A new metric for rotating charged Gauss–Bonnet black holes in AdS space*

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In this paper, we study a new metric for slowly rotating charged Gauss-Bonnet black holes in higher-dimensional anti-de Sitter space. Taking the angular momentum parameter a up to second order, the slowly rotating charged black hole solutions are obtained by working directly in the action.

Keywords: Gauss–Bonnet gravity, slow rotation, anti-de Sitter spaces

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1. Introduction

Higher-derivative gravity theories have attracted increasing interest in recent years. It is believed that Einstein's action is only a valid effective gravitational action for small curvatures or low energies.[1] Considering the fundamental nature of quantum gravity, one sees that the action will be modified by higher-derivative interactions in the renormalization of quantum field theory in curved spacetimes.[2−4] These higher-derivative terms can be seen in the construction of the low-energy effective action of strings.[5−8] The so-called Lovelock gravity is quite special.[9] Apart from the advantage that its equations of motion, as in the case of Einstein's general relativity, do not contain terms with more than second derivatives of the metric, the Lovelock gravity has been shown to be free of ghosts during expansion on a flat space, thus avoiding any problems with unitarity.[10,11] The Gauss–Bonnet terms appear as the first higher-derivative curvature correction terms to general relativity. The black hole solutions in Gauss–Bonnet gravity were first discovered by Boulware and Deser[12] and Wheeler,[13] independently. The thermodynamics of the uncharged static spherical Gauss–Bonnet black hole solutions have been considered in Refs. [14]–[16] and of the charged solutions in Ref. [17]. Recently the quasinormal mode of a scalar field in five-dimensional Lovelock black hole spacetime for different angular quantum numbers l was obtained in Ref. [18]. Liu[19] studied the electromagnetic perturbations of black holes in Gauss–Bonnet gravity.

It is of interest to generalize these static Gauss–Bonnet solutions by including the effects of rotation. This problem has been considered recently in Ref. [20] within a perturbative approach. It is worth mentioning that, for the Gauss–Bonnet action, the resulting field equations, obtained after variation with respect to the metric tensor, have seven terms, while the resulting field equation for third-order Lovelock gravity contains 34 terms. However, by working directly in the action, the exact static Gauss–Bonnet black hole solutions were obtained,[12−16] along with third-order Lovelock black hole solutions, in Refs. [21]–[23]. Using the same approach, in this paper, we obtain the slowly rotating black hole solutions in Lovelock gravity. Apparently the lowest-level contribution of rotation should be proportional to a^2. Hence, being linearly dependent on a, the metric demonstrated in Ref. [20] cannot be adopted in this case. So we need to find a proper metric ansatz up to a^2.

This paper is organized as follows. In Section 2, we present a new metric form and obtain slowly rotating charged black hole solutions by working directly in the action. Then we discuss some related thermodynamic properties of black holes. Section 3 is devoted

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to a summary of the results.

2. Slowly rotating charged Gauss–Bonnet black holes

2.1. A new metric and rotating black holes

In general relativity, the higher-dimensional generalization of the Kerr–AdS black hole solutions can be found in Refs. [24] and [25]. In Boyer–Lindpuist-type coordinates, it is expressed as

$$\begin{aligned} \mathrm{d}s^2 = & -\frac{\Delta_r}{\Sigma}\left(\mathrm{d}t-\frac{a\sin^2\theta}{\Xi}\mathrm{d}\phi\right)^2 \\ & +\frac{\Sigma}{\Delta_r}\mathrm{d}r^2+\frac{\Sigma}{\Delta_\theta}\mathrm{d}\theta^2 \\ & +\frac{\Delta_\theta\sin^2\theta}{\Sigma}\left(a\,\mathrm{d}t-\frac{r^2+a^2}{\Xi}\mathrm{d}\phi\right)^2 \\ & +r^2\cos^2\theta\,\mathrm{d}\Omega_{D-4}^2, \end{aligned} \tag{1}$$

where

$$\begin{aligned} &\Sigma = r^2+a^2\cos^2\theta, \\ &\Delta_r = (r^2+a^2)\left(1+\frac{r^2}{l^2}\right)-mr^{5-D}, \\ &\Delta_\theta = 1-\frac{a^2}{l^2}\cos^2\theta, \quad \Xi = 1-\frac{a^2}{l^2}. \end{aligned}$$

The parameters m and a are related to the mass and angular momentum of the black hole, as we shall see.

