Half-plane diffraction of Gaussian beams carrying two vortices of equal charges*

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This paper derives explicit expressions for the propagation of Gaussian beams carrying two vortices of equal charges $m = \pm 1$ diffracted at a half-plane screen, which enables the study of the dynamic evolution of vortices in the diffraction field. It shows that there may be no vortices, a pair or several pairs of vortices of opposite charges $m = +1, -1$ in the diffraction field. Pair creation, annihilation and motion of vortices may appear upon propagation. The off-axis distance additionally affects the evolutionary behaviour. In the process the total topological charge is equal to zero, which is unequal to that of the vortex beam at the source plane. A comparison with the free-space propagation of two vortices of equal charges and a further extension are made.

Keywords: two vortices of equal charges, diffraction at a half-plane screen, dynamic evolution

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1. Introduction

Optical beams carrying phase singularities (vortices and/or edge dislocations) and their attractive applications are of substantial current interest.[1−7] The free-space propagation properties of an array of optical vortices including vortices of equal charges and pairs of vortices of opposite charges embedded in a Gaussian beam were analysed by Indebetouw and Freund in Refs. [8] and [9]. The creation and annihilation of the phase singularities of Gaussian beams focused by an aperture lens were studied theoretically and experimentally by Karman *et al.*[10,11] Masajada dealt with the half-plane diffraction of Gaussian background vortex beams.[12,13] Vasnetsov *et al.* discovered the self-reconstruction of an optical vortex in the diffraction field when the screen cuts off part of the incident beam.[14] Half-plane diffraction is of interest in singular optics because for such a case analytical results are often available,[15,16] and it can be easily realized in experiments. The motivation of the present paper is to study the dynamic evolution of two optical vortices of equal charges nested in a Gaussian beam diffracted at a half-plane screen. In Section 2 analytical expressions for the propagation of Gaussian beams containing two vortices of equal charges m=±1 diffracted at a half-plane screen are derived. The dynamic evolution of vortices in the diffraction field is studied and illustrated numerically in Section 3. Finally, the main results obtained in this paper are summarized and a comparison with the free-space propagation of two vortices of equal charges and a further extension are made in Section 4.

2. Theoretical formulation

Assume that two optical vortices of equal topological charges $m = +1$ or $m = -1$ (shortly written as $m = \pm 1$) are nested in a Gaussian beam whose electric field at the plane $z = 0$ reads as

$$E'_{0\pm}(x_0, y_0, 0) = E_0 \left[\frac{x_0 - s_1 \pm \mathrm{i}(y_0 - s_2)}{w_0}\right] \times \left[\frac{x_0 + s_1 \pm \mathrm{i}(y_0 + s_2)}{w_0}\right] \times \exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right), \quad (1)$$

where s_1, s_2 denote the off-axis distances of vortices in the x and y directions, and $m = \pm 1$ correspond to ± 1, respectively, w_0 is the waist width of the Gaussian background beam, E_0 is a constant.

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If a half-plane screen with transmissivity

$$T(x_0, y_0) = \begin{cases} 1, & y_0 \geq 0, \\ 0, & y_0 < 0, \end{cases} \tag{2}$$

is positioned at the plane $z = 0$, the electric field of the Gaussian vortex beam just behind the screen is

$$\begin{aligned} E_{0\pm}(x_0, y_0, 0) = {} & T(x_0, y_0) \left[\frac{x_0 - s_1 \pm \mathrm{i}(y_0 - s_2)}{w_0} \right] \\ & \times \left[\frac{x_0 + s_1 \pm \mathrm{i}(y_0 + s_2)}{w_0} \right] \\ & \times \exp\left(-\frac{x_0^2 + y_0^2}{w_0^2} \right). \end{aligned} \tag{3}$$

Within the framework of the paraxial approximation, the diffraction field in the half space $z > 0$ obeys the Fresnel diffraction integral of the form

$$\begin{aligned} E_{\pm}(x, y, z) = {} & \frac{-\mathrm{i}k}{2\pi z} \int_0^{+\infty} \int_{-\infty}^{+\infty} E_{0\pm}(x_0, y_0, 0) \\ & \times \exp\left\{ \frac{\mathrm{i}k}{2z} \left[(x - x_0)^2 \right.\right. \\ & \left.\left. + (y - y_0)^2 \right] \right\} \mathrm{d}x_0 \mathrm{d}y_0, \end{aligned} \tag{4}$$

where k is the wave number related to the wavelength λ by $k = \lambda/2\pi$.

Substituting Eq. (3) into Eq. (4), and recalling the integral formula[17]

$$\begin{aligned} & \int_0^{\infty} x^{v-1} \exp\left(-sx^2 - qx\right) \mathrm{d}x \\ & = (2s)^{-v/2} \Gamma(v) \exp\left(\frac{q^2}{8s}\right) D_{-v}\left(\frac{q}{\sqrt{2s}}\right), \\ & \quad \mathrm{Re}(s) > 0, \quad \mathrm{Re}(v) > 0, \end{aligned} \tag{5}$$

with Re being the real part of s and v, by tedious but straightforward integral calculations we obtain

$$\begin{aligned} & E_{\pm}(x, y, z) \\ & = \frac{-\mathrm{i}}{\lambda z w_0^2} \sqrt{\frac{\pi}{p}} \exp\left[\mathrm{i}kz + \frac{\mathrm{i}k(x^2 + y^2)}{2z} + \frac{q_x^2}{p} + \frac{q_y^2}{8p} \right] \\ & \times \left\{ \left[\frac{(q_x^2 + p/2)}{p^2} \mp 2\mathrm{i}s_1 s_2 - s_1^2 + s_2^2 \right] \right. \\ & \times (2p)^{-1/2} D_{-1}\left(\frac{q_y}{\sqrt{2p}}\right) \\ & \pm 2\mathrm{i}\frac{q_x}{p}(2p)^{-1} D_{-2}\left(\frac{q_y}{\sqrt{2p}}\right) \\ & \left. - 2(2p)^{-3/2} D_{-3}\left(\frac{q_y}{\sqrt{2p}}\right) \right\}, \end{aligned} \tag{6}$$

