Distribution of non-Markovian intervals for open qubit systems*

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We study the non-Markovianity of open qubit systems using the measure $\mathcal{N}$ proposed by Breuer, Laine and Piilo [*Phys. Rev. Lett.* **103** 210401 (2009)]. We find that for the three types of quantum noises, amplitude-damping, dephasing and depolarizing noises, there exist some non-Markovian time intervals whose distribution is independent of the selection of the pair of initial states. Therefore, the maximization in the definition of measure $\mathcal{N}$ can be actually removed without influencing the detection of non-Markovianity.

Keywords: open quantum system, non-Markovianity, non-Markovian interval

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1. Introduction

The evolution of open quantum systems can be divided into two basic types, Markovian and non-Markovian processes.[1] For Markovian processes, the correlation time between the system and environment is considered to be infinitesimally small so that the dynamical map does not carry any memory effects, leading to a monotonic flow of the information from system to environment. In contrast, non-Markovian processes with memory have different dynamical traits which give rise to the back flow of information from environment to the system.[2,3] Recently, it has been found that non-Markovian processes can lead to distinctly different effects on the decoherence and disentanglement[4−6] of open systems compared with Markovian processes. Many relevant physical systems, such as the quantum optical system,[1] quantum dot,[7] superconductor system[8] and colour-core spin in semiconductor,[9] could not be described simply by Markovian dynamics. In addition, quantum chemistry[10] and excitation transfer of biological system[11] also need to be treated as non-Markovian processes. Because of these distinct properties and extensive applications, more and more attention and interest have been devoted to the study of non-Markovian processes of open systems, including the measures of non-Markovianity,[2,3,12−15] the positivity,[16−18] and some other dynamical properties[19−24] and approaches[25,26] of non-Markovian processes. Experimentally, the simulation[27,28] of non-Markovian environment has been realized.

The measure of non-Markovianity of quantum evolution is a fundamental problem which aims to detect whether a quantum process is non-Markovian and how much degree it deviates from a Markovian one. So far, almost all measures can only be served to settle the former, i.e., served as the sufficient (not necessary) condition for the emergence of non-Markovianity. Therefore, the problem for measuring the non-Markovianity of quantum processes remain to be elusive and, in some sense, controversial. Based on the distinguishability of quantum states, Breuer *et al.*[2] proposed a measure to detect the non-Markovianity of quantum processes which is linked to the flow of information between system and environment. Alternatively, Rivas *et al.*[2] also presented a measure of non-Markovianity by exploiting the dynamical behaviour of quantum entanglement under the local trace-preserving completely positive (CP) maps. Other measures of non-Markovianity include one proposed by Wolf *et al.*[13] based on the breakdown of the semigroup property, one proposed by Lu *et al.*[15] using quantum Fisher information flow and the ones proposed by Usha Devi *et al.*[14] using relative

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entropy difference and fidelity difference.

It is noted that the measure proposed in Ref. [2] includes a maximization which gives rise to enormous trouble in the practical calculations. Several studies[29,30] were aimed at this problem. In this paper, we show that for the three types of quantum noises, amplitude-damping, dephasing and depolarizing noises, there exist some non-Markovian time intervals whose distribution is independent of the selection of the pair of initial states. Therefore, the maximization in the definition of measure $\mathcal{N}$ can be actually removed without influencing the detection of non-Markovianity. In other words, for a quantum non-Markovian process, any pair of initial states will consistently lead to $\mathcal{N} > 0$. The three types of quantum noises involved here are most fundamental in the theoretical and experimental studies of dynamics of open qubit systems. Furthermore, the result is valid for arbitrary-structured spectral density. Thus, the result is important and adaptable.

The article is organized as follows. In Section 2, we briefly review the definition of measure $\mathcal{N}$ proposed by Breuer *et al.*[2] And in Sections 3, 4 and 5, we study the distribution properties of non-Markovian intervals respectively for an open qubit system coupled to amplitude damping, dephasing and depolarizing non-Markovian noises. A conclusion is arranged in Section 6.