If we consider Eq. (1) up to second order in the parameter a, the components of the metric are expressed as follows:

$$\begin{aligned} g_{tt} = & -\left[1-\frac{m}{r^{D-3}}+\frac{r^2}{l^2}+\frac{a^2}{l^2}\sin^2\theta \right. \\ & \left. +\frac{a^2m\cos^2\theta}{r^{D-1}}\right]+\mathcal{O}(a^4), \\ g_{rr} = & \frac{1+a^2\cos^2\theta/r^2}{1-m/r^{D-3}+r^2/l^2} \\ & -\frac{a^2}{(1-m/r^{D-3}+r^2/l^2)^2}\left(\frac{1}{l^2}+\frac{1}{r^2}\right)+\mathcal{O}(a^4), \\ g_{\theta\theta} = & (r^2+a^2\cos^2\theta+\frac{a^2r^2}{l^2}\cos^2\theta)+\mathcal{O}(a^4), \\ g_{\phi\phi} = & \left[r^2+a^2+a^2\sin^2\theta\left(\frac{m}{r^{D-3}}-\frac{r^2}{l^2}\right)\right. \\ & \left.+\frac{2a^2r^2\sin^2\theta}{l^2}\right]\sin^2\theta+\mathcal{O}(a^4), \\ g_{t\phi} = & 2a\sin^2\theta\left(\frac{r^2}{l^2}-\frac{m}{r^{D-3}}\right)+\mathcal{O}(a^3). \end{aligned} \tag{2}$$

Setting $a=0$, we find that equation (2) reduces to the static case. It is known that the static black hole solution in D-dimensional spacetimes in general relativity is $f(r)=1-m/r^{D-3}+r^2/l^2$. Replacing $1-m/r^{D-3}+r^2/l^2$ with $f(r)$, we obtain a new metric when all terms involving a^3 and higher powers in a are ignored

$$\begin{aligned} \mathrm{d}s^2 = & -\left[f(r)+\frac{a^2}{l^2}+\frac{a^2(1-f(r))\cos^2\theta}{r^2}\right]\mathrm{d}t^2 \\ & +\left[\frac{1}{f(r)}+\frac{a^2(\cos^2\theta f(r)-1)}{r^2f(r)^2}-\frac{a^2}{l^2f(r)^2}\right]\mathrm{d}r^2 \\ & +(r^2+a^2\cos^2\theta+\frac{a^2r^2}{l^2}\cos^2\theta)\mathrm{d}\theta^2+\left[r^2+a^2\right. \\ & \left.+a^2\big(1-f(r)\big)\sin^2\theta+\frac{2a^2r^2\sin^2\theta}{l^2}\right]\sin^2\theta\,\mathrm{d}\phi^2 \\ & +2ar^2p(r)\sin^2\theta\,\mathrm{d}t\,\mathrm{d}\phi+r^2\cos^2\theta\,\mathrm{d}\Omega_{D-4}^2, \end{aligned} \tag{3}$$

where parameter a^2 is a small quantity and the functions $f(r)$ and $p(r)$ satisfy the relation $p(r)=[-1+f(r)]/r^2$. For future simplification, let $f(r)$ and $p(r)$ be two independent variables. Taking the parameter a up to the first order, equation (3) becomes its counterpart in Ref. [20].

The action for Gauss–Bonnet theory with negative cosmological constant $\Lambda=-(D-1)(D-2)/2l^2$ in D dimensions is given by[20]

$$\begin{aligned} \mathscr{I} = & \frac{1}{16\pi G}\int \mathrm{d}^Dx\sqrt{-g}(-2\Lambda+R \\ & +\alpha\mathscr{L}_{\mathrm{GB}}-4\pi GF_{\mu\nu}F^{\mu\nu}), \end{aligned} \tag{4}$$

where α is the Gauss–Bonnet coefficient with dimension (length)^2 and is henceforth rescaled with $\tilde{\alpha}/(D-3)(D-4)$. Note that the Lagrangian of Lovelock gravity is the sum of dimensionally extended Euler densities, and so the Lagrangian of Gauss–Bonnet term also can be expressed as[15]

$$\mathscr{L}_{\mathrm{GB}}=\frac{1}{4}\delta^{a_1b_1a_2b_2}_{c_1b_1c_2d_2}R^{c_1d_1}{}_{a_1b_1}R^{c_2d_2}{}_{a_2b_2}. \tag{5}$$

