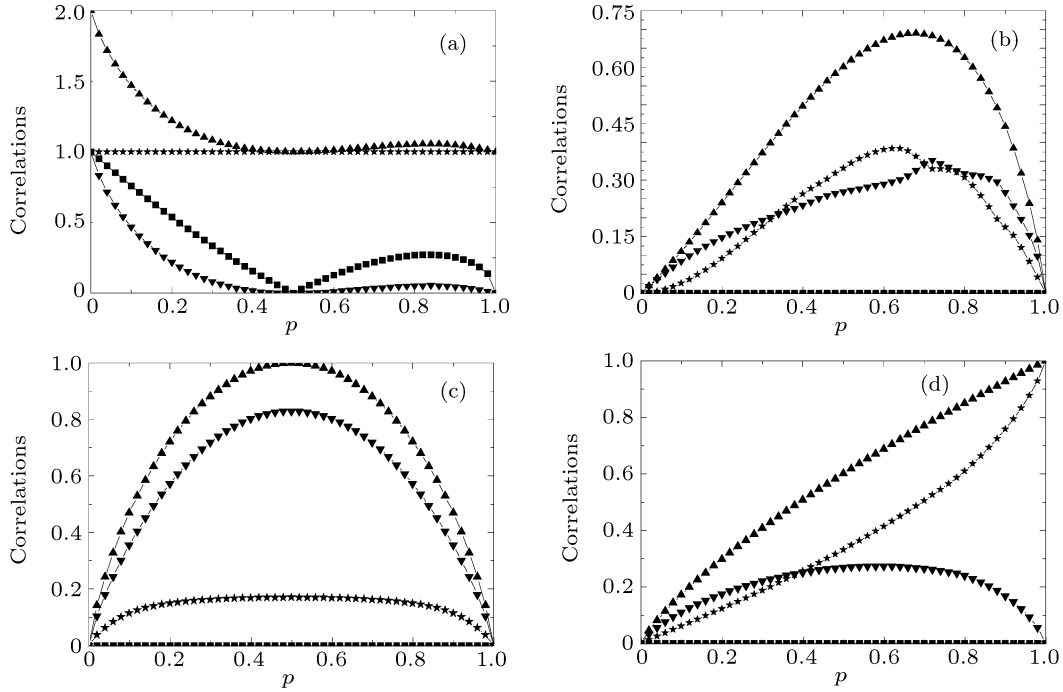


Fig. 9. Correlation dynamics for partitions AB (a), E_AE_B (b), AE_B (c), and BE_A (d) with the combined phase-damping and phase-flip environments for the Bell initial state ($a = 1$). The lines marked with triangle, flipped triangle, star, and square represent total, quantum, classical correlations and the entanglement, respectively.

4. Summary

We study the quantum and the classical correlation (including the entanglement) dynamics of two qubits A and B, which are independently and locally coupled to two non-identical environments E_A and E_B, respectively. Three different combinations of the noisy environments are considered, i.e., the amplitude plus phase damping environments (APE), the amplitude-damping plus the bit-flip environments (ABE), and the phase-damping plus the phase-flip environments (PPE). We find that contrary to the single-type noisy environment case, the entanglement and the quantum correlation transfer direction can be controlled by combining different noisy environments. This may provide us a way of manipulating the correlation dynamics of a composite system. The amplitude-damping environment plays a very important role in the correlation dynamics and determines whether there exists the entanglement transfer among bi-partitions of a composite system. If there is no amplitude damping, e.g., in the PPE, the initial entanglement is completely evaporated. In the ABE case, we show that all the quantum and the classical correlations, and even the entanglement, which exist in the initial state of the two qubits, can be completely transferred to bi-partition BE_A without any loss. When the initial state is the Bell one, in the APE case, the quantum correlation between the two qubits can remain the initial value, while the classical correlation and the entanglement decay to zero. In the PPE case, the classical correlation keeps a constant, while the quantum correlation and the entanglement decay to zero. This result shows that it is possible to distinguish the quantum correlation from the classical one and the entanglement by combining different kinds of environments. In the combined environment of ABE and PPE, even for the Bell state, the entanglement can display the sudden death (ESD) and the revival phenomena. We also notice that the quantum correlation can appear in arbitrary bi-partitions and has no sudden death behaviour during the evolution, which indicates the robustness of the quantum correlation to the decoherence. Finally, we should point out a possible way to realize the suggested model. We take two spins as the qubits. The coupling of one spin with an amplitude-damping reservoir may be realized with the aid of the NMR and the quantum optical techniques in solid-state quantum systems.[45,46] In those systems, the relaxation of electron (nuclear) spins to their surroundings is a typical amplitude-damping process.[47] The coupling of a spin with a phase-damping reservoir may be experimentally engineered by using atoms in optical lattices.[48−50] In that case, one qubit transversely interacts with an Ising spin environment and