2. Review of the non-Markovianity $\mathcal{N}$

In Ref. [2], the authors presented a measure of non-Markovianity for quantum processes of open systems, which is based on the idea that Markovian processes tend to continuously reduce the trace distance between any two states of a quantum system. Thus, an increase in trace distance during any time intervals implies the emergence of non-Markovianity. In quantum information science, the trace distance is defined as[31]

$$D(\rho_1, \rho_2) = \frac{1}{2} \operatorname{tr} |\rho_1 - \rho_2|, \tag{1}$$

with trace norm defined as $|A| = \sqrt{A^+ A}$. Considering that the measure should reveal the total feature of a whole quantum process, the authors thus suggest the quantity

$$\mathcal{N} = \max_{\rho_{1,2}(0)} \int_{\sigma>0} \mathrm{d}t \sigma(t, \rho_{1,2}(0)) \tag{2}$$

as the measure of non-Markovianity of a quantum process. Here

$$\sigma(t, \rho_{1,2}(0)) = \frac{\mathrm{d}}{\mathrm{d}t} D(\rho_1(t), \rho_2(t)) \tag{3}$$

denotes the time derivative of trace distance for a pair of dynamical states $\rho_1(t)$ and $\rho_2(t)$ with initial values $\rho_{1,2}(0)$. The time integration is extended over all intervals in which σ is positive and the maximum is taken over all pairs of initial states. For any Markovian process, $\mathcal{N} = 0$. And if $\mathcal{N} > 0$, the process must be non-Markovian. The authors further linked the changes of trace distance to the flow of information between a system and its environment, and concluded that the back flow of information from environment to the system is the key feature of a non-Markovian dynamics.

3. Non-Markovian amplitude damping

The existence of non-Markovianity[2,3] for an amplitude-damping qubit system has been demonstrated already. Here, we only want to show that, when one uses $\mathcal{N}$ to measure the non-Markovianity of this amplitude-damping qubit system, there exist some non-Markovian time intervals whose distribution is independent of the selection of the pair of initial states. Thus, the maximization in the definition of $\mathcal{N}$ can be removed without influencing the detection of non-Markovianity. For this end, we should first derive the expression of the measure $\mathcal{N}$. Let us consider a qubit interacting with its environment via amplitude-damping model. The environment is assumed to be initially in the vacuum state, but its spectral density is arbitrary. This interaction model can be solved exactly. For any pair of initial qubit states, $\rho_1(0)$ and $\rho_2(0)$, one can obtain the time derivative of trace distance of the corresponding dynamical states as[3]

$$\sigma(t, \rho_{1,2}(0)) = \frac{2|G(t)|^2\alpha^2 + |\beta|^2}{\sqrt{|G(t)|^2\alpha^2 + |\beta|^2}} \frac{\mathrm{d}}{\mathrm{d}t}|G(t)|, \tag{4}$$

where $\alpha = \langle 1|\rho_1(0)|1\rangle - \langle 1|\rho_2(0)|1\rangle$ and $\beta = \langle 1|\rho_1(0)|0\rangle - \langle 1|\rho_2(0)|0\rangle$ are the differences of populations and of coherences respectively between the two given initial states. The function $G(t)$ is defined as the solution of the integro-differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t} G(t) = -\int_0^t \mathrm{d}t_1 f(t - t_1) G(t_1), \tag{5}$$

with initial condition $G(0) = 1$. The kernel $f(t - t_1)$ denotes the two-point reservoir correlation function

which is the Fourier transformation of the spectral density. Introducing the time-dependent decay rate

$$\gamma(t) = -2\mathrm{Re}\left[\frac{\dot{G}(t)}{G(t)}\right] = -\frac{2}{|G(t)|}\frac{\mathrm{d}}{\mathrm{d}t}|G(t)|, \qquad (6)$$

then the derivative of the trace distance may be rewritten as

$$\sigma(t, \rho_{1,2}(0)) = -\gamma(t)F(t). \qquad (7)$$

Here, the completely positive real function $F(t)$ is defined as

$$F(t) = \frac{\alpha^2 \mathrm{e}^{-\frac{3}{2}\Gamma(t)} + |\beta|^2 \mathrm{e}^{-\frac{1}{2}\Gamma(t)}}{\sqrt{\alpha^2 \mathrm{e}^{-\Gamma(t)} + |\beta|^2}}, \qquad (8)$$

with $\Gamma(t) = \int_0^t \mathrm{d}t'\gamma(t')$.