where $D_v(\cdot)$ denotes the parabolic cylinder functions ($v = -1, -2, -3$), $q_x = -\mathrm{i}kx/2z$, $q_y = \mathrm{i}ky/z$, and $p = 1/w_0^2 - \mathrm{i}k/2z$. Equation (6) is explicit expressions for the propagation of Gaussian beams carrying two vortices of equal charges ($m = \pm 1$) diffracted at a half-plane screen, and indicates that $E_{\pm}(x, y, z)$ depends on the off-axis distances s_1, s_2 of vortices and propagation distance z. Equation (6) is valid for both Fresnel and Fraunhofer zones, and for the latter case p=$1/w_0^2$.

The position of vortices in the diffraction field is given by[1]

$$\begin{aligned} & \mathrm{Re}\left[E(x, y, z)\right] = 0, \\ & \mathrm{Im}\left[E(x, y, z)\right] = 0, \end{aligned} \tag{7}$$

where Im is the imaginary part of $E_{\pm}(x, y, z)$. The topological charge and its sign of vortices are determined by the sign principle proposed by Freund *et al.*[18]

3. Dynamic evolution of vortices in the diffraction field

Numerical calculations were performed by using Eqs. (6) and (7) to illustrate the dynamic evolution of vortices in the diffraction field. In the following calculations $\lambda = 1.06 \times 10^{-3}$ mm and $w_0 = 1$ mm are kept fixed.

3.1. Two vortices of $m = -1$ at the plane $z = 0$ located on the y axis ($s_1 = 0$)

The phase contour lines of a Gaussian beam carrying two vortices of $m = -1$ and propagating through a half-plane screen are given in Figs. 1(a)–1(d) for selected values of the propagation distances: (a) $z = 0.5z_{\mathrm{R}}$ ($z_{\mathrm{R}} = \pi w_0^2/\lambda$-Rayleigh range); (b) $z = 0.8087z_{\mathrm{R}}$; (c) $z = 2.08z_{\mathrm{R}}$; (d) $z = 10z_{\mathrm{R}}$, and $s_1 = 0$, $s_2 = 0.1$ mm. The analysis of vorticity of phase contours around singularities (where the arrow indicates the direction of increasing phase) shows that two pairs of vortices of opposite charges $m = +1$, -1 corresponding to anti-clockwise and clockwise rotational arrows, respectively, are present at the plane $z = 0.5z_{\mathrm{R}}$ in Fig. 1(a), and their positions are (−0.0010012 mm, 0.0291778 mm), (4.17813 mm, 1.69536 mm), (−1.13846 mm, 2.12622 mm), and (1.05894 mm, 2.37956 mm), respectively. At the plane $z = 0.8087z_{\mathrm{R}}$ in Fig. 1(b) vortices 3 and 4 of opposite charge $m = -1$, $+1$ annihilate each other, and vortices 1 and 2 move to (0.00352615 mm, 0.0276511 mm), and (2.2138 mm, 2.21986 mm), respectively. With a further increase of the propagation distance to $z = 2.08z_{\mathrm{R}}$ in Fig. 1(c) a pair of

vortices 5 and 6 of opposite charge $m = -1, +1$ appears and vortices 1 and 2 shift to (0.01682 mm, 0.02658 mm), and (2.10525 mm, 5.53074 mm), respectively. In the far field $z = 10z_{\rm R}$ in Fig. 1(d), the positions of vortices 2 and 6 are (7.19602 mm, 137.692 mm) and (−7.19602 mm, −137.692 mm) (not shown), and vortices 1 and 5 move to (0.0876 mm, 0.0264 mm), (−11.3294 mm, −2.5485 mm), respectively. The corresponding three-dimensional (3D) trajectory is plotted in Fig. 2, which indicates clearly the evolution process of vortices in free-space propagation.

