Optical time-domain differentiation based on intensive differential group delay*

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An optical time-domain differentiation scheme is proposed and demonstrated based on the intensive differential group delay in a high birefringence fibre waveguide. Results show that the differentiation waveforms agree well with the mathematically calculated derivatives. Both error and efficiency will increase when the birefringence fibre becomes longer, and the error rises up more quickly while the efficiency approaches to a maximum of ∼0.25. By using a 1-m birefringence fibre a lower error of ∼0.26% is obtained with an efficiency of 1% for the first-order differentiation of 10-ps Gaussian optical pulses, and the high-order optical differentiation up to 4th order is achieved with an error less than 3%. Due to its compact structure being easy to integrate and cascade into photonic circuits, our scheme has great potential for ultrafast signal processing.

Keywords: optical signal processing, differentiation, differential group delay

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1. Introduction

Photonic circuits, in which signals can be optically processed rapidly at a higher speed far beyond the limitations currently imposed by electronics-based system, are a long-standing goal for future information technology and especially for ultrafast signal processing and communications.[1−4] All-optical differentiators are fundamental devices used to build photonic circuits for ultrafast optical processing, which are receiving much more attention recently following the development of ultrahigh-speed communication technology.[5−8] Optical differentiators that implement the temporal differentiation of the optical field or an input optical signal with a complex envelope, have been demonstrated during the past few years for many wide-ranging applications, for example, complete optical signal characterization in ultrahigh-bit-rate fibre-optics communications,[6] the generation of picosecond Hermite–Gaussian waveforms,[7] and dark-soliton detection,[8] which shows their great capacity in application to future optical information technology.

To realize the differentiation of optical signals, one can use the energy coupling in a long-period fibre grating,[9] the cross modulation effect in a semiconductor optical amplifier[10,11] or the spectral filtering in conventional interferometers.[12] However, to further investigate its physical mechanism, one can find that the high birefringence fibre (HBF) waveguide is a promising candidate for optical differentiation. In this paper we propose and demonstrate an efficient scheme of optical time-domain differentiation (OTD) based on the intensive differential group delay (DGD) in HBF waveguides, while analyzing its performance in detail in the first order optical differentiation and further investigating the high-order optical differentiation up to 4th order, the results of which show that the OTD scheme works efficiently and the output waveforms agree well with the mathematically calculated derivatives.

2. Principle of optical time-domain differentiation

Figure 1 shows a schematic description of the OTD system. The input optical signal or optical field is firstly 45° linearly polarized by a fibre polarizer (P_1) with a compact polarization controller (PC) and then enters into a short length of HBF whose birefringent axes are axis-aligned in the coordinate system. Owing to the DGD effect in the HBF,[13,14] a small time

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delay is generated between the horizontal (x axis) and vertical (y axis) polarization components. After coupling of these two polarization modes and then being filtered out by a 135° polarizer (P_2), we will obtain the temporal differentiation of the input signals.

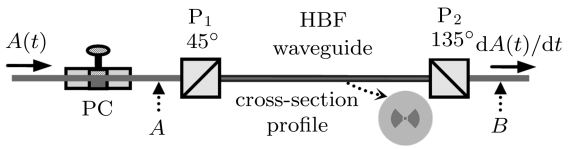


Fig. 1. Principle diagram of the OTD system, PC: polarization controller, P_1, P_2: fibre polarizer.

In general, the input optical field can be described by $E(t) = A(t)\,\mathrm{e}^{-\mathrm{i}\omega t}$ where ω is the carrier frequency and A(t) is the slowly varying amplitude or envelope of the optical signal. As linearly polarized by a PC, the optical field (A in Fig. 1) is $E_A(t) = A(t)\,\mathrm{e}^{-\mathrm{i}\omega t}[\cos 45°, \sin 45°]^{\mathrm{T}}$. While in the HBF waveguide, two polarization components become $E_{x,y}(z,t) = A_{x,y}(z,t)\,\mathrm{e}^{\mathrm{i}\beta_{x,y}z-\mathrm{i}\omega_0 t}$ in which $\beta_{x,y}$ denotes the carrier wave number for the x and y polarization components, respectively, while z is the length of the HBF. Since the nonlinear effect can be neglected in most cases, the propagation equations governing the evolution of the two polarization components along the HBF fibre can be written as[15]