From Eq. (7), one can see that, due to the complete positivity of function $F(t)$ for any time and any pair of initial states, the time intervals in which the trace distance increases monotonously are uniquely determined by the condition $\gamma(t) < 0$ (Note that $\gamma(t)$ is uniquely determined by the spectral density of environment). The initial states $\rho_{1,2}(0)$ can alter the relative magnitude of σ in each interval, but would not change the distribution of these intervals, i.e., if these intervals exist, they will never vanish and their positions and lengths will not change, no matter how the pair of initial states changes. For convenience, we call them non-Markovian intervals in what follows. Due to the initial-state independence of the distribution of non-Markovian intervals, the maximization in the definition of Eq. (2) may be removed, without influencing the detection of non-Markovianity.

It is worthwhile to make clear that the removing of the maximization only does not influence the detection of non-Markovianity. If we want to quantify the degrees of non-Markovianity of a quantum process, then the maximization may be necessary. Though it is not yet very clear whether $\mathcal{N}$ can act as this quantitative part, many researchers are trying to find the ways[29,30] for the maximization of $\mathcal{N}$.

In the above process, we did not assume any specific spectral density, thus the result is valid in general for randomly structured environments. As an example, we discuss below the case of frequently-used Lorentzian spectral density

$$J(\omega) = \frac{1}{2\pi}\frac{\gamma_0\lambda^2}{(\omega_0 - \omega - \Delta)^2 + \lambda^2}, \qquad (9)$$

where ω_0 denotes the transition frequency of the qubit, $\Delta = \omega_0 - \omega_\mathrm{c}$ is the detuning between the frequencies of qubit transition and cavity mode, λ is the width of Lorentzian distribution and is connected to the reservoir correlation time $\tau_\mathrm{R} = \lambda^{-1}$. The parameter γ_0 can be regarded as the decay rate for the excited qubit in the Markovian limit of flat spectrum which is related to the relaxation time $\tau_\mathrm{S} = \gamma_0^{-1}$. For this Lorentzian spectrum, the correlation function can be easily evaluated as

$$f(t - t_1) = \frac{1}{2}\gamma_0\lambda\, \mathrm{e}^{-(\lambda - \mathrm{i}\Delta)(t - t_1)}, \qquad (10)$$

and thus the function $G(t)$ is

$$G(t) = \mathrm{e}^{-(\lambda - \mathrm{i}\Delta)t/2}\left[\cosh\left(\frac{\mathrm{d}t}{2}\right) + \frac{\lambda - \mathrm{i}\Delta}{\mathrm{d}}\sinh\left(\frac{\mathrm{d}t}{2}\right)\right], \qquad (11)$$

with $d = \sqrt{(\lambda - \mathrm{i}\Delta)^2 - 2\gamma_0\lambda}$. Finally, the expression of $\gamma(t)$ can be obtained from Eq. (6) as

$$\gamma(t) = \mathrm{Re}\left[\frac{2\gamma_0\lambda\sinh(\mathrm{d}t/2)}{\mathrm{d}\cosh(\mathrm{d}t/2) + (\lambda - \mathrm{i}\Delta)\sinh(\mathrm{d}t/2)}\right]. \qquad (12)$$

For the Lorentzian spectrum, two typical coupling regimes, weak and strong regimes, can be distinguished conventionally. For the weak coupling regime, $\gamma_0 < \lambda/2$, the correlation time τ_B of reservoir is smaller than the relaxation time τ_R of the qubit. Thus, in this regime, the effect of non-Markovianity is weaker and the time length of each non-Markovian interval as well as the total time that the non-Markovianity lasts are shorter (see the solid line in Fig. 1). In contrast, for the strong coupling regime, $\gamma_0 > \lambda/2$, the correlation time of reservoir is greater than or of the same order as the relaxation time of the qubit. Thus, the non-Markovian effect

