A nearly deterministic scheme for generation of multiphoton GHZ states with weak cross-Kerr nonlinearity*

Wang Yi(王 奕)[a)b)†], Ye Liu(叶 柳)[a)‡], and Fang Bao-Long(方保龙)[a)]

[a)] School of Physics and Material Science, Anhui University, Hefei 230039, China

[b)] Teaching and Research Section of Physics, School of Life Sciences, Anhui Medical University, Hefei 230032, China

(Received 19 April 2011; revised manuscript received 9 June 2011)

We propose a scheme to generate polarization-entangled multiphoton Greenberger–Horne–Zeilinger (GHZ) states based on weak cross-Kerr nonlinearity and subsequent homodyne measurement. It can also be generalized to produce maximally N-qubit entangled states. The success probabilities of our schemes are almost equal to 1.

Keywords: GHZ state, cross-Kerr nonlinearity, quantum entanglement

PACS: 03.67.Mn, 03.65.Ud, 42.50.Dv **DOI:** 10.1088/1674-1056/20/10/100313

Entanglement, one of the most striking features of quantum mechanics, is not only an essential ingredient for tests of quantum nonlocality against local hidden variable theories,[1−3] but also one of the most useful phenomena for application to quantum information theory and quantum computation.[4−6] In contrast with two-particle entangled states, Greenberger–Horne–Zeilinger (GHZ) states provide the possibility of testing quantum nonlocality without using Bell inequalities[2,3] and have practical applications in quantum cryptographic conference and quantum secret sharing.[7] Hence, preparation of the GHZ entangled state has become a critical technique in quantum information processing. Such states have been produced experimentally with microwave cavity QED,[8] photons,[9] and trapped ions.[10−13] Compared with most massive qubits, flying photon qubits are more suitable for long-distance quantum communication and more robust against decoherence because of their high speed and weak interaction with the environment.[14−17] Photon GHZ states have been successfully demonstrated by the multifold coincidence-measurement method.[18−20] In these experiments, the success of the schemes is conditional upon the detection of one or two photons leaking out of the cavities. Therefore, the schemes are probabilistic and the success probability decreases as the detection efficiency of the photodetectors decreases. So the output entangled states cannot be used as input states for further experimental operations. This drawback limits their efficient application to large-scale quantum information processing.[21]

Here, we propose a simple and efficient scheme by which it is comparatively easy to realize GHZ states of light fields, with a probability almost equal to 1. Our scheme has several other advantages. First, it uses only a coherent state instead of a coherent state superposition as in other methods.[22] This makes our scheme simpler to implement. Second, our scheme uses X homodyne measurement, which enhances experimental operability and makes simplifies the process.

Before presenting our protocol, we first review the cross-Kerr nonlinear interaction between a mode probe and a mode signal. Cross-Kerr nonlinearity provides a good tool to construct quantum nondemolition detection (QND), which has the potential of being able to condition the evolution of our system without having to destroy the single photons,[23−26] and which, with a cross-Kerr medium and a coherent state, can be used for checking the parity of the polarizations of two photons, operating as a controlled-Not (CNOT)

*Project supported by the National Natural Science Foundation of China (Grant No. 11074002), the Doctoral Foundation of the Ministry of Education of China (Grant No. 20103401110003), and the Personal Development Foundation of Anhui Province of China (Grant No. 2008Z018).

†E-mail: myonee@sina.com

‡Corresponding author. E-mail: yeliu@ahu.edu.cn

© 2011 Chinese Physical Society and IOP Publishing Ltd

gate,[23] and analysing the Bell states.[27] The Hamiltonian of a cross-Kerr nonlinear medium can be described by

$$H_{\rm QND} = \hbar\chi \hat{n}_{\rm pr}\hat{n}_{\rm si}. \tag{1}$$

The symbol $\hat{n}_{\rm pr}(\hat{n}_{\rm si})$ denotes the number operator for the mode probe (signal), and $\hbar\chi$ is the coupling strength of the nonlinearity, which is determined by the property of the material used. If we consider the signal state to have the form