$$\begin{aligned}\frac{\partial A_x}{\partial z}+\delta\frac{\partial A_x}{\partial t}+\frac{\mathrm{i}}{2}\beta_2\frac{\partial^2 A_x}{\partial t^2}=0,\\ \frac{\partial A_y}{\partial z}-\delta\frac{\partial A_y}{\partial t}+\frac{\mathrm{i}}{2}\beta_2\frac{\partial^2 A_y}{\partial t^2}=0,\end{aligned}\tag{1}$$

where β_2 is the dispersion coefficient and $2\delta l = l(1/v_{gx}-1/v_{gy})$ denotes the DGD in HBF with length of l. Transforming the time frame of each polarization component into a dynamic time frame ($\tau_x = t-\delta z$, $\tau_y = t+\delta z$) which moves with the pulse at its group velocity, we obtain

$$\begin{aligned}\mathrm{i}\frac{\partial A_x(\tau_x,z)}{\partial z}=\frac{1}{2}\beta_2\frac{\partial^2 A_x(\tau_x,z)}{\partial \tau_x^2},\\ \mathrm{i}\frac{\partial A_y(\tau_y,z)}{\partial z}=\frac{1}{2}\beta_2\frac{\partial^2 A_y(\tau_y,z)}{\partial \tau_y^2}.\end{aligned}\tag{2}$$

Generally the envelope of the input pulses has a Gaussian profile and can be expressed as $A(z=0,t) = A_0\,\mathrm{e}^{(-t^2/2T_0^2)}$ in which T_0 is the initial pulse width, thus its two polarization components after propagating through an HBF with length l are given by

$$\begin{aligned}&A_{x,y}(l,\tau_{x,y})\\ &=\frac{A_0}{\sqrt{2}}\frac{1}{\sqrt{1-\mathrm{i}l/l_{\mathrm{D}}}}\exp\left(-\frac{\tau_{x,y}}{2(1-\mathrm{i}l/l_{\mathrm{D}})}\right),\end{aligned}\tag{3}$$

where $l_{\mathrm{D}} = T_0^2/\beta_2$ is the dispersion length. Given $(\beta_x-\beta_y)l = 2n\pi$ (n an integer number), the two polarization components will constructively interfere in P_2. Define $\sigma = \sqrt{1+(l/l_{\mathrm{D}})^2}$, then the output intensity (B in Fig. 1) is

$$\begin{aligned}I_{\mathrm{out}}(t) = \frac{|A_0|^2}{4}\frac{1}{\sigma}\Bigg\{&\exp\left(-\left(\frac{t-\delta l}{T_0\sigma}\right)^2\right)\\ &+\exp\left(-\left(\frac{t+\delta l}{T_0\sigma}\right)^2\right)\\ &-2\exp\left(-\frac{\left(t^2+(\delta l)^2\right)}{T_0^2\sigma^2}\right)\\ &\times\cos\left(\frac{2(\delta l)(\beta_2 l)t}{T_0^4\sigma^2}\right)\Bigg\}.\end{aligned}\tag{4}$$

Considering that a very short length of HBF will produce the required DGD, that is, $l \ll l_{\mathrm{D}}$ is easy to be satisfied, thus we have $\sigma \approx 1$ and $\mu = \cos\left(2(\delta l)(\beta_2 l)t/T_0^4\sigma^2\right) \sim 1$, and then equation (4) becomes

$$\begin{aligned}I_{\mathrm{out}}(t) \approx \frac{|A_0|^2}{4}\Bigg[&\exp\left(-\frac{(t-\delta l)^2}{2T_0^2}\right)\\ &-\exp\left(-\frac{(t+\delta l)^2}{2T_0^2}\right)\Bigg]^2.\end{aligned}\tag{5}$$