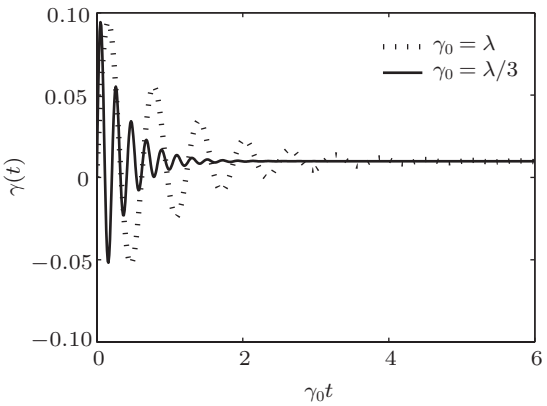


Fig. 1. The decay rate $\gamma(t)$ for non-Markovian amplitude damping model as a function of the dimensionless time $\gamma_0 t$ for $\Delta = 10\lambda$. The solid line ($\gamma_0 = \lambda/3$) and the dotted line ($\gamma_0 = \lambda$) represent the weak and strong coupling cases respectively. The time intervals in which $\gamma(t) < 0$ are the non-Markovian one.

becomes more relevant and the time length of each non-Markovian interval as well as the total time that the non-Markovianity lasts are longer (dotted line in Fig. 1). In Fig. 1, the selection of detuning $\Delta = 10\lambda$ is in principle random, but in this detuning vicinity, the non-Markovian effect is more evident.[2] At other detunings, similar features for the distribution of non-Markovian intervals may also exist.

4. Non-Markovian dephasing

As the second illustrative example, we consider a qubit which is coupled to a reservoir of harmonic oscillators via the dephasing model. The Hamiltonian in Schrödinger picture is taken to be

$$H = \frac{\omega_0}{2}\sigma_z + \sum_k \omega_k b_k^\dagger b_k + \sum_k \sigma_z(\lambda_k b_k^\dagger + \lambda_k^* b_k), \quad (13)$$

where ω_0 is the transition frequency and $\sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0|$ is the Pauli operator of the qubit; ω_k and b_k are respectively the frequency and annihilation operator for the k-th harmonic oscillator of the reservoir. The coupling strength λ_k is assumed to be complex in general. This dephasing model, which is extensively used to simulate the decoherence of a qubit coupling to its environment in quantum information science, can be solved exactly. For the initial state of the total system

$$\rho_{\rm SB}(0) = \rho(0) \otimes \rho_{\rm B}, \quad (14)$$

with $\rho(0)$ being the initial state of the qubit and $\rho_{\rm B}$ the initial thermal equilibrium state of the reservoir, the evolution for the elements of the reduced density matrix of the qubit may be written as[1]

$$\rho_{11}(t) = \rho_{11}(0), \rho_{00}(t) = \rho_{00}(0),$$
$$\rho_{10}(t) = \rho_{01}^*(t) = \rho_{10}(0)\,{\rm e}^{\Gamma_{\rm p}(t)}. \quad (15)$$

Here, the negative dephasing function $\Gamma_{\rm p}(t)$ is defined as

$$\Gamma_{\rm p}(t) = -\int_0^\infty {\rm d}\omega J(\omega) {\rm coth}(\omega/2k_{\rm B}T)\frac{1-\cos\omega t}{\omega^2}, \quad (16)$$

where $J(\omega)$ is the spectral density of the reservoir, $k_{\rm B}$ is the Boltzmann constant and T is the reservoir temperature in thermal equilibrium. It is not difficult from Eq. (15) to verify that the dynamical master equation for this dephasing-qubit system can be written as

$$\frac{{\rm d}\rho}{{\rm d}t} = \gamma_{\rm p}(t)[\sigma_z\rho\sigma_z - \rho], \quad (17)$$

where the dephasing rate is defined as $\gamma_{\rm p}(t) = -\frac{1}{2}\dot{\Gamma}_{\rm p}(t)$.