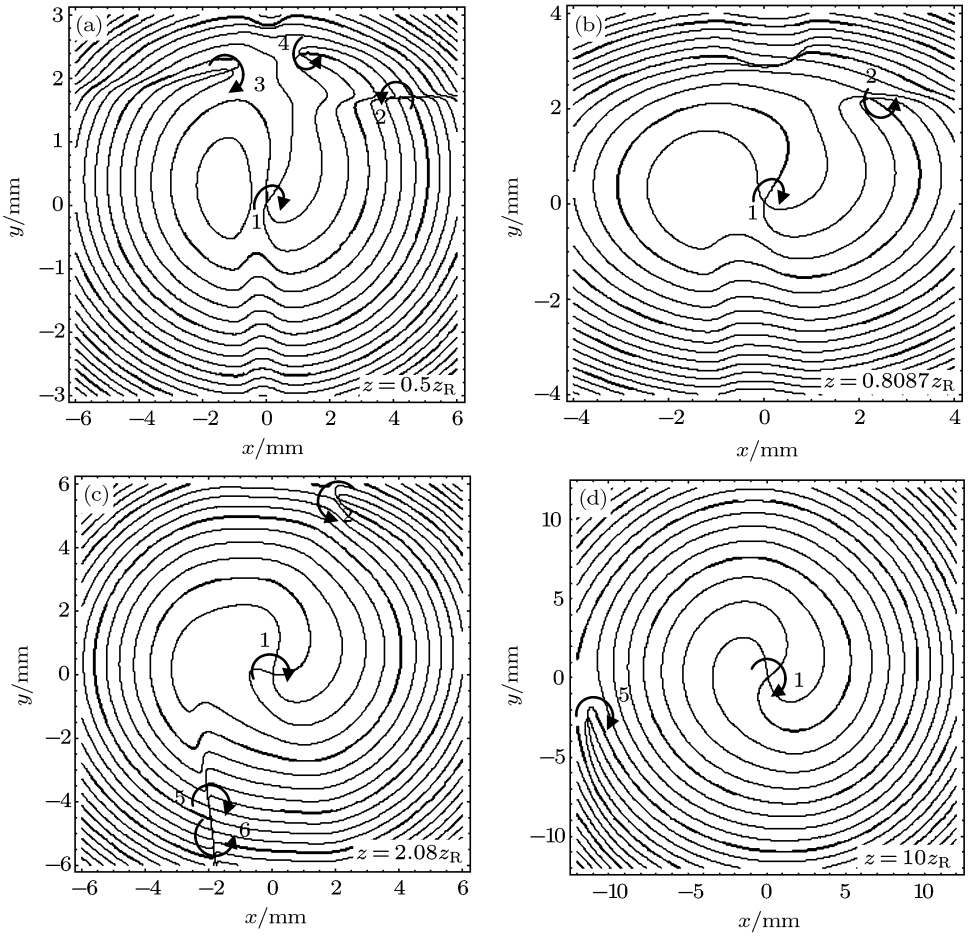


Fig. 1. Contour lines of phase of a diffracted Gaussian beam carrying two vortices of equal charges $m = -1$ for selected values of the propagation distance (a) $z = 0.5z_{\rm R}$, (b) $z = 0.8087z_{\rm R}$, (c) $z = 2.08z_{\rm R}$, (d) $z = 10z_{\rm R}$, and $s_1 = 0$, $s_2 = 0.1$ mm.

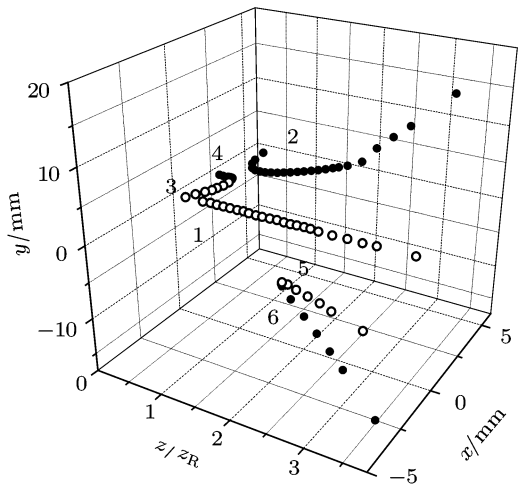


Fig. 2. The 3D trajectory of vortices in the diffraction field. "•" $-m = +1$, "○" $-m = -1$.

Figures 3(a)–3(e) show the evolution of vortices in the diffraction field when the off-axis distance is changed to $s_2 = 1.2$ mm. At the plane $z = 0.5z_{\rm R}$ in Fig. 3(a) there are a pair of vortices 1 and 2 of opposite charge $m = -1, +1$ located at (1.18363 mm, 1.44464 mm), and (3.48337 mm, 1.67227 mm), respectively. At the plane $z = 0.6022z_{\rm R}$ in Fig. 3(b) the annihilation of vortices 1 and 2 of $m = -1, +1$ takes place, and no vortices are found in the diffraction field. With increasing propagation distance to $z = 1.2849z_{\rm R}$ in Fig. 3(c) a pair of vortices 3, 4 of $m = -1, +1$ appears, which are located at (1.8612 mm, 2.3775 mm), and (1.8695 mm, 2.3853 mm), respectively. A further increase of $z = 4.624z_{\rm R}$ in Fig. 3(d) leads to the creation of a pair of vortices 5 and 6 of

$m = -1$, $+1$ located at (-7.5218 mm, -7.8146 mm) and (-7.4527 mm, -8.1364 mm), and vortices 3 and 4 move to (3.68332 mm, 2.19164 mm) and (6.9186 mm, 11.8729 mm), respectively. Finally, in the far field $z = 10z_{\rm R}$ in Fig. 3(e) the positions of vortices 3 and 5 are (7.69177 mm, 2.19179 mm), and (-18.5919 mm, -5.07842 mm), and 4 and 6 move to (14.1173 mm, 43.4461 mm), (-14.1173 mm, -43.446 mm) (not shown), respectively. Therefore, in comparison with Figs. 1 and 2 the evolution behaviour of vortices in the diffraction field depends on the off-axis distance, and in the evolution process the total topological charge is zero, and is unequal to that of vortices at the source plane $z = 0$.