indeed undergoes a phase-damping process. A challenge problem in that realization is to combine the two processes together.

Appendix A: reduced density matrices for various bi-partitions under the action of amplitude-damping and bit-flip environments

The reduced density matrix for partition AB obtained by taking the partial trace of $\rho_{ABE_AE_B}$ over the degrees of freedom of the reservoir is given by

$$\rho_{AB} = \frac{1}{4}\begin{pmatrix} 2p^2+qa_- & 0 & 0 & \sqrt{q}(pb_-+qb_+) \\ 0 & 2p^2+qa_+ & \sqrt{q}(pb_++qb_-) & 0 \\ 0 & \sqrt{q}(pb_++qb_-) & q(pa_-+qa_+) & 0 \\ \sqrt{q}(pb_-+qb_+) & 0 & 0 & q(qa_-+pa_+) \end{pmatrix}. \tag{A1}$$

For partitions AE_A, BE_B, AE_B and BE_A, the reduced density operators are

$$\rho_{AE_A} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & p & \sqrt{pq} & 0 \\ 0 & \sqrt{pq} & q & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A2}$$

$$\rho_{BE_B} = \frac{1}{2}\begin{pmatrix} p & 0 & 0 & \sqrt{pq} \\ 0 & q & \sqrt{pq} & 0 \\ 0 & \sqrt{pq} & p & 0 \\ \sqrt{pq} & 0 & 0 & q \end{pmatrix}, \tag{A3}$$

$$\rho_{AE_B} = \frac{1}{2}\begin{pmatrix} (p+p^2) & 0 & 0 & q\sqrt{p}c_1 \\ 0 & (1+p)q & q\sqrt{p}c_1 & 0 \\ 0 & q\sqrt{p}c_1 & pq & 0 \\ q\sqrt{p}c_1 & 0 & 0 & q^2 \end{pmatrix}, \tag{A4}$$

$$\rho_{BE_A} = \frac{1}{4}\begin{pmatrix} (1+p)D & 0 & 0 & \sqrt{p}B \\ 0 & pC & \sqrt{p}A & 0 \\ 0 & \sqrt{p}A & (1+p)C & 0 \\ \sqrt{p}B & 0 & 0 & pD \end{pmatrix}, \tag{A5}$$

where $A = (qb_- + pb_+)$, $B = (pb_- + qb_+)$, $C = (pa_- + qa_+)$, and $D = (pa_+ + qa_-)$.

After tracing out the degrees of freedom of the two qubits, we obtain the reduced density matrix for the two environments E_A and E_B,

$$\rho_{E_AE_B} = \frac{1}{2}\begin{pmatrix} p(1+q) & 0 & 0 & p\sqrt{q}c_1 \\ 0 & q(2-p) & p\sqrt{q}c_1 & 0 \\ 0 & p\sqrt{q}c_1 & p^2 & 0 \\ p\sqrt{q}c_1 & 0 & 0 & pq \end{pmatrix}. \tag{A6}$$

Appendix B: density matrices for various bi-partitions under the action of phase-damping and phase-flip environments