For any pair of initial states $\rho_1(0)$ and $\rho_2(0)$ of the qubit, one can easily obtain the dynamical states $\rho_1(t)$ and $\rho_2(t)$ and then the dynamical trace distance can be written as

$$D(t, \rho_{1,2}(0)) = \sqrt{\alpha^2 + |\beta|^2\,{\rm e}^{2\Gamma_{\rm p}(t)}}, \quad (18)$$

where $\alpha = \langle 1|\rho_1(0)|1\rangle - \langle 1|\rho_2(0)|1\rangle$ and $\beta = \langle 1|\rho_1(0)|0\rangle - \langle 1|\rho_2(0)|0\rangle$ are the differences of the populations and coherences for the two given initial states respectively. Thus, the time derivative of this trace distance is

$$\sigma(t, \rho_{1,2}(0)) = -2\gamma_{\rm p}(t)\frac{|\beta|^2\,{\rm e}^{2\Gamma_{\rm p}(t)}}{\sqrt{\alpha^2 + |\beta|^2\,{\rm e}^{2\Gamma_{\rm p}(t)}}}. \quad (19)$$

It again shows that the non-Markovian intervals are uniquely determined by $\gamma_{\rm p}(t) < 0$ and the selection of different pairs of initial states does not change the distribution of these intervals.

Again, the result is valid for any spectral density of environment. As an example, we consider the Ohmic spectrum, $J(\omega) = \omega\,{\rm e}^{-\omega/\Omega}$. It has a linear increase for small frequencies and an exponential frequency cutoff at the cutoff frequency Ω. Such a form for the spectral density is typically obtained in the quantum optical regime.[1] For this special spectrum, the dephasing rate can be written as

$$\gamma_{\rm p}(\tau) = \frac{1}{2\tau_{\rm B}}\int_0^\infty {\rm d}\widetilde{\omega}\,{\rm e}^{-\widetilde{\omega}/\widetilde{\Omega}}\,{\rm coth}\left(\frac{\widetilde{\omega}}{2}\right)\sin(\widetilde{\omega}\tau), \quad (20)$$

where the thermal correlation time is defined as $\tau_{\rm B} = 1/k_{\rm B}T$, $\widetilde{\omega} = \omega\tau_{\rm B}$ and $\widetilde{\Omega} = \Omega\tau_{\rm B}$ are the dimensionless frequencies and $\tau = t/\tau_{\rm B}$ is the dimensionless time.

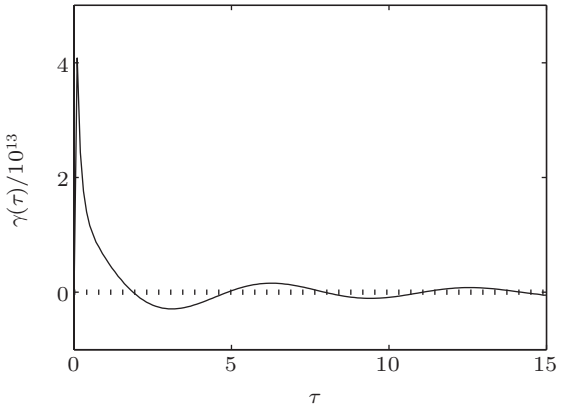


Fig. 2. The dephasing rate $\gamma_{\rm p}(\tau)$ of Eq. (20) as a function of the dimensionless time τ for parameters $\tau_{\rm B} = 10^{-13}$ s and $\widetilde{\Omega} = 20$. The time intervals in which $\gamma_{\rm p}(\tau) < 0$ are non-Markovian one.

In Fig. 2, we plot this dephasing rate as a function of dimensionless time for the parameters $\tau_{\rm B} = 10^{-13}$ s and $\widetilde{\Omega} = 20$, where the non-Markovian intervals can be observed visibly.