According to the Taylor expansion we take the first-order approximation and obtain

$$I_{\mathrm{out}}(t) \approx \left(\frac{\partial A(z=0,t)}{\partial t}\right)^2(\delta l)^2 \sim \left(\frac{\mathrm{d}A(t)}{\mathrm{d}t}\right)^2,$$

or

$$A_{\mathrm{out}}(t) \approx \left|\frac{\partial A(z=0,t)}{\partial t}\right|(\delta l) \sim \left|\frac{\mathrm{d}A(t)}{\mathrm{d}t}\right|.\tag{6}$$

Equation (6) reveals that the output optical amplitude is approximately proportional to the absolute value of the first derivative of the input optical field envelope or signal. Thus by reasonably selecting the length of the HBF waveguide, we can gain proper differentiation of the input signals.

3. Demonstration of ps-pulse differentiation

The picosecond optical pulse plays an important role in optical signal processing and communications.[16] Here we choose the ps-pulse with a full width at half maximum (FWHM) of 10 ps to demonstrate our OTD scheme. The HBF being used is a bow-tie type fibre with a beat length of 2.5 mm,

DGD parameter (δ) 1.0333 ps/m, dispersion coefficient (β_2) –21.7 ps^2/km, and nonlinear index of refraction parameter $n_2 = 2.6 \times 10^{-20}$ m^2/W at the wavelength around 1550 nm. According to Ref. [15], the dispersion length $L_{\rm D}$ of the HBF is 1.662 km, which is much longer than the length of the HBF employed, thus the approximation condition can be well satisfied. The input pulse has a lower peak power of ∼20 mW and a carrier wavelength of 1550 nm, so that the nonlinear effect contributes very little and can be neglected.

After such consideration and preparation of the material for OTD implementation, a series of HBF waveguides with different lengths from 0.5 m to 5 m have been investigated in detail using the scheme described in Fig. 1. For each sample, we estimate the differentiation error by computing the mean deviation between normalized OTD output pulse $d_{\rm o}(t)$ and mathematically calculated derivative $d_{\rm c}(t)$ of the input pulse during the time interval T with $\varepsilon = \int_0^T |d_{\rm o}(t) - d_{\rm c}(t)|\,{\rm d}t/T$. While we further evaluate the differentiation efficiency by calculating the ratio of input and output pulse intensities with $\eta = \max(I_{\rm out})/\max(I_{\rm in})$. Figure 2 illustrates some of the representative results.