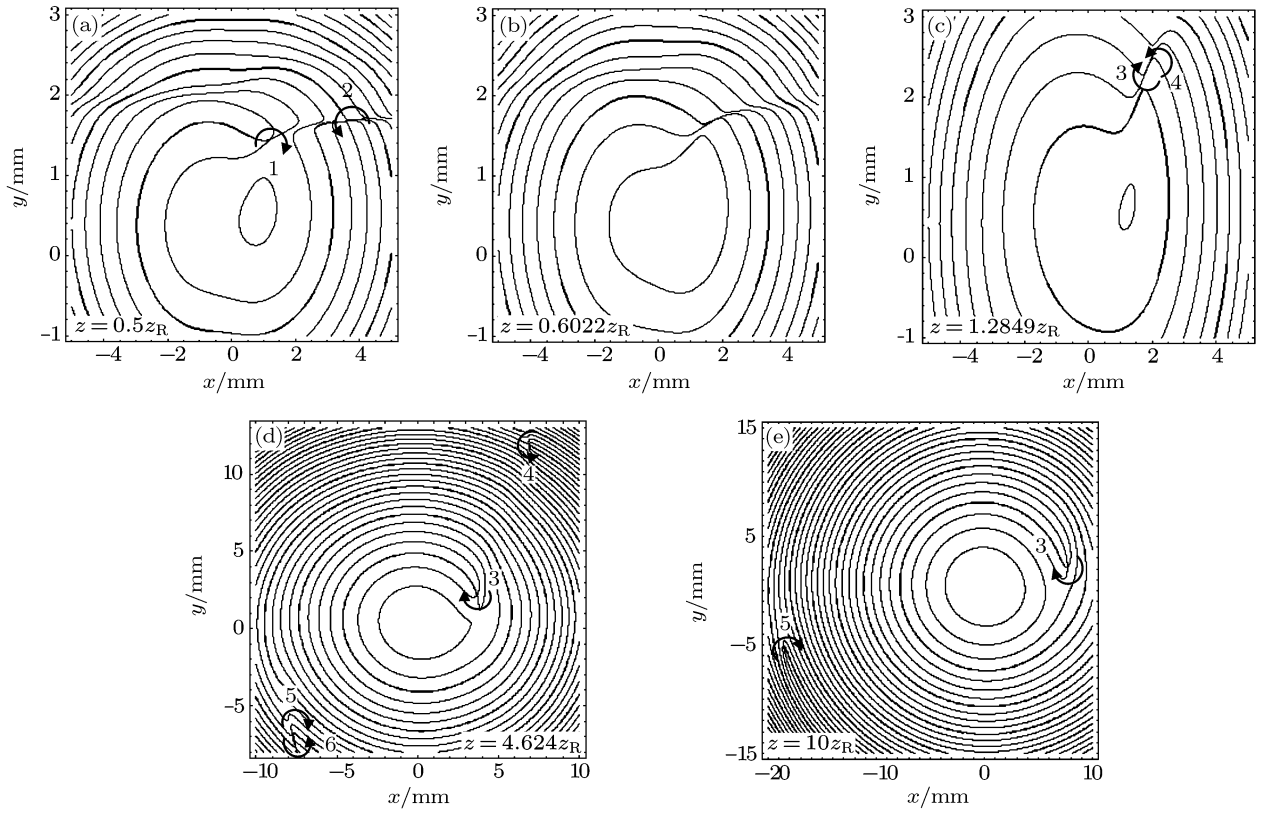


Fig. 3. Contour lines of phase of a diffracted Gaussian beam carrying two vortices of equal charges $m = -1$ for selected values of the propagation distance: (a) $z = 0.5z_{\rm R}$; (b) $z = 0.6022z_{\rm R}$; (c) $z = 1.2849z_{\rm R}$; (d) $z = 4.624z_{\rm R}$; (e) $z = 10z_{\rm R}$, and $s_1 = 0$, $s_2 = 1.2$ mm.

3.2. Two vortices of $m = -1$ at the plane $z = 0$ are located on the x axis ($s_2 = 0$)

The contour lines of phase of a Gaussian beam carrying two vortices of $m = -1$ and propagating through a half-plane screen are plotted in Figs. 4(a)–4(d), where $s_1 = 0.2$ mm, $s_2 = 0$, and (a) $z = 0.5z_{\rm R}$, (b) $z = 0.8665z_{\rm R}$, (c) $z = 1.88z_{\rm R}$, (d) $z = 10z_{\rm R}$. As can be seen, there are two pairs of vortices 1, 2 and 3, 4 of opposite charge $m = -1$, $+1$ at the plane $z = 0.5z_{\rm R}$ in Fig. 4(a), whose positions are (0.0110118 mm, -0.123316 mm), (4.19627 mm, 1.69589 mm), (-1.1531 mm, 2.13085 mm), and (1.07893 mm, 2.38931 mm), respectively. At the plane $z = 0.8665z_{\rm R}$ in Fig. 4(b) vortices 3 and 4 of $m = -1$, $+1$ annihilate each other, and vortices 1 and 2 shift to (-0.0133169 mm, -0.116358 mm), and (2.13588 mm, 2.33768 mm), respectively. With a further increase of $z = 1.88z_{\rm R}$ in Fig. 4(c) a pair of vortices 5 and 6 of $m = -1$, $+1$ is present, and the positions of vortices 1 and 2 become (-0.0598611 mm, -0.111971 mm), and (1.96638 mm, 4.94241 mm), respectively. In the far field $z = 10z_{\rm R}$ in Fig. 4(d) the positions of vortices 2 and 6 are (6.8305 mm, 161.749 mm) and (-6.8305 mm, -161.749 mm) (not shown), and vortices 1 and 5 are located at (-0.364653 mm, -0.110428 mm) and (-10.8869 mm, -2.35775 mm), respectively.