5. Non-Markovian depolarizing

The dynamical property for the coloured depolarizing noise and especially the conditions of complete positivity have been studied in detail.[32] The time-dependent Hamiltonian for a qubit system subjected to random telegraphic noise can be modeled as

$$H(t) = \sum_{i=1}^{3} \Gamma_i(t)\sigma_i, \tag{21}$$

where $\Gamma_i(t) = a_i n_i(t)$ are independent random variables and σ_i are the usual Pauli operators. The term $n_i(t)$ has a Poisson distribution with a mean of $t/2\tau_i$, while a_i is an independent random variable taking values $\pm a_i$. Under the assumptions of correlation function $\langle \Gamma_j(t)\Gamma_k(t')\rangle = a_k^2\, {\rm e}^{-|t-t'|/\tau_k}\delta_{jk}$ and of decorrelation of the state from the random variables, the Hamiltonian induces a completely positive dynamical map which can be described by Kraus decomposition[32]

$$\rho(\nu) = \sum_{k=1}^{4} A_k^{+}\rho(0)A_k, \tag{22}$$

with dimensionless time $\nu = t/2\tau$ (assuming $\tau_i = \tau$). The Kraus operators $A_k = \sqrt{\xi_k(\nu)}\sigma_k$, where $\sigma_{1,2,3}$ are the usual Pauli operators and $\sigma_4 = I$ is the identity operator. The non-negative parameters $\xi_k(\nu)$ are defined by

$$\begin{aligned}
\xi_1(\nu) &= [1 + \Lambda_1(\nu) - \Lambda_2(\nu) - \Lambda_3(\nu)]/4,\\
\xi_2(\nu) &= [1 - \Lambda_1(\nu) + \Lambda_2(\nu) - \Lambda_3(\nu)]/4,\\
\xi_3(\nu) &= [1 - \Lambda_1(\nu) - \Lambda_2(\nu) + \Lambda_3(\nu)]/4,\\
\xi_4(\nu) &= [1 + \Lambda_1(\nu) + \Lambda_2(\nu) + \Lambda_3(\nu)]/4.
\end{aligned} \tag{23}$$

Here,

$$\Lambda_i(\nu) = {\rm e}^{-\nu}[\cos(\mu_i\nu) + \sin(\mu_i\nu)/\mu_i] \tag{24}$$

describe a set of damped harmonic oscillators having frequencies $\mu_i = \sqrt{(4\kappa_i\tau)^2 - 1}$ with $\kappa_i^2 = a_j^2 + a_k^2$ for $i \neq j \neq k$.

For simplicity, we only discuss the homogeneous depolarizing noise. In this case, $a_1 = a_2 = a_3 \equiv a$, thus, $\Lambda_1(\nu) = \Lambda_2(\nu) = \Lambda_3(\nu) \equiv \Lambda(\nu)$, $\xi_1(\nu) = \xi_2(\nu) = \xi_3(\nu) \equiv (1 - \Lambda(\nu))/4$ and $\xi_4(\nu) = (1 + 3\Lambda(\nu))/4$. The Kraus decomposition of Eq. (22) becomes

$$\rho(\nu) = \frac{1 - \Lambda(\nu)}{4}[\sigma_1\rho(0)\sigma_1 + \sigma_2\rho(0)\sigma_2 + \sigma_3\rho(0)\sigma_3] + \frac{1 + 3\Lambda(\nu)}{4}\rho(0). \tag{25}$$

It is not difficult to show that $\rho(\nu)$ fits the master equation

$$\frac{{\rm d}\rho}{{\rm d}\nu} = \gamma(\nu)[\sigma_1\rho\sigma_1 + \sigma_2\rho\sigma_2 + \sigma_3\rho\sigma_3 - 3\rho], \tag{26}$$

with $\gamma(\nu) = -\frac{1}{4\Lambda(\nu)}\frac{{\rm d}\Lambda(\nu)}{{\rm d}\nu}$. Here, we employ the formula[31] $[\rho + \sigma_1\rho\sigma_1 + \sigma_2\rho\sigma_2 + \sigma_3\rho\sigma_3]/4 = I/2$ for any density operator ρ. For any pair of initial states $\rho_1(0)$ and $\rho_2(0)$, the dynamical trace distance for the evolved states $\rho_1(\nu)$ and $\rho_2(\nu)$ can be derived as

$$D(\rho_1(\nu), \rho_2(\nu)) = |\Lambda(\nu)|(\alpha^2 + |\beta|^2), \tag{27}$$

where $\alpha = \langle 1|\rho_1(0)|1\rangle - \langle 1|\rho_2(0)|1\rangle$ and $\beta = \langle 1|\rho_1(0)|0\rangle - \langle 1|\rho_2(0)|0\rangle$ denote the differences of the populations and of coherences for the two initial states respectively. The derivative of the dynamical trace distance is thus

$$\frac{\rm d}{{\rm d}\nu}D(\rho_1(\nu), \rho_2(\nu)) = -4(\alpha^2 + |\beta|^2)|\Lambda(\nu)|\gamma(\nu). \tag{28}$$