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{\rm si} + |1\rangle_{\rm si})$$

with the probe beam initially in a coherent state $|\alpha_{\rm pr}\rangle$, then the cross-Kerr interaction causes the combined signal-probe system to evolve as

$$\begin{aligned}\psi(t)_{\rm out} &= U_{\rm CK}\,|\varphi\rangle_{\rm si}\,|\alpha\rangle_{\rm pr}\\ &= {\rm e}^{{\rm i}\,H_{\rm K}t/\hbar}\frac{1}{\sqrt{2}}(|0\rangle_{\rm si} + |1\rangle_{\rm si})\,|\alpha\rangle_{\rm pr}\\ &= \frac{1}{\sqrt{2}}(|0\rangle_{\rm si}\,|\alpha\rangle_{\rm pr} + |1\rangle_{\rm si}\,|\alpha\,{\rm e}^{{\rm i}\,\theta}\rangle_{p}),\end{aligned} \tag{2}$$

where $\theta = \chi t$ with t being the interaction time. We observe immediately that the Fock state $|n_{\rm a}\rangle$ is unaffected by the interaction with the cross-Kerr nonlinearity but the coherent state $|\alpha_{\rm pr}\rangle$ picks up a phase shift directly proportional to the number of photons $n_{\rm si}$ in the signal $|n_{\rm si}\rangle$ state. If we could measure this phase shift we could then infer the number of photons in the signal mode. This can be achieved simply with a homodyne measurement and conditioned on the results of the homodyne measurement as in Ref. [27].

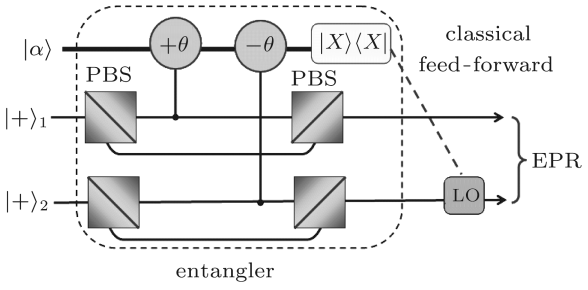


Fig. 1. Schematic setup for generation of an EPR state using two cross-Kerr nonlinearities and a coherent laser probe beam $|\alpha\rangle$. PBS: polarization beam splitters. LO: local operation. Signal mode: $|+\rangle_j = (|H\rangle + |V\rangle)_j/\sqrt{2}$ with $j = 1, 2$.

Now we study the generation of entangled EPR states using the weak-nonlinearity-based method, as shown in Fig. 1. Photons in the signal modes are prepared in a superposition state of horizontal and vertical polarizations while the probe beam is initially set in a coherent state. Then the initial state of the system that consists of two photons in the signal modes and a coherent probe beam is

$$\begin{aligned}|\psi\rangle_{\rm in} &= |+\rangle_1 \otimes |+\rangle_2 \otimes |\alpha\rangle_{\rm pr}\\ &= \frac{1}{2}(|H\rangle + |V\rangle)_1 \otimes (|H\rangle + |V\rangle)_2 \otimes |\alpha\rangle_{\rm pr}\,,\end{aligned} \tag{3}$$

where $|H\rangle_k\,(|V\rangle_k)$ is the horizontal (vertical) polarization state of the k-th photon in the signal mode. These qubits are split on polarizing beam splitters (PBS) into spatial modes which then interact with cross-Kerr nonlinearities. Using the weak cross-Kerr nonlinearity interactions with the phase shift angle θ and $-\theta$ (i.e., $\theta \sim 10^{-2}$[22]), the coherent state interacts with four qubits, thus an initial system state $|\psi\rangle_{\rm in}$ evolves to

$$\begin{aligned}|\psi\rangle_a = \frac{1}{2}[&(|HH\rangle + |VV\rangle)\,|\alpha\rangle_{\rm pr} + |HV\rangle\,|\alpha\,{\rm e}^{{\rm i}\,\theta}\rangle_{\rm pr}\\ &+ |VH\rangle\,|\alpha\,{\rm e}^{-{\rm i}\,\theta}\rangle_{\rm pr}].\end{aligned} \tag{4}$$