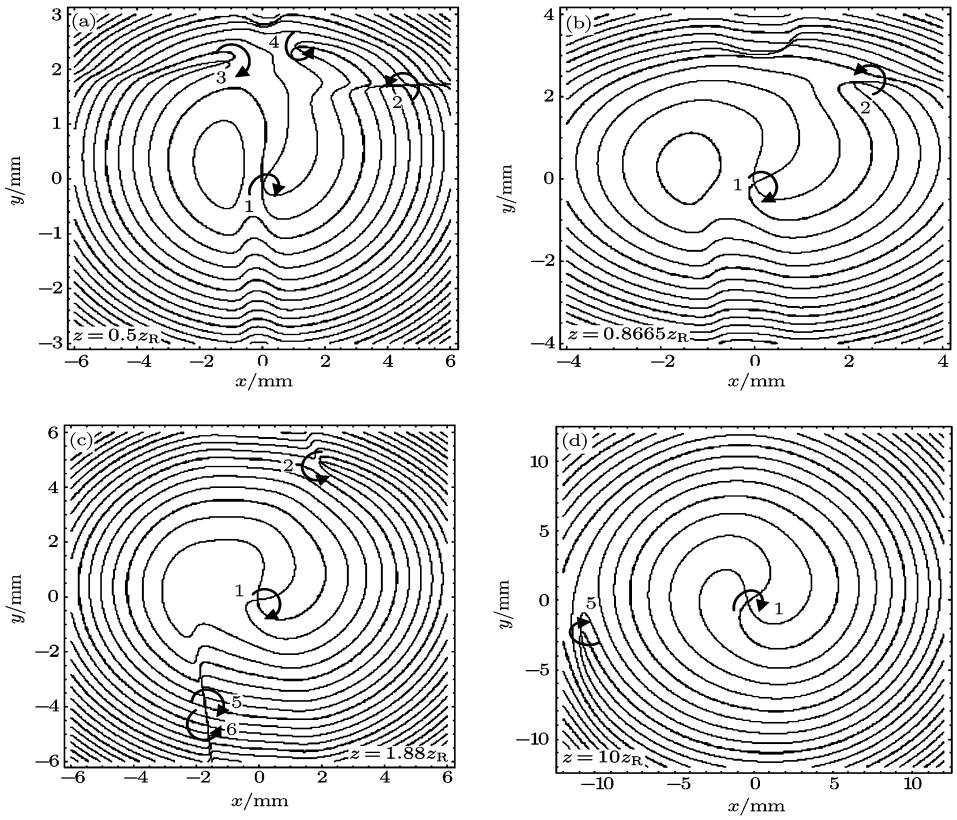


Fig. 4. Contour lines of phase of a diffracted Gaussian beam carrying two vortices of equal charges $m = -1$ for selected values of the propagation distance: (a) $z = 0.5z_R$, (b) $z = 0.8665z_R$, (c) $z = 1.88z_R$, (d) $z = 10z_R$, and $s_1 = 0.2$ mm, $s_2 = 0$.

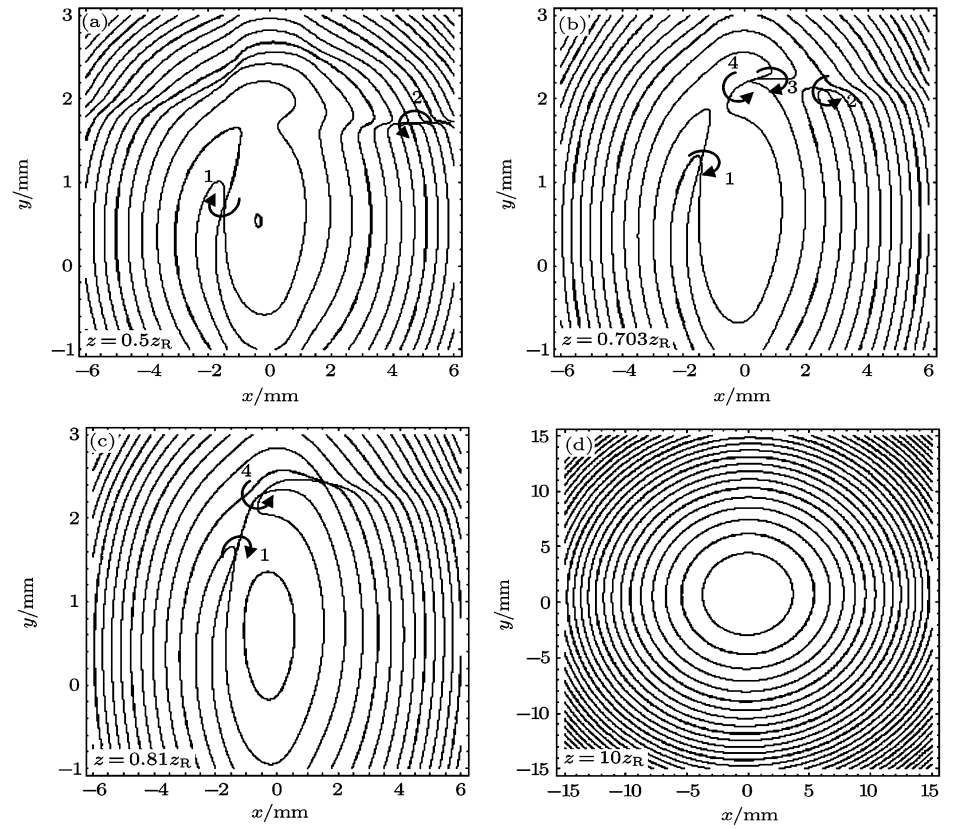


Fig. 5. Contour lines of phase of a diffracted Gaussian beam carrying two vortices of equal charges $m = -1$ for selected values of the propagation distance: (a) $z = 0.5z_R$; (b) $z = 0.703z_R$; (c) $z = 0.81z_R$; (d)$z = 10z_R$, and $s_1 = 1.2$ mm, $s_2 = 0$.

Figures 5(a)–5(d) illustrate the evolution of vortices in the diffraction field when the off-axis distance is changed to s_1=1.2 mm. At the plane $z = 0.5z_R$ we see a pair of vortices 1 and 2 of $m = -1, +1$ located at (−1.4701 mm, 0.757333 mm), and (4.62628 mm, 1.70763 mm), respectively. At the plane $z = 0.703z_R$ in Fig. 5(b) a pair of vortices 3 and 4 of $m = -1, +1$ is present and vortices 1 and 2 move to (−1.43344 mm, 1.24885 mm), and (2.56654 mm, 2.11021 mm), respectively. With increasing $z = 0.81z_R$ in Fig. 5(c), vortices 2 and 3 of opposite charges annihilate each other and vortices 1 and 4 move to (−1.36968 mm, 1.55209 mm) and (−0.464727 mm, 2.29032 mm), respectively. Finally, in the far field $z = 10z_R$ in Fig. 5(d) all vortices disappear in the diffraction field. In the evolution process of Figs. 4 and 5 the total topological charge is zero.