The reduced density matrix for AB under the influence of the mixed phase damping and phase flip environments is

$$\rho_{AB} = \frac{1}{4}\begin{pmatrix} a_+ & 0 & 0 & \sqrt{q}(p-q)b_- \\ 0 & a_- & \sqrt{q}(p-q)b_+ & 0 \\ 0 & \sqrt{q}(p-q)b_+ & a_- & 0 \\ \sqrt{q}(p-q)b_- & 0 & 0 & a_+ \end{pmatrix}. \tag{B1}$$

For partitions AE_A, BE_B, AE_B, and BE_A, the reduced density operators are analytically given by

$$\rho_{AE_A} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & q & \sqrt{pq} \\ 0 & 0 & \sqrt{pq} & p \end{pmatrix}, \tag{B2}$$

$$\rho_{BE_B} = \frac{1}{2}\begin{pmatrix} p & \sqrt{pq} & 0 & 0 \\ \sqrt{pq} & q & 0 & 0 \\ 0 & 0 & p & -\sqrt{pq} \\ 0 & 0 & -\sqrt{pq} & q \end{pmatrix}, \tag{B3}$$

$$\rho_{AE_B} = \frac{1}{2}\begin{pmatrix} p & \sqrt{pq}c_3 & 0 & 0 \\ \sqrt{pq}c_3 & q & 0 & 0 \\ 0 & 0 & p & -\sqrt{pq}c_3 \\ 0 & 0 & -\sqrt{pq}c_3 & q \end{pmatrix}, \tag{B4}$$

$$\rho_{BE_A} = \frac{1}{4}\begin{pmatrix} a_+ + qa_- & 0 & \sqrt{pq}a_- & 0 \\ 0 & a_- + qa_+ & 0 & \sqrt{pq}a_+ \\ \sqrt{pq}a_- & 0 & pa_- & 0 \\ 0 & \sqrt{pq}a_+ & 0 & pa_+ \end{pmatrix}. \tag{B5}$$

Similarly, the reduced density matrix for partition E_AE_B is

$$\rho_{E_AE_B} = \frac{1}{2}\begin{pmatrix} p(1+q) & -p\sqrt{pq}c_3 & p\sqrt{pq} & -pqc_3 \\ -p\sqrt{pq}c_3 & q(1+q) & -pqc_3 & q\sqrt{pq} \\ p\sqrt{pq} & -pqc_3 & p^2 & -p\sqrt{pq}c_3 \\ -pqc_3 & q\sqrt{pq} & -p\sqrt{pq}c_3 & pq \end{pmatrix}. \tag{B6}$$

References

[1] Neilsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)

[2] Ollivier H and Zurek W H 2001 *Phys. Rev. Lett.* **88** 017901

[3] Henderson L and Vedral V 2011 *J. Phys.* A **34** 68899

[4] Vedral V 2003 *Phys. Rev. Lett.* **90** 050401

[5] Lanyon B P, Barbieri M, Almeida M P and White A G 2008 *Phys. Rev. Lett.* **101** 200501

[6] Knill E and Laflamme R 1992 *Phys. Rev. Lett.* **81** 5672

[7] Oppenheim J, Horodecki M, Horodecki P and Horodecki R 2002 *Phys. Rev. Lett.* **89** 180402

[8] Groisman B, Popescu S and Winter A 2005 *Phys. Rev.* A **72** 032317

[9] Luo S 2008 *Phys. Rev.* A **77** 022301

[10] Modi K, Paterek T, Son W, Vedral V and Williamson M 2010 *Phys. Rev. Lett.* **104** 080501

[11] Chen Q Y, Fang M F, Xiao X and Zhou X F 2011 *Chin. Phys.* B **20** 050302

[12] Wang L C, Yan J Y and Yi X X 2011 *Chin. Phys.* B **20** 040305

[13] Maziero J, Céleri L C, Serra R M and Vedral V 2009 *Phys. Rev.* A **80** 044102

[14] Mazhar Ali, Rau A R P and Alber G 2010 *Phys. Rev.* A **81** 042105

[15] Dillenschneider R 2008 *Phys. Rev.* B **78** 224413

[16] Sarandy M S 2009 *Phys. Rev.* A **80** 022108

[17] Werlang T and Rigolin G 2010 *Phys. Rev.* A **81** 044101

[18] Werlang T, Trippe C, Ribeiro G A P and Rigolin G 2010 *Phys. Rev. Lett.* **105** 095702