A homodyne detector can distinguish different coherent states. Therefore, when we find that the homodyne detector is in the coherent state $|\alpha\rangle_{\rm pr}$, photon 1 and photon 2 will be projected into the entangled state $(|HH\rangle+|VV\rangle)/\sqrt{2}$. If the result of the homodyne detector is $|\alpha\,{\rm e}^{{\rm i}\,\theta}\rangle_{\rm pr}$ ($|\alpha\,{\rm e}^{-{\rm i}\,\theta}\rangle_{\rm pr}$), the state of these two photons collapses into $|HV\rangle$ ($|VH\rangle$). In order to obtain the EPR state deterministically, we choose the local oscillator phase $\pi/2$ offset from the probe phase (we will call this an X quadrature measurement), then the states $|\alpha\,{\rm e}^{\pm{\rm i}\,\theta}\rangle_{\rm pr}$ cannot be distinguished.[27] More specifically, with α real, an X homodyne measurement condition is

$$\begin{aligned}|\psi\rangle_b = \frac{1}{2}[&f(X,\alpha)(|HH\rangle + |VV\rangle)\\ &+ f(X,\alpha\cos\theta)({\rm e}^{{\rm i}\,\phi(X)}\,|HV\rangle\\ &+ {\rm e}^{-{\rm i}\,\phi(X)}\,|VH\rangle)],\end{aligned} \tag{5}$$

where $f(x,\beta) = \exp[-\frac{1}{4}(x-2\beta)]/(2\pi)^{1/4}$ and $\phi(X) = \alpha x\sin\theta - \alpha^2\sin 2\theta({\rm Mod}\,2\pi)$.

We see that $f(X,\alpha)$ and $f(X,\alpha\cos\theta)$ are two Gaussian curves with the midpoint between the peaks located at $X_0 = \alpha(1+\cos\theta)$, and the peaks separated by a distance $X_{\rm d} = 2\alpha(1-\cos\theta)$. As long as this difference is large $\alpha\theta^2 \gg 1$, there is little overlap between these curves. When $X > X_0$, we have

$$|\psi_{X>X_0}\rangle_b \sim \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle), \tag{6}$$

while for $X < X_0$,

$$|\psi_{X<X_0}\rangle_b \sim \frac{1}{\sqrt{2}}({\rm e}^{{\rm i}\,\theta}\,|HV\rangle + {\rm e}^{-{\rm i}\,\theta}\,|VH\rangle). \tag{7}$$

The action of this two-mode polarization nondemolition parity detector is now very clear; the even

($|HH\rangle$ and $|VV\rangle$) and odd ($|HV\rangle$ and $|VH\rangle$) parity terms can be distinguished without being destroyed, by a high efficiency X quadrature homodyne measurement, i.e., the even parity terms can be split almost deterministically from the odd parity terms.

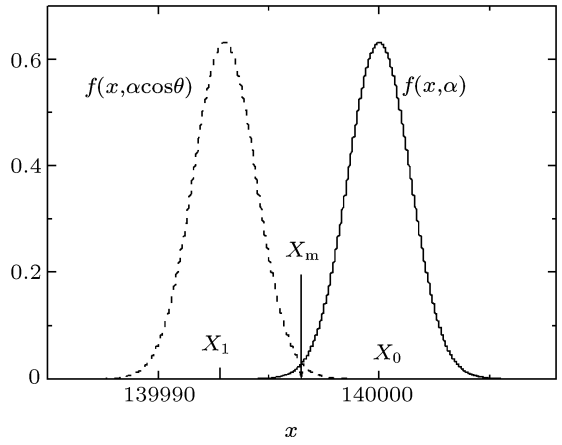


Fig. 2. Curves of the homodyne measurement result with $\theta = 0.01$ and $\alpha = 70000$.

We have used the approximate symbol ($\sim$) in these equations as there is a small but finite probability that the state (6) can occur for $X < X_0$. The probability of this error occurring is given by $P_{\text{error}} = \frac{1}{2}(1 - \text{er}[X_{\text{d}}/2\sqrt{2}])$ which is less than 10^{-5} when the distance ($X_{\text{d}} \sim \alpha\theta^2 > 9$) meets the conditions. This shows that it is still possible to operate in the regime of weak cross-Kerr nonlinearities, $\theta \ll \pi$. In general, natural Kerr media have extremely small nonlinearities with a typical dimensionless magnitude of $\theta < 10^{-18}$, thus a large Kerr nonlinearity at the single-photon level is almost impossible.

It is also interesting to look at the $X < X_0$ solution given by Eq. (7). We observe immediately that this state is dependent on the measured X homodyne value and hence it is conditioned depending on our measurement result X. Simple local rotations using phase shifters depending on the X measurement result via a feed-forward process (the dashed line in Fig. 1 can be performed to transform this state to $(|HV\rangle + |VH\rangle)/\sqrt{2}$ which is independent of X. A simple bit flip σ_x on the second polarization qubit transforms it into the state of $(|HH\rangle + |VV\rangle)/\sqrt{2}$. This gate allows us to take two separable polarization qubits and efficiently entangle them (nearly deterministically). Thus our two mode parity QNDs can be configured to act as an entangler by which the entangled EPR state can be generated nearly deterministically.