Here, all non-vanishing components of Riemann tensors (up to a^2) can be expressed as

$$R^{ij}{}_{kl}=\bar{R}^{ij}{}_{kl}+\tilde{R}^{ij}{}_{kl},$$

where $\bar{R}^{ij}{}_{kl}$ is absent from a and $\tilde{R}^{ij}{}_{kl}$ is proportional to a^2. Moreover, based on the relationship of indices, these Riemann tensors can also be classified into three groups: (I) diagonal form $R^{\hat{i}\hat{j}}{}_{\hat{i}\hat{j}}$, (II) off-diagonal form proportional to a, and (III) off-diagonal form proportional to a^2. For group (I), we find that the Riemann tensors $\bar{R}^{12}{}_{12}$, $\bar{R}^{1i}{}_{1i}$, $\bar{R}^{2i}{}_{2i}$, and $\bar{R}^{ij}{}_{ij}$ are all absent from parameter a. They can be represented as $\bar{R}^{12}{}_{12}=-f''(r)/2$, $\bar{R}^{1i}{}_{1i}=-f'(r)/2r$, $\bar{R}^{2i}{}_{2i}=-f(r)/2r$, and $\bar{R}^{ij}{}_{ij}=[1-f(r)]/r^2$ $(3\leq i<j\leq D)$ which is

independent of the dimensions D. Some key steps are given in Appendix A. Denoting the Lagrangian of Gauss–Bonnet term from the contribution of case (I) by $\mathscr{L}_d$, it can be written as

$$\begin{aligned}\mathscr{L}_d &= \bar{\mathscr{L}}_d + \tilde{\mathscr{L}}_d \\ &= \frac{1}{4}\delta^{a_1b_1a_2b_2}_{c_1b_1c_2d_2}(\bar{R}^{c_1d_1}{}_{a_1b_1}\bar{R}^{c_2d_2}{}_{a_2b_2} \\ &\quad + 2\bar{R}^{c_1d_1}{}_{a_1b_1}\tilde{R}^{c_2d_2}{}_{a_2b_2}) \\ &= \bar{A}_0 + \bar{A}_1 + \bar{A}_2 + \bar{A}_3 + \bar{A}_4 \\ &\quad + \tilde{B}_1 + \tilde{B}_2 + \tilde{B}_3 + \tilde{B}_4, \end{aligned} \tag{6}$$

where

$$\begin{aligned}\bar{A}_0 &= \frac{1}{4}\delta^{a_1b_1a_2b_2}_{c_1b_1c_2d_2}\bar{R}^{c_1d_1}{}_{a_1b_1}\bar{R}^{c_2d_2}{}_{a_2b_2} \\ &= (D-2)(D-3)(D-4)(D-5)(\bar{R}^{ij}{}_{ij})^2, \\ &\quad (i,j\geq 3), \\ \bar{A}_1 &= 2\delta^{a_1b_112}_{c_1b_112}\bar{R}^{c_1d_1}{}_{a_1b_1}\bar{R}^{12}{}_{12} \\ &= 4(D-2)(D-3)\bar{R}^{12}{}_{12}\bar{R}^{ij}{}_{ij}, \\ \bar{A}_2 &= \frac{1}{4}\delta^{1i2j}_{1i2j}\bar{R}^{1i}{}_{1i}\bar{R}^{2j}{}_{2j} \\ &= 8(D-2)(D-3)\bar{R}^{1i}{}_{1i}\bar{R}^{2j}{}_{2j}, \\ \bar{A}_3 &= \frac{1}{4}\delta^{1ijk}_{1ijk}\bar{R}^{jk}{}_{jk}\bar{R}^{1i}{}_{1i} \\ &= 4(D-2)(D-3)(D-4)\bar{R}^{1i}{}_{1i}\bar{R}^{jk}{}_{jk}, \\ \bar{A}_4 &= \frac{1}{4}\delta^{2ijk}_{2ijk}\bar{R}^{jk}{}_{jk}\bar{R}^{2i}{}_{2i} \\ &= 4(D-2)(D-3)(D-4)\bar{R}^{2i}{}_{2i}\bar{R}^{jk}{}_{jk}, \\ &\quad (i,j,k\geq 3), \end{aligned} \tag{7}$$