3.3. Two vortices of $m = +1$ at the plane $z = 0$ located on the y axis ($s_1 = 0$)

The contour lines of phase of a Gaussian beam carrying two vortices of $m = +1$ and propagating through a half-plane screen are depicted in Figs. 6(a)–6(d) for selected values of the propagation distance: (a) $z = 0.5z_R$; (b) $z = 0.8087z_R$; (c) $z = 2.08z_R$; (d) $z = 10z_R$, and $s_1 = 0$, $s_2 = 0.1$ mm. It is seen that at the plane $z = 0.5z_R$ there exist two pairs of vortices of $m = +1, -1$ located at (0.0010012 mm, 0.0291778 mm), (−4.17813 mm, 1.69536 mm), (1.13846 mm, 2.12622 mm), and (−1.05894 mm, 2.37956 mm), respectively. At $z = 0.8087z_R$ in Fig. 6(b) the annihilation of vortices 3 and 4 of $m = +1, -1$ takes place and vortices 1 and 2 move to (−0.00352615 mm, 0.0276511 mm), and (−2.2138 mm, 2.21986 mm), respectively. A further increase of the distance to $z = 2.08z_R$ in Fig. 6(c) a pair of vortices 5 and 6 of $m = +1, -1$ is present and vortices 1 and 2 shift to (−0.01682 mm, 0.02658 mm), and (−2.10525 mm, 5.53074 mm) respectively. In the far field $z = 10z_R$ in Fig. 6(d) vortices 1, 5 move to (−0.0876 mm, 0.0264 mm), (11.3294 mm, −2.5485 mm), and vortices 2, 6 move to (−7.19602 mm, 137.692 mm), (7.19602 mm, −137.692 mm), (not shown), respectively.

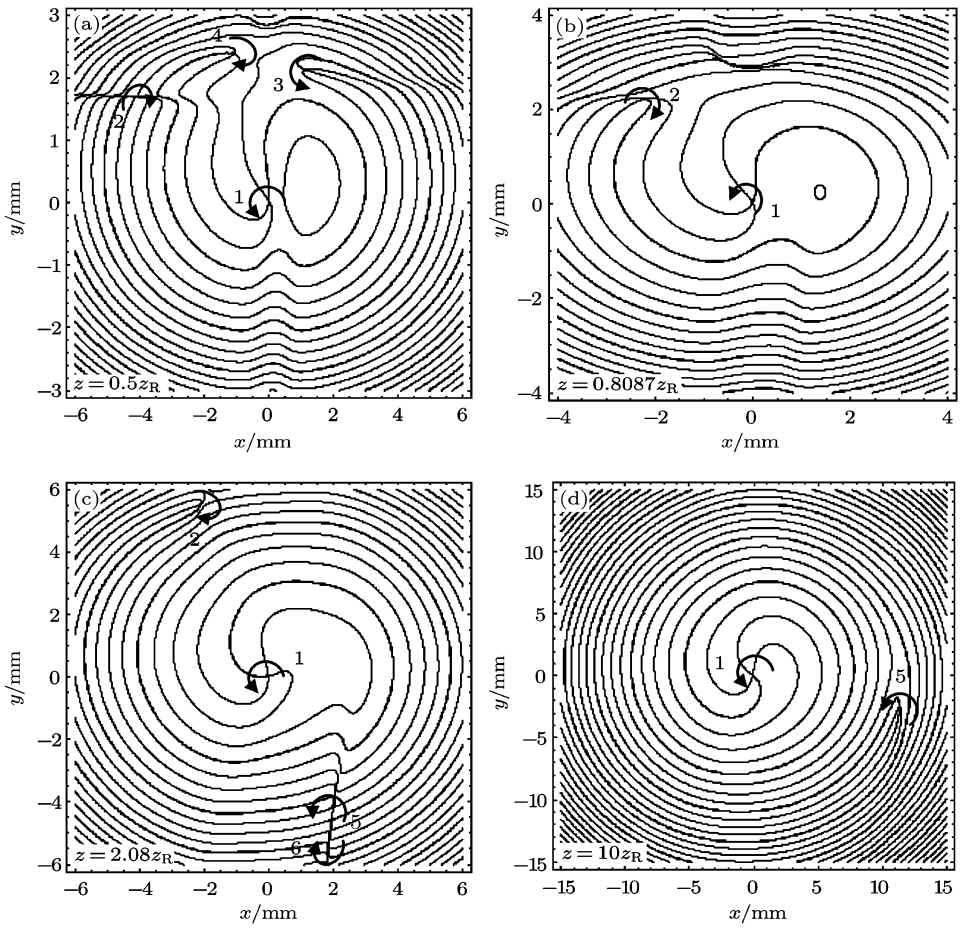


Fig. 6. Contour lines of phase of a diffracted Gaussian beam carrying two vortices of equal charges $m = +1$ for selected values of the propagation distance: (a) $z = 0.5z_R$; (b) $z = 0.8087z_R$; (c) $z = 2.08z_R$; (d) $z = 10z_R$, and $s_1 = 0$, $s_2 = 0.1$ mm.

In comparison with Figs. 1 and 6 we see that when a Gaussian beam carrying two on-axis vortices of $m = -1$ is diffracted at a half-plane screen, the evolution behaviour of vortices in the diffraction field is similar to that of the diffracted Gaussian beam with same beam parameters containing two on-axis vortices of $m = +1$, i.e., the pair creation and annihilation of vortices appear at the same plane, and the positions of vortices in the diffraction field are symmetric about the y axis. In fact, from Eq. (6) it turns out that for the on-axis case the term $\mp 2\,\mathrm{i}\,s_1 s_2 = 0$, and the term $\pm 2\,\mathrm{i}(q_x/p)\,(2p)^{-1}\,D_{-2}\left(q_y/\sqrt{2p}\right)$ lead to the symmetry about the y axis of vortices.