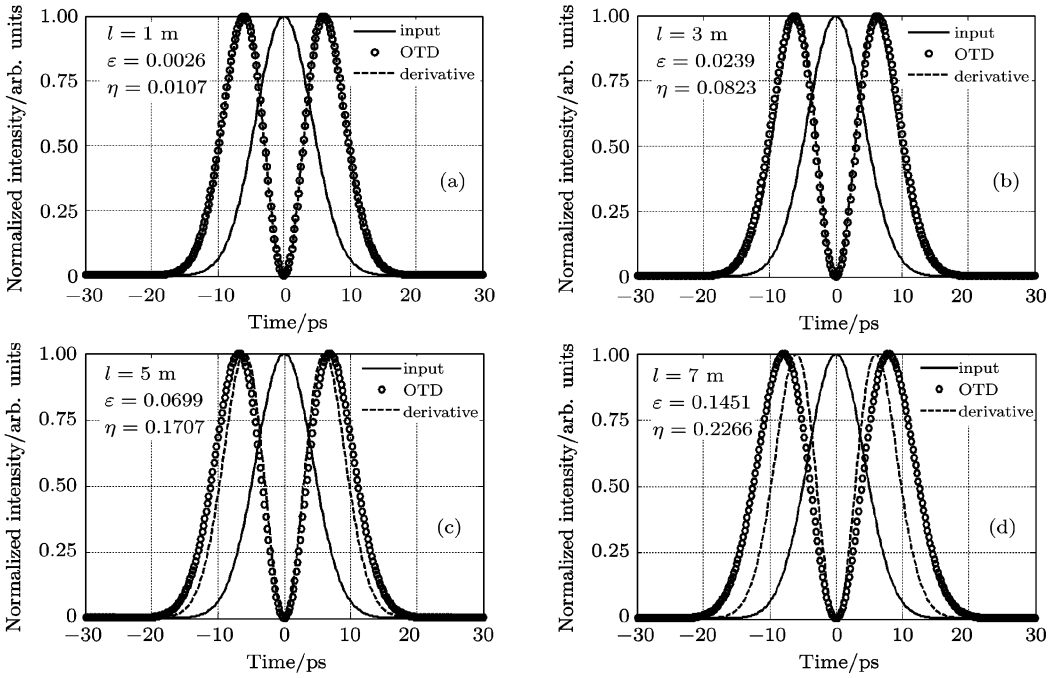


Fig. 2. Temporal waveforms (open circles) from the OTD with different lengths of HBF and corresponding derivative calculated (dashed curves) for a 10-ps Gaussian optical pulse input (solid curves).

Figure 2 shows that both error and efficiency are small when using a shorter HBF, while a lower error of ∼0.26% is achieved by a 1-m HBF; however, the efficiency drops to 1%. When the HBF becomes longer, the differential pulse width will broaden and the error increases rapidly. To further reveal the relationship, we investigate in detail the dependence of these parameters on the length of the HBF waveguide. Figure 3 presents relatively comprehensive results, which reveals that the error rises up quickly when the HBF becomes longer and longer, while the efficiency approaches a certain maximum ∼0.25. In addition, to confirm the approximation conditions in Eq. (5), we calculate the differences between σ and μ, and the required value "1". Figure 3 shows that they ($|1-\sigma|$ and $|1-\mu|$) are always less than 9.35×10^{-4}, very close to zero in our investigation, so it is reasonable to set $\sigma = \mu = 1$, and then the theoretical model in Section 2 is valid in practical applications.

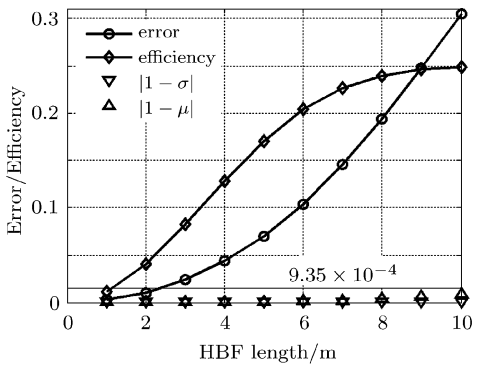


Fig. 3. Differentiation error and efficiency versus HBF length, and confirmation of approximation conditions for Eq. (5).

4. Further investigation on high-order optical differentiation

According to the mathematical definition of high-order differentiation, one can find that n-th-order time-domain optical differentiation can be realized by simply cascading n OTD units, as shown in Fig. 1, where the n-th polarizer P_n should have its axis $[2-(-1)^n]\pi/4$ oriented. Based on this consideration, we demonstrate high-order optical differentiation up to 4th order by using a cascaded OTD. To maintain consistency, a ps-pulse with FWHM of 10 ps is still used as the input with the same peak power of ∼20 mW and carrier wavelength of 1550 nm. In addition, the HBF we utilized has the same parameters as described in Section 2. In the first-order unit, we employ a 2-m HBF, while following it we use 0.5-m, 0.2-m, and 0.05-m HBFs, respectively, for the 2nd, 3rd, and 4th order differentiations. We investigate the output waveforms from every high-order stage and estimate the discrepancy between the OTD differentiation and the calculated derivative. Some of the results are presented in Fig. 4.