and

$$\begin{aligned}\tilde{B}_1 &= 4(D-2)(D-3)\tilde{R}^{12}{}_{12}\times\bar{R}^{34}{}_{34}, \\ \tilde{B}_2 &= 4\sum_{i=3}^{D}\tilde{R}^{1i}{}_{1i} \\ &\quad \times[2(D-3)\bar{R}^{24}{}_{24} + (D-3)(D-4)\bar{R}^{45}{}_{45}], \\ \tilde{B}_3 &= 4\sum_{i=3}^{D}\tilde{R}^{2i}{}_{2i} \\ &\quad \times[2(D-3)\bar{R}^{14}{}_{14} + (D-3)(D-4)\bar{R}^{45}{}_{45}], \\ \tilde{B}_4 &= 8\sum_{i>j=3}^{D}\tilde{R}^{ij}{}_{ij}\Big[\bar{R}^{12}{}_{12} + (D-4)(\bar{R}^{13}{}_{13} + \bar{R}^{25}{}_{25}) \\ &\quad + \frac{(D-4)(D-5)}{2}\bar{R}^{kl}{}_{kl}\Big], \quad (k>l\geq 4). \end{aligned} \tag{8}$$

Inspecting the off-diagonal Riemann tensors, we calculate the parts of the Lagrangian from the off-diagonal Riemann tensors in case (II)

$$\mathscr{L}_{\rm od} = \mathscr{L}_{\rm od1} + \mathscr{L}_{\rm od2} + \mathscr{L}_{\rm od3}, \tag{9}$$

where

$$\begin{aligned}\mathscr{L}_{\rm od1} &= 4(R^{12}{}_{34}R^{34}{}_{12} + R^{13}{}_{24}R^{24}{}_{13} + R^{14}{}_{23}R^{23}{}_{14}), \\ \mathscr{L}_{\rm od2} &= \sum_{i=3}^{D}4(R^{12}{}_{24}R^{i4}{}_{1i} + R^{24}{}_{12}R^{1i}{}_{i4}), \\ \mathscr{L}_{\rm od3} &= \sum_{i,j=3}^{D}4(R^{1i}{}_{i4}R^{j4}{}_{1j} + R^{1j}{}_{j4}R^{i4}{}_{1i}), \\ &\quad (i>j\geq 3). \end{aligned} \tag{10}$$

It is interesting to note that the contribution of off-diagonal Riemann tensors in case (III) vanishes following the properties of the Kronecker delta symbol. In addition, the Ricci scalar R is given by

$$R = \bar{R} + \tilde{R}, \tag{11}$$

where $\bar{R}$ is equal to $-6f'(r)/r + 6(1-f(r))/r^2 - f''(r)$ and $\tilde{R}$ is proportional to a^2 and shown in Aappendix A.

Since the black hole rotates along the direction ϕ, it will generate a magnetic field. If we consider this effect, the gauge potential can be chosen as

$$A_\mu \mathrm{d}x^\mu = A_t \mathrm{d}t + A_\phi \mathrm{d}\phi, \tag{12}$$

where

$$A_t = Qh(r) + a^2\cos^2\theta Qk(r), \quad A_\phi = -a\sin^2\theta Qc(r).$$

Therefore the nonzero contravariant components of the electromagnetic field tensor can be written as

$$\begin{aligned}&F_{tr} = -Qh'(r) - a^2\cos^2\theta Qk'(r), \\ &F_{t\theta} = a^2\sin(2\theta)Qk(r), \quad F_{r\phi} = -aQc'(r)\sin^2\theta, \\ &F_{\theta\phi} = -aQc(r)\sin(2\theta). \end{aligned} \tag{13}$$