Once again, the non-Markovian intervals are uniquely determined by $\gamma(\nu) < 0$, independent of the selection of the pair of initial states. As $\gamma(\nu)$ has singularities at $\Lambda(\nu) = 0$, we plot $-{\rm d}|\Lambda(\nu)|/{\rm d}\nu$ as a function of dimensionless time ν in Fig. 3. In fact, from Eq. (27), one find that, up to an initial state-dependant constant, ${\rm d}|\Lambda(\nu)|/{\rm d}\nu$ determines the increasing rate of trace distance. Figure 3 shows that when μ increases, the oscillation frequency becomes faster and the time length

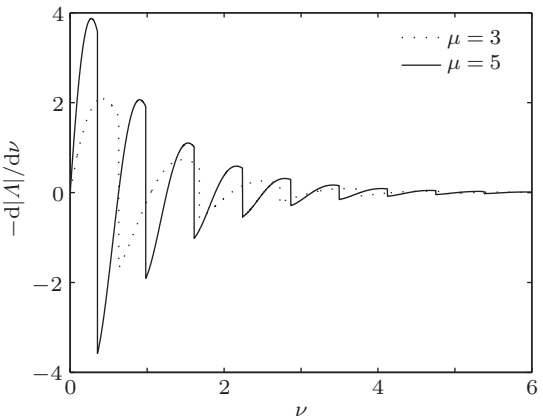


Fig. 3. The quantity $-{\rm d}|\Lambda(\nu)|/{\rm d}\nu$ as a function of the dimensionless time ν for parameters $\mu = 3, 5$. The time intervals in which $-{\rm d}|\Lambda(\nu)|/{\rm d}\nu < 0$ are non-Markovian.

of each non-Markovian interval becomes shorter. Note that the derivative ${\rm d}|\Lambda(\nu)|/{\rm d}\nu$ does not exist at some discrete values $\nu_{\rm s}$ that satisfy $\Lambda(\nu_{\rm s}) = 0$, this causes

the sudden transitions of the curves at the instances. In addition, from Eq. (27) combined with the definition $\mathscr{N}$ of Eq. (2), one can easily find that the total area enclosed by the curve and all the non-Markovian time intervals in Fig.3 are proportional to the measure $\mathscr{N}$.

6. Conclusion

In conclusion, we have studied the non-Markovianity of open qubit systems using the measure $\mathscr{N}$ proposed by Breuer, Laine and Piilo.[2] We found that for the three types of quantum noises, amplitude-damping, dephasing and depolarizing noises, there exist some non-Markovian time intervals whose distribution is independent of the selection of the pair of initial states. Therefore, the maximization in the definition of measure $\mathscr{N}$ can be actually removed without influencing the detection of non-Markovianity. This result is important due to wholly avoiding the complicated mathematical calculations. Of course, the existence of non-Markovian intervals only indicates the non-Markovianity of a quantum process, rather than the degree of non-Markovianity. If we want to quantify the degree of an open quantum process deviating from a Markovian one, the maximization may be necessary. Unfortunately, at the present stage, the quantification of the non-Markovianity of a quantum process is still a mystery and all the existing approaches can only be used to detect effectively the non-Markovianity. The dynamical models considered here, i.e., amplitude damping, dephasing and depolarizing models, represent fundamental coupling forms in studying problems of open systems. Thus, our results have good adaptability. In this paper, we have only discussed the three types of fundamental quantum noises. For more general cases, especially for the case of multi-channel quantum noises, we do not know what the result may be. This is a more challenging task.

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