For the cases that two vortices of $m = +1$ with $s_2 = 1.2$ mm at the plane $z = 0$ are located on the y axis and two vortices of $m = +1$ with $s_1 = 0.2$ mm, 1.2 mm are located on the x axis, the results are similar to Figs. 2, 3 and 4, respectively, except for that the positions of vortices are symmetric about the y axis, which are not shown.

4. Conclusion

In this paper, analytical expressions for the propagation of Gaussian beams containing two vortices of equal charges $m = \pm 1$ diffracted at a half-plane screen have been derived and used to study the dynamic evolution of vortices in the diffraction field. It has been found that there may exist no vortices, a pair or several pairs of vortices of opposite charges $m = +1$, -1 in the diffraction field, due to the half-screen diffraction and interaction of vortices. The pair creation, annihilation and motion of vortices take place by varying propagation distance and/or off-axis parameter. In the evolution process the total topological charge is zero, which is unequal to that of the vortex beam at the source plane $z = 0$.

In comparison with the free-space propagation of two vortices of equal charges $m = \pm 1$ nested in a Gaussian beam ($T(x_0,\ y_0) = 1$), by straightforward calculations of the Fresnel diffraction integral we obtain

$$\begin{aligned} E_{\pm}(x,y,z) = {} & \frac{2\pi \exp\left[\mathrm{i}\,kz + \dfrac{\mathrm{i}(x^2+y^2)}{kw_0^2 + 2z\,\mathrm{i}}\right]}{(kw_0^2 + 2z\,\mathrm{i})^3 \lambda} \\ & \times \{kw_0^4(x^2 - y^2 - s_1^2 + s_2^2) \\ & \pm 8ks_1 s_2 w_0^2 z + 4z^2(s_1^2 - s_2^2) \\ & - 2\,\mathrm{i}[\pm kw_0^4(s_1 s_2 - xy) \\ & + 2zkw_0^2(s_1^2 - s_2^2) \mp 4s_1 s_2 z^2]\}. \quad (8) \end{aligned}$$

Substituting Eq. (8) into Eqs. 7(a) and 7(b) yields

$$\begin{aligned} & kw_0^4\left(x^2 - y^2 - s_1^2 + s_2^2\right) \\ & \quad \pm 8ks_1 s_2 w_0^2 z + 4z^2\left(s_1^2 - s_2^2\right) = 0, \quad (9a) \\ & 2[\pm kw_0^4\left(s_1 s_2 - xy\right) \\ & \quad + 2zkw_0^2\left(s_1^2 - s_2^2\right) \mp 4s_1 s_2 z^2] = 0. \quad (9b) \end{aligned}$$

From Eqs. (9a) and (9b) the positions of vortices are given by

$$x_{1,\pm} = \pm s_1 - \frac{s_2 z}{z_R}, \quad y_{1,\pm} = \pm s_2 + \frac{s_1 z}{z_R}, \quad (10a)$$

$$x_{2,\pm} = \mp s_1 + \frac{s_2 z}{z_R}, \quad y_{2,\pm} = \mp s_2 - \frac{s_1 z}{z_R}. \quad (10b)$$

It can be shown by numerical calculations and vorticity analysis that there are two vortices of equal charges $m = \pm 1$ in free-space propagation, which are located at $(x_{1,\pm},\ y_{1,\pm})$, and $(x_{2,\pm},\ y_{2,\pm})$ symmetrically about the origin (0, 0), when the two vortices at the source plane $z = 0$ are positioned symmetrically at $(s_1,\ s_2)$ and $(-s_1,\ -s_2)$, respectively. The total topological charge is equal to $m = +2$ for two vortices of $m = +1$, or equal to $m = -2$ for two vortices of $m = -1$ in free-space propagation, and is conserved. In addition, in the far field, Eqs. (10a) and (10b) reduce to

$$X_{1,\pm} = \frac{x_{1,\pm}}{z/z_R} = -s_2, \quad Y_{1,\pm} = \frac{y_{1,\pm}}{z/z_R} = s_1, \quad (11a)$$

$$X_{2,\pm} = \frac{x_{2,\pm}}{z/z_R} = s_2, \quad Y_{2,\pm} = \frac{y_{2,\pm}}{z/z_R} = -s_1, \quad (11b)$$

respectively. Equations (11a) and (11b) imply that a 90°-rotation of the pattern takes place in the far field, but there is no change in the vortex separation normalized by z/z_{R}, which is consistent with Refs. [8] and [9]. Therefore, the evolutionary behaviour of two vortices of equal charges in the diffraction field is quite different from that in free-space propagation.

The results for $m = \pm 1$ can be extended to the case of $|m| > 1$. For example, for two vortices of equal charge m=+2 nested in a Gaussian beam the electric field at the plane $z = 0$ is

$$E_0(x_0, y_0, 0) = T(x_0, y_0)\left[\frac{x_0 - s_1 + \mathrm{i}\,(y_0 - s_2)}{w_0}\right]^2 \left[\frac{x_0 + s_1 + \mathrm{i}\,(y_0 + s_2)}{w_0}\right]^2 \exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right). \quad (12)$$