Substituting Eqs. (3) and (13) into Eq. (4), we find that the action becomes

$$\mathscr{I} = \mathscr{I}(f(r), p(r), h(r), k(r), c(r)). \tag{14}$$

First, varying Eq. (14) with regard to the $k(r)$, we find the solution

$$h''(r)r + (D-2)h'(r) = 0. \tag{15}$$

Hence, we obtain

$$h(r) = \frac{C_2}{r^{D-3}} + C_1, \tag{16}$$

where C_1 and C_2 are two integration constants. We choose $C_1 = 0$ and $C_2 = 1/4(D-3)\pi$ and then the function $h(r)$ can be written as $h(r) = 1/4\pi(D-3)r^{D-3}$. Then, the variation of the action Eq. (14) with regard to $c(r)$ becomes

$$[r^{D-4}f(r)c'(r)]' - 2(D-3)r^{D-6}c(r) + \frac{p'(r)}{4\pi} = 0. \tag{17}$$

Taking the variation of Eq. (14) with regard to the function $p(r)$, we arrive at

$$
\begin{aligned}
0 = {} & [2\tilde{\alpha}f(r) - 2\tilde{\alpha} - r^2]r^2 f(r)p''(r) \\
& + [2(D-2)\tilde{\alpha}(f(r)-1) - Dr^2 + 2\tilde{\alpha}rf'(r)]rf(r)p'(r) \\
& + \{(D-2)\Big[- f'(r)r^3 + (1-f(r))((D-3)r^2 - 2\tilde{\alpha}rf'(r)) + (D-5)\tilde{\alpha}(1-f(r))^2 + \frac{(D-1)r^4}{l^2}\Big] \\
& - 8\pi GQ^2 r^4 h'(r)\}p(r) + 16\pi GQ^2 r h'(r)c'(r). \qquad (18)
\end{aligned}
$$

Supposing the coefficient of function $p(r)$ vanishes, we find that, for $h(r) = 1/4\pi(D-3)r^{D-3}$, the function $f(r)$ is obtained as

$$
f(r) = 1 + \frac{r^2}{2\tilde{\alpha}}\Bigg(1 - \sqrt{1 - \frac{4\tilde{\alpha}}{l^2} + \frac{4\tilde{\alpha}m}{r^{D-1}} - \frac{2\tilde{\alpha}}{\pi(D-2)(D-3)}\frac{GQ^2}{r^{2D-4}}}\Bigg). \qquad (19)
$$

Moreover, equation (18) reduces to

$$
r^D[1 + 2\tilde{\alpha}\omega(r)]p'(r) + 4GQ^2 c(r) + C_3 = 0, \qquad (20)
$$

where $\omega(r) = [1 - f(r)]/r^2$. Based on Ref. [20], the function $p(r)$ is given by

$$
p(r) = \frac{1}{2\tilde{\alpha}}\Bigg(1 - \sqrt{1 - \frac{4\tilde{\alpha}}{l^2} + \frac{4\tilde{\alpha}m}{r^{D-1}} - \frac{2\tilde{\alpha}}{\pi(D-2)(D-3)}\frac{GQ^2}{r^{2D-4}}}\Bigg), \qquad (21)
$$

and $c(r)$ is

$$
c(r) = -\frac{1}{4(D-3)\pi r^{D-3}}, \qquad (22)
$$

by taking constant $C_3 = (D-1)m$. It is apparent that the functions $f(r)$ and $p(r)$ satisfy the relationship $p(r) = -[1 - f(r)]/r^2$.

Varying the action with regard to $h(r)$, we have

$$
\begin{aligned}
& (D-1) + 2(D-2)(D-3)\pi r^D k'(r) \\
& \quad + 2(D-3)\pi r^{D+1}k''(r) = 0. \qquad (23)
\end{aligned}
$$

We can obtain $k(r) = -1/4\pi(D-3)r^{D-1}$ where integral constants are all ignored. Therefore, the tensor of electromagnetic field are given as

$$
\begin{aligned}
F_{tr} &= \frac{Q}{4\pi r^{D-2}} - \frac{(D-1)}{4\pi(D-3)r^D}a^2 Q\cos^2\theta, \\
F_{t\theta} &= -\frac{a^2 Q}{4\pi(D-3)r^{D-1}}\sin(2\theta), \\
F_{r\phi} &= -\frac{aQ}{4\pi r^{D-2}}\sin^2\theta, \\
F_{\theta\phi} &= \frac{aQ}{4\pi(D-3)r^{D-3}}\sin(2\theta). \qquad (24)
\end{aligned}
$$

It is easy to verify that the expressions for functions $f(r)$ [Eq. (19)], $p(r)$ [Eq. (21)], $c(r)$ [Eq. (22)], and $h(r) = -k(r)r^2 = 1/4\pi(D-3)r^{D-3}$ still satisfy the variation of Eq. (4) with respect to $f(r)$.