[19] Sun Z Y, Li L, Yao K L, Du G H, Liu J W, Luo B, Li N and Li H N 2010 *Phys. Rev.* A **82** 032310

[20] Giorda P and Paris M G A 2010 *Phys. Rev. Lett.* **105** 020503

[21] Adesso G and Datta A 2010 *Phys. Rev. Lett.* **105** 030501

[22] Werlang T, Souza S, Fanchini F F and Villas Boas C J 2009 *Phys. Rev.* A **80** 024103

[23] Fanchini F F, Werlang T, Brasil C A, Arruda L G E and Caldeira A O 2010 *Phys. Rev.* A **81** 052107

[24] Mazzola L, Piilo J and Maniscalco S 2010 arXiv: 1006.1805

[25] Mazzola L, Piilo J and Maniscalco S 2010 *Phys. Rev. Lett.* **104** 200401

[26] Céleri L C, Landulfo A G S, Serra R M and Matsas G E A 2010 *Phys. Rev.* A **81** 062130

[27] Fanchini F F, Castelano L K and Caldeira A O 2010 *New J. Phys.* **12** 073009

[28] Maziero J, Werlang T, Fanchini F F, Céleri L C and Serra R M 2010 *Phys. Rev.* A **81** 022116

[29] Xu J S, Xu X Y, Li C F, Zhang C J, Zou X B and Guo G C 2010 *Nat. Commun.* **1** 7

[30] Xu J S, Li C F, Zhang C J, Xu X Y, Zhang Y S and Guo G C 2010 *Phys. Rev.* A **82** 042328

[31] Soares-Pinto D O, Céleri L C, Auccaise R, Fanchini F F, de Azevedo E R, Maziero J, Bonagamba T J and Serra R M 2010 *Phys. Rev.* A **81** 062118

[32] López C E, Romero G, Lastra F, Solano E and Retamal J C 2008 *Phys. Rev. Lett.* **101** 080503

[33] Groisman B, Popescu S and Winter A 2005 *Phys. Rev.* A **72** 032317

[34] Schumacher B and Westmorel M D 2006 *Phys. Rev.* A **74** 042305

[35] Terhal B M, Horodecki M, Leung D W and DiVincenzo D P 2002 *J. Math. Phys.* **43** 4286

[36] DiVincenzo D P, Horodecki M, Leung D W, Smolin J A and Terhal B M 2004 *Phys. Rev. Lett.* **92** 067902

[37] Piani M, Horodecki P and Horodecki R 2008 *Phys. Rev. Lett.* **100** 090502

[38] Piani M, Christandl M, Mora C E and Horodecki P 2009 *Phys. Rev. Lett.* **102** 250503

[39] Leung D W 2003 *J. Math. Phys.* **44** 528

[40] Salles A, de Melo F, Almeida M P, Hor-Meyll M, Walborn S P, Souto Ribeiro P H and Davidovich L 2008 *Phys. Rev.* A **78** 022322

[41] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245

[42] Yu T and Eberly J H 2004 *Phys. Rev. Lett.* **93** 140404

[43] Yu T and Eberly J H 2006 *Phys. Rev. Lett.* **96** 140403

[44] Yu T and Eberly J H 2009 *Science* **323** 598

[45] Stepanenko D, Burkard G, Giedke G and Imamoglu A 2006 *Phys. Rev. Lett.* **96** 136401

[46] Lai C W, Maletinsky P, Badolato A and Imamoglu A 2006 *Phys. Rev. Lett.* **96** 167403

[47] Scala M, Migliore R and Messina A 2008 *J. Phys.* A **41** 435304

[48] Rossini D and Calarco T 2007 *Phys. Rev.* A **75** 032333

[49] Cai J M, Zhou Z W and Guo G C 2005 *Phys. Rev.* A **72** 022312

[50] Duan L M, Demler E and Lukin M D 2003 *Phys. Rev. Lett.* **9** 090402