Substituting Eq. (12) into Eq. (4), by straightforward integral calculations, we obtain

$$E(x,y,z) = \frac{-\mathrm{i}}{\lambda z w_0^2}\sqrt{\frac{\pi}{p}}\exp\left[\mathrm{i}kz + \frac{\mathrm{i}k\left(x^2+y^2\right)}{2z} + \frac{q_x^2}{p} + \frac{q_y^2}{8p}\right] \times \left\{\left[\frac{\left(4q_x^4+12q_x^2p+3p^2\right)}{4p^4} + \frac{\left(4q_x^2s_2^2+2s_2^2p\right)}{2q_x^2} + s_2^4\right](2p)^{-1/2} D_{-1}\left(\frac{q_y}{\sqrt{2p}}\right) + 4\,\mathrm{i}\left(\frac{q_x^3}{p^3} + \frac{q_x}{4p^2} + \frac{s_2^2q_x}{p}\right)(2p)^{-1} D_{-2}\left(\frac{q_y}{\sqrt{2p}}\right) - 2\left(6q_x^2 + \frac{3}{p} + 2s_2^2\right)(2p)^{-3/2} D_{-3}\left(\frac{q_y}{\sqrt{2p}}\right) - 6\left(\frac{4\,\mathrm{i}\,q_x}{p}\right)(2p)^{-2} D_{-4}\left(\frac{q_y}{\sqrt{2p}}\right) + 24\,(2p)^{-5/2} D_{-5}\left(\frac{q_y}{\sqrt{2p}}\right)\right\} \quad (\text{for } s_1 = 0), \tag{13a}$$

$$E(x,y,z) = \frac{-\mathrm{i}}{\lambda z w_0^2}\sqrt{\frac{\pi}{p}}\exp\left[\mathrm{i}kz + \frac{\mathrm{i}k\left(x^2+y^2\right)}{2z} + \frac{q_x^2}{p} + \frac{q_y^2}{8p}\right] \times \left\{\left[\frac{\left(4q_x^4+12q_x^2p+3p^2\right)}{4p^4} - \frac{\left(4q_x^2s_1^2+2s_1^2p\right)}{2q_x^2} + s_1^4\right](2p)^{-1/2} D_{-1}\left(\frac{q_y}{\sqrt{2p}}\right) + 4\,\mathrm{i}\left(\frac{q_x^3}{p^3} + \frac{q_x}{4p^2} - \frac{s_1^2q_x}{p}\right)(2p)^{-1} D_{-2}\left(\frac{q_y}{\sqrt{2p}}\right) - 2\left(6q_x^2 + \frac{3}{p} - 2s_1^2\right)(2p)^{-3/2} D_{-3}\left(\frac{q_y}{\sqrt{2p}}\right) - 6\left(\frac{4\,\mathrm{i}\,q_x}{p}\right)(2p)^{-2} D_{-4}\left(\frac{q_y}{\sqrt{2p}}\right) + 24\,(2p)^{-5/2} D_{-5}\left(\frac{q_y}{\sqrt{2p}}\right)\right\} \quad (\text{for } s_2 = 0). \tag{13b}$$

As can be readily shown in a similar way, there may exist no vortices, a pair or several pairs of vortices of opposite charges $m = +1$, -1, and the creation–annihilation process may occur, but the total topological charge is equal to zero, and is not conserved in the diffraction field.

For the half-plane diffraction of Gaussian beams the propagation equation is expressed as

$$E(x,y,z) = \frac{-\mathrm{i}}{\lambda z}\sqrt{\frac{\pi}{p}}\exp\left[\mathrm{i}kz + \frac{\mathrm{i}k\left(x^2+y^2\right)}{2z} + \frac{q_x^2}{p} + \frac{q_y^2}{8p}\right](2p)^{-1/2} D_{-1}\left(\frac{q_y}{\sqrt{2p}}\right). \tag{14}$$

Figures 7(a) and 7(b) give the phase contour lines at (a) $z = 0.5z_{\mathrm{R}}$ and (b) $z = 10z_{\mathrm{R}}$. The other calculation parameters are the same as those in Fig. 1. It is seen that no vortices appear, because, as shown in Ref. [19], the physical origin of the presence of vortices in the diffraction field is the vortices embedded in the Gaussian beam at the source plane $z = 0$ rather than the half-plane diffraction.

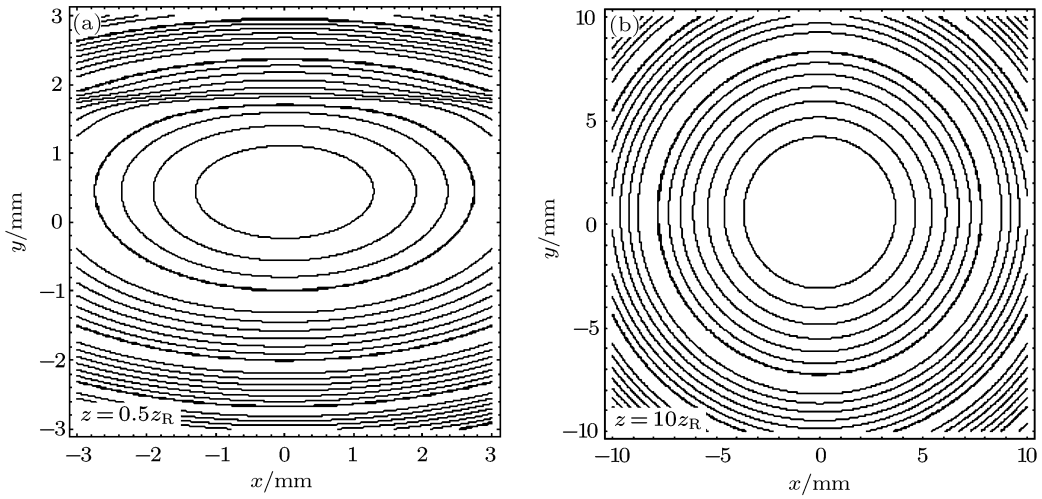


Fig. 7. Contour lines of phase of a diffracted Gaussian beam for selected values of the propagation distance: (a) $z = 0.5z_{\mathrm{R}}$; (b) $z = 10z_{\mathrm{R}}$.

The results obtained in this paper would be beneficial for understanding the dynamic evolution behaviour of the vortex array of equal charges in the diffraction field.

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