2.2. Thermodynamics of black holes

Killing vectors can be used to give a physical interpretation of the parameters m and a. Following the analysis given in Refs. [25]–[28], one can obtain coordinate-independent definitions for these parameters. We have the integral

$$
\begin{aligned}
M &= -\frac{1}{16\pi G}\frac{D-2}{D-3}\oint \xi^{\mu;\nu}_{(t)}\,\mathrm{d}^{D-2}\Sigma_{\mu\nu}, \\
J &= \frac{1}{16\pi G}\oint \xi^{\mu;\nu}_{(\phi)}\,\mathrm{d}^{D-2}\Sigma_{\mu\nu}, \qquad (25)
\end{aligned}
$$

where the integrals are taken over the $(D-2)$-sphere at spatial infinity,

$$
\begin{aligned}
& \mathrm{d}^{D-2}\Sigma_{\mu\nu} \\
& = \frac{1}{(D-2)!}\sqrt{-g}\epsilon_{\mu\nu\alpha\beta\gamma}\,\mathrm{d}x^{i_1}\wedge\mathrm{d}x^{i_2}\wedge\cdots\mathrm{d}x^{i_{D-2}}. \qquad (26)
\end{aligned}
$$

To justify Eq. (25), we can calculate the integrands in the asymptotic region $r \to \infty$. For the dominant terms in the asymptotic expansion we have

$$
\begin{aligned}
\xi^{t;r}_{(t)} &= \frac{m(D-3)}{2r^{D-2}} + \mathcal{O}\Big(\frac{1}{r^D}\Big), \\
\xi^{t;r}_{(\phi)} &= -\frac{am(D-1)\sin^2\theta}{2r^{D-2}} + \mathcal{O}\Big(\frac{1}{r^D}\Big). \qquad (27)
\end{aligned}
$$

We perform the integration over a $(D-2)$-sphere at $r \to \infty$. It gives

$$
M = \frac{m(D-2)\Sigma_k}{16\pi G}, \quad J = \frac{2Ma}{D-2}. \qquad (28)
$$

In addition, if $a = 0$, the black hole does not rotate and the metric represents a spherically symmetric charged black hole with a spherically symmetric electric field. If the black hole rotates $a \neq 0$, the electric field is supplemented by a magnetic field due to

the dragging of the inertial reference frames into rotational motion around the black hole.

3. Concluding remarks

We have proposed a new metric and obtained the slowly rotating charged Gauss–Bonnet black hole solutions in anti-de Sitter space by working directly at the action. Besides the function $f(r)$, the diagonal components of the metric also involve a^2. Moreover, the $g_{t\phi}$ is proportional to $r^2p(r)$, thus making the equation for $p(r)$ much simpler. Since the black hole rotates along the direction ϕ, the vector potential has an extra nonradial component $A_\phi = -a\sin^2\theta Qc(r)$ and the t component is written as $A_t = Qh(r) + a^2\cos^2\theta Qk(r)$. By taking variation of the action with respect to the functions $p(r)$, $f(r)$, $c(r)$, $h(r)$, and $k(r)$, respectively, we obtained the exact form for function $p(r)$, while the function $f(r)$ retains the form of the static solutions. The exact expressions for $c(r)$, $h(r)$, and $k(r)$ have been expressed as $c(r) = -h(r) = k(r)r^2 = -1/4\pi(D-3)r^{D-3}$.

Appendix A

From Eq. (3), we obtain some of the intermediate steps of the calculation.

Riemann tensors The non-vanishing Riemann tensors can be classified into three groups: (I) diagonal form $R^{\hat{i}\hat{j}}{}_{\hat{i}\hat{j}}$, (II) off-diagonal form proportional to a, and (III) off-diagonal form proportional to a^2. We give them explicitly below.