Figure 4 shows that the OTD waveforms closely match the calculated derivatives in the cases of high-order differentiation, while the error is 1.21% for the 2nd, 2.62% for the 3rd, and 2.89% for the 4th order differentiations, which demonstrates that the HBF also works efficiently in high-order optical differentiation.

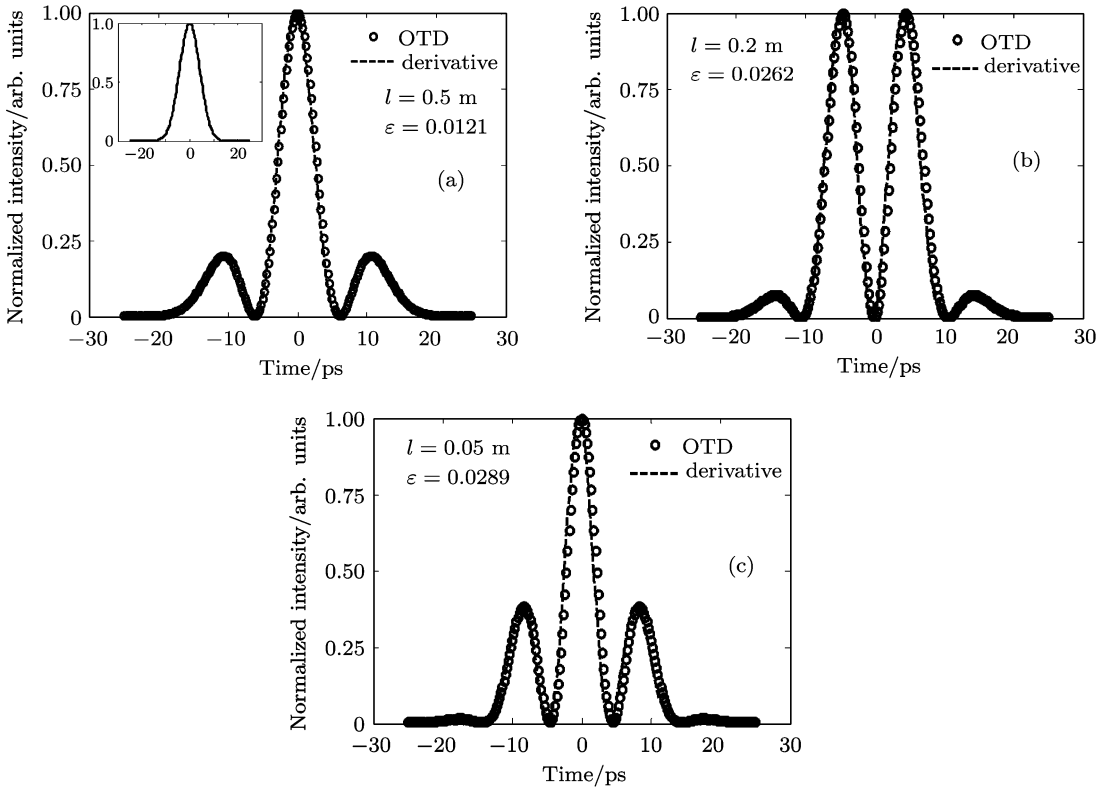


Fig. 4. OTD waveforms (open circles) from the 2nd (panel (a), inset: 10-ps Gaussian pulse input), 3rd (b), and 4th (c) order differentiations with corresponding calculated derivatives (dashed curves).

5. Conclusions

In summary, an all-optical differentiation scheme is proposed and demonstrated based on DGD in an HBF waveguide. Analysis of the first-order OTD shows that both error and efficiency will increase when the HBF gets longer, and the error rises up quickly while the efficiency gradually approaches a maximum of ∼0.25. For a 1-m HBF the error is ∼0.26% with efficiency of 1% at 10-ps Gaussian input. Further investigations on high-order differentiation demonstrate that the OTD scheme still performs well, with 2nd to 4th order optical differentiations being realized with errors of less than 3%. Due to the compact structure for photonic circuits to be easily integrated and cascaded, our scheme is valid for ultrafast signal processing.

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