The feed-forward technique plays a very important role in EPR state generation, in that the future local rotation basis depends on earlier measurement results. This technique is crucial for achieving deterministic quantum state generation. It has been demonstrated that,[28] for an entangled state and no photon loss, the feed-forward components would operate at very high speed and low error for detected photons. With current technology, the individual computational step (in this case the individual feed-forward cycle) can be operated in less than 150 ns using electro-optical modulators.

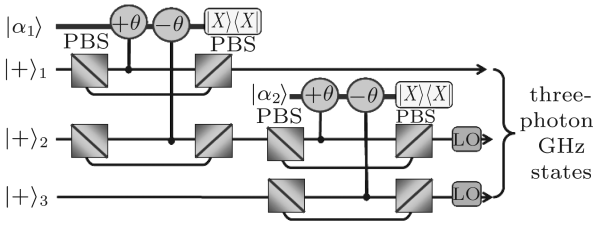


Fig. 3. Experimental scheme for generating the three-photon GHZ state.

We now describe how to implement three-photon GHZ state generation using such entanglers. The schematic setup of our proposal is shown in Fig. 3. The photon in the signal mode is prepared in a superposition state of horizontal and vertical polarizations while the probe beams are initially set in a coherent state. Then the initial state of the system that consists of three photons in the signal mode and the coherent probe beams is

$$\begin{aligned}|\psi\rangle_0 &= \overset{3}{\underset{k=1}{\otimes}} \frac{1}{\sqrt{2^3}}(|H\rangle + |V\rangle)_k \otimes |\alpha\rangle_{\text{p1}} \otimes |\alpha\rangle_{\text{p2}} \\ &= \frac{1}{2\sqrt{2}}(|HH\rangle + |HV\rangle + |VH\rangle + |VV\rangle)_{12} \otimes |\alpha\rangle_{\text{p1}} \\ &\quad \otimes (|H\rangle + |V\rangle)_3 \otimes |\alpha\rangle_{\text{p2}}. \end{aligned} \tag{8}$$

In the first step, photons 1 and 2 are both injected into entangler 1. Let photons 1 and 2 interact with the cross-Kerr medium. Thus the state of the total system becomes

$$\begin{aligned}|\psi\rangle_1 &= \frac{1}{2\sqrt{2}}[(|HH\rangle + |VV\rangle)_{12}|\alpha\rangle_{\text{p1}} \\ &\quad + |HV\rangle_{12}|\alpha\,\text{e}^{\text{i}\,\theta}\rangle_{\text{p1}} + |VH\rangle_{12}|\alpha\,\text{e}^{-\text{i}\,\theta}\rangle_{\text{p1}}] \\ &\quad \otimes (|H\rangle + |V\rangle)_3 \otimes |\alpha\rangle_{\text{p2}}. \end{aligned} \tag{9}$$

Then photons 2 and 3 are both injected into entangler 2. Let photons 2 and 3 interact with the cross-Kerr medium. Then the state of the total system becomes

$$\begin{aligned}|\psi\rangle_1 &= \frac{1}{2\sqrt{2}}[(|HHH\rangle + |VVV\rangle)_{12}|\alpha\rangle_{\text{p1}}|\alpha\rangle_{\text{p2}} \\ &\quad + |VVH\rangle_{12}|\alpha\rangle_{\text{p1}}|\alpha\,\text{e}^{-\text{i}\,\theta}\rangle_{\text{p2}} \\ &\quad + |HHV\rangle|\alpha\rangle_{\text{p1}}|\alpha\,\text{e}^{\text{i}\,\theta}\rangle_{\text{p2}} \end{aligned}$$

$$
\begin{aligned}
&+|VHH\rangle_{12}|\alpha\,\mathrm{e}^{-\mathrm{i}\,\theta}\rangle_{\mathrm{p1}}|\alpha\rangle_{\mathrm{p2}}\\
&+|HVV\rangle_{12}|\alpha\,\mathrm{e}^{\mathrm{i}\,\theta}\rangle_{\mathrm{p1}}|\alpha\rangle_{\mathrm{p2}}\\
&+|VHV\rangle_{12}|\alpha\,\mathrm{e}^{-\mathrm{i}\,\theta}\rangle_{\mathrm{p1}}|\alpha\,\mathrm{e}^{\mathrm{i}\,\theta}\rangle_{\mathrm{p2}}\\
&+|HVH\rangle_{12}|\alpha\,\mathrm{e}^{\mathrm{i}\,\theta}\rangle_{\mathrm{p1}}|\alpha\,\mathrm{e}^{-\mathrm{i}\,\theta}\rangle_{\mathrm{p2}}]. \qquad (10)
\end{aligned}
$$

In the next step, we implement the X-quadrature homodyne measurement on the probe beams. If we choose the local oscillator phase $\pi/2$ offset from the probe phases then the states $|\alpha\,\mathrm{e}^{\pm\mathrm{i}\,\theta}\rangle_{\mathrm{pr}}$ cannot be distinguished.[27] There are two independent X-quadrature homodyne measurements in Fig. 3, which cannot affect each other at all. The resulting three-photon state in the signal mode is then

$$
\begin{aligned}
|\psi\rangle_1 = \frac{1}{2}[&f_1(X,\alpha)f_2(X,\alpha)|\mathrm{GHZ}\rangle_0\\
&+f_1(X,\alpha)f_2(X,\alpha\cos\theta)|\mathrm{GHZ}\rangle_1\\
&+f_1(X,\alpha\cos\theta)f_2(X,\alpha)|\mathrm{GHZ}\rangle_2\\
&+f_1(X,\alpha\cos\theta)f_2(X,\alpha\cos\theta)|\mathrm{GHZ}\rangle_3], \qquad (11)
\end{aligned}
$$

where

$$
|\mathrm{GHZ}\rangle_0 \sim |HHH\rangle + |VVV\rangle, \qquad (12)
$$

$$
|\mathrm{GHZ}\rangle_1 \sim |HHV\rangle + |VVH\rangle, \qquad (13)
$$

$$
|\mathrm{GHZ}\rangle_2 \sim |HVV\rangle + |VHH\rangle, \qquad (14)
$$

$$
|\mathrm{GHZ}\rangle_3 \sim |HVH\rangle + |VHV\rangle. \qquad (15)
$$

For example, when the two X-quadrature homodyne measurements yield the results of phase shift 0 for the coherent states, the state collapses into $|\mathrm{GHZ}\rangle_0$. Finally, according to the measurement result X, the state of three-photons collapses into $|\mathrm{GHZ}\rangle_j$ $(j = 0, 1, 2, 3)$, respectively. Thus we have proposed a simple method for the generation of three-photon GHZ states via weak cross-Kerr nonlinearity and subsequent homodyne measurement, which is independent of X, with a probability of 1.

The above scheme can be easily extended to generate n-photon GHZ states. The general initial state is

$$
|\psi\rangle_0 = \bigotimes_{k=1}^{n} \frac{1}{\sqrt{2^k}}(|H\rangle + |V\rangle)_k \bigotimes_{j=1}^{n-1} |\alpha\rangle_{\mathrm{pj}}. \qquad (16)
$$

After $n-1$ successive cross-Kerr nonlinearity interactions and feed-forward cycles, as shown in Fig. 4, the whole system becomes

$$
\begin{aligned}
|\psi\rangle' = \frac{1}{\sqrt{2^k}}[&(|HH\cdots H\rangle + |VV\cdots V\rangle)\otimes|\alpha\rangle_{\mathrm{p1}}\cdots|\alpha\rangle_{\mathrm{p(n-1)}}\\
&+\cdots+|HH\cdots V\rangle\otimes|\alpha\rangle_{\mathrm{p1}}\cdots|\alpha\rangle_{\mathrm{p(n-2)}}|\alpha\,\mathrm{e}^{\mathrm{i}\,\theta}\rangle_{\mathrm{p(n-1)}}\\
&+|VV\cdots H\rangle|\alpha\rangle_{\mathrm{p1}}\cdots|\alpha\rangle_{\mathrm{p(n-2)}}|\alpha\,\mathrm{e}^{-\mathrm{i}\,\theta}\rangle_{\mathrm{p(n-1)}}]. \qquad (17)
\end{aligned}
$$

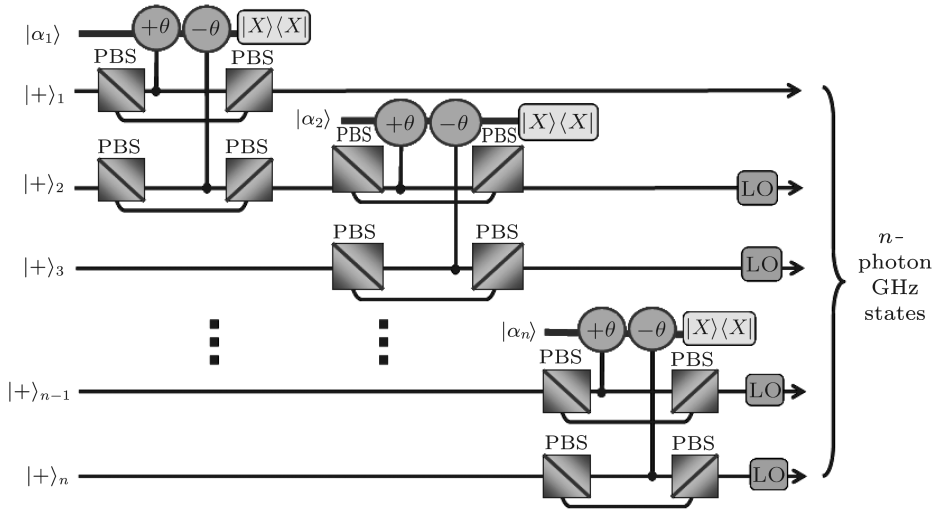


Fig. 4. Experimental scheme for generating the n-photon GHZ states.

If the X-quadrature homodyne measurements obtain the results of phase shift 0 for all the coherent states, the state collapses into $|\mathrm{GHZ}\rangle_n = |HH\cdots H + VV\cdots V\rangle$. If not, the simple bit flip σ_x on the polarization qubits can transform it into the state of $|\mathrm{GHZ}\rangle_n$. Similar to the above steps, we can prepare the n-atom GHZ state. The success probability approaches 1 with nearly perfect fidelity.

To summarize, we have presented an effective scheme to generate multiphoton GHZ states with high success probabilities and near-perfect fidelities. This scheme takes advantage of the nonlinear Kerr effect. It uses only the basic tools in quantum optical laboratories and can be implemented in the regime of the weak cross-Kerr nonlinearity. In addition, our scheme is not limited to the all-optical implication and can also be used in other bus-mediated QIP architectures with a cross-Kerr-like interaction Hamiltonian.[29,30]

References

[1] Bell J S 1965 *Physics* **1** 195

[2] Greenberger D M, Horne M A and Zeilinger A 1989 in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe* ed. Kafatos M (Dordrecht: Kluwer)

[3] Greenberger D M, Horne M A, Shimony A and Zeilinger A 1990 *J. Phys.* **58** 1131

[4] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)

[5] Pan J W, Simon C, Brukner C and Zeilinger A 2001 *Nature* (London) **410** 1067

[6] Horodecki M, Oppenheim J and Winter A 2005 *Nature* (London) **436** 673

[7] Hillery M, Buzek V and Berthiaume A 1999 *Phys. Rev.* A **59** 1829

[8] Rauschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond J M and Haroche S 2000 *Science* **288** 2024

[9] Zhao Z, Chen Y A, Zhang A N, Yang T, Briegel H and Pan J W 2004 *Nature* (London) **430** 54

[10] Leibfried D D, Knill E, Seidelin S, Britton J, Blakestad R B, Chiaverini J, Hume D B, Itano W M, Jost J D, Langer C, Ozeri R, Reichle R and Wineland D J 2005 *Nature* (London) **438** 639

[11] Sackett C A, Kielpinski D, King B E, Langer C, Meyer V, Myatt C J, Rowe M, Turchette Q A, Itano W M, Wineland D J and Monroe C 2000 *Nature* (London) **404** 256

[12] Leibfried D, Barrett M D, Schaetz T, Britton J, Chiaverini J, Itano W M, Jost J D, Langer C and Wineland D J 2004 *Science* **304** 1476

[13] Christian F R, Mark R, Hartmut H, Wolfgang H, Jan B, Gavin P T L, Christoph B, Ferdinand S K and Rainer B 2004 *Science* **304** 1478

[14] For recent reviews about optical QIP, see, e.g., Ralph T C 2006 *Rep. Prog. Phys.* **69** 853

[15] DellAnno F, de Siena S and Illuminati F 2006 *Phys. Rep.* **428** 53

[16] Kok P, Munro W J, Nemoto K, Ralph T C, Dowling J P and Milburn G J