Group (I)

$$
\begin{aligned}
R^{12}{}_{12} &= \bar{R}^{12}{}_{12} + \tilde{R}^{12}{}_{12}, \quad \bar{R}^{12}{}_{12} = -\frac{f(r)''}{2},\\
\tilde{R}^{12}{}_{12} &= a^2\Bigg\{\Bigg[\Bigg(-1+\frac{rf(r)'+r^2f(r)''}{2f(r)}-\frac{r^2f(r)'^2}{4f(r)^2}\Bigg)p(r)^2\\
&\quad+\Bigg(\frac{r^2f(r)'p(r)'}{f(r)}-5rp(r)'-r^2p(r)''\Big)\frac{p(r)}{2}-\frac{r^2p(r)'^2}{4}+\frac{f(r)'-rf(r)''}{2r^3f(r)}\\
&\quad+\frac{f(r)'^2}{4r^2f(r)^2}\Bigg]\sin^2\theta+\Bigg[\frac{-2f(r)'}{r^3}-\frac{3(1-f(r))}{r^4}+\frac{f(r)''}{2r^2}\Bigg]\cos^2\theta\Bigg\},\\
R^{1i}{}_{1i} &= \bar{R}^{1i}{}_{1i} + \tilde{R}^{1i}{}_{1i}, \quad \bar{R}^{1i}{}_{1i} = -\frac{f(r)'}{2r}, \quad (3\le i\le D),\\
\tilde{R}^{13}{}_{13} &= \Bigg\{-\frac{p(r)^2}{f(r)}+\frac{1}{f(r)r^4}+\frac{f(r)'}{r^3}-\frac{f(r)}{r^4}+\Big[-\frac{rp(r)^2f(r)'}{2f(r)}+\frac{rp(r)p(r)'}{2}\\
&\quad-\frac{1+f(r)}{r^4}-\frac{f(r)'}{2f(r)r^3}+\frac{2}{r^4f(r)}-\frac{2p(r)^2}{f(r)}+\frac{f(r)'}{r^3}+p(r)^2\Big]\sin^2\theta\Bigg\}a^2,\\
\tilde{R}^{14}{}_{14} &= \Bigg\{\Bigg[-p(r)^2-rp(r)'p(r)-\frac{r^2p(r)'^2}{4}+\frac{f(r)'^2}{r^4}-\frac{f(r)'}{2r^3f(r)}+\frac{f(r)'}{4r^2}\\
&\quad-\frac{f(r)'f(r)}{2r^3}+\frac{rf(r)'p(r)^2}{2f(r)}\Bigg]\sin^2\theta+\frac{f(r)'}{r^3}+\frac{-p(r)^2r^4+1-f(r)^2}{f(r)r^4}\cos^2\theta\Bigg\}a^2,\\
\tilde{R}^{1j}{}_{1j} &= \Bigg\{\Bigg[\frac{p(r)^2}{f(r)}-p(r)^2-\frac{f(r)'}{2r^3f(r)'}-\frac{1}{r^4f(r)}+\frac{f(r)'p(r)^2}{2f(r)}-\frac{rp(r)'p(r)}{2}\Bigg]\sin^2\theta\\
&\quad+\Bigg[\frac{f(r)'}{2r^3}-\frac{f(r)}{r^4}\Bigg]\cos^2\theta+\frac{1}{r^4}\Bigg\}a^2, \quad (5\le j\le D),\\
R^{2i}{}_{2i} &= \bar{R}^{2i}{}_{2i} + \tilde{R}^{2i}{}_{2i}, \quad \bar{R}^{2i}{}_{2i} = -\frac{f(r)}{2r}, \quad (3\le i\le D),\\
\tilde{R}^{23}{}_{23} &= \tilde{R}^{2j}{}_{2j} = a^2\Bigg[\frac{f(r)'}{r^3}+\frac{2(1-f(r))}{r^4}\Bigg]\cos^2\theta, \quad (5\le j\le D),\\
\tilde{R}^{24}{}_{24} &= \Bigg\{\Bigg[\frac{f(r)'^2-2f(r)''f(r)}{4r^2}-1-\frac{r^2p(r)p(r)''}{2}-\frac{3f(r)f(r)'}{2r^3}+\frac{f(r)^2}{r^4}\\
&\quad-\frac{r^2p(r)'^2}{4}-\frac{3rp(r)p(r)'}{2}\Bigg]\sin^2\theta+4(2-2f(r)+f(r)'r)\Bigg\}a^2,\\
R^{ij}{}_{ij} &= \bar{R}^{ij}{}_{ij} + \tilde{R}^{ij}{}_{ij}, \quad \bar{R}^{ij}{}_{ij} = \frac{1-f(r)}{r^2}, \quad (3\le i<j\le D),
\end{aligned}
$$

$$\tilde{R}^{34}{}_{34} = \left\{\left[\frac{6-5f(r)-f(r)^2}{r^4}+\frac{f(r)f(r)'}{2r^3}+\frac{10-rp(r)p(r)'}{2}\right]\sin^2\theta-\frac{9}{r^2l^2}-\frac{6(1-f(r))}{r^4}\right\}a^2,$$

$$\tilde{R}^{3k}{}_{3k} = 2a^2\cos^2\theta\left(-\frac{1}{r^2l^2}+\frac{f(r)-1}{r^4}\right),\quad (5\le k\le D),$$

$$\tilde{R}^{4l}{}_{4l} = \left\{\left[\frac{f(r)f(r)'}{2r^3}-\frac{rp(r)p(r)'}{2}+\frac{3}{r^2l^2}+\frac{-f(r)^2+2-f(r)}{r^4}\right]\sin^2\theta + \frac{2(f(r)-1)}{r^4}-\frac{2}{r^2l^2}\right\}a^2,\quad (5\le l\le D),$$

$$\tilde{R}^{kl}{}_{kl} = a^2\cos^2\theta\frac{f(r)-1}{r^4},\quad (5\le k<l\le D).$$

Group (II):

$$R^{12}{}_{34} = -ar^2\sin\theta\cos\theta p'(r),\quad R^{13}{}_{24} = -\frac{a\cos\theta\sin\theta p'(r)}{2f(r)},\quad R^{14}{}_{32} = -\frac{a\cos\theta p'(r)}{2f(r)\sin\theta},$$

$$R^{12}{}_{24} = -\frac{ar\sin^2\theta}{2}(3p'(r)+rp''(r)),\quad R^{1i}{}_{i4} = -\frac{ar\sin^2\theta}{2}p'(r),\quad (3\le i\le D).$$

Group (III):

$$R^{12}{}_{13} = \left[-\frac{f'(r)}{2f(r)r^2}-p(r)^2r-\frac{3}{r^3}+\frac{p(r)^2f'(r)r^2}{2f(r)}-p(r)p'(r)r^2+\frac{3f(r)}{r^3}-\frac{f'(r)}{r^2}\right]a^2\sin\theta\cos\theta,$$

$$R^{24}{}_{34} = \left[-p(r)p'(r)r^2+\frac{3f(r)(1-f(r))}{r^3}+\frac{3f'(r)f(r)}{2r^2}\right]a^2\sin\theta\cos\theta.$$

With regard to the Ricci scalar, the term $\tilde{R}$ which is proportional to a^2 is given by

$$R = \left\{\left[\left(\frac{r^2f(r)''}{f(r)}-8-\frac{r^2f(r)'^2}{2r^2f(r)^2}+\frac{4rf(r)'}{f(r)}+\frac{8}{f(r)}\right)p(r)^2+(14r-\frac{r^2f(r)'}{f(r)})p(r)' - \frac{3r^2p(r)'^2}{2}-\frac{2f(r)'}{f(r)r^3}-\frac{2f(r)f(r)'}{r^3}+\frac{f(r)'^2}{r^2}-\frac{8}{r^4f(r)}+\frac{f(r)f(r)''}{r^2}-\frac{2f(r)'}{r^3} + \frac{f(r)'^2}{2f(r)^2r^2}-\frac{f(r)''}{r^2}-\frac{2f(r)^2}{r^4}+\frac{20}{r^4}-\frac{f(r)''}{f(r)r^2}-\frac{10f(r)}{r^4}\right]\sin^2\theta\frac{4p(r)^2}{f(r)}-\frac{14}{r^4}+\frac{4}{f(r)r^4} + \frac{6f(r)'}{r^3}+\frac{f(r)''}{r^2}+\frac{10f(r)}{r^4}+\frac{30\sin^2\theta-26}{r^2l^2}\right\}a^2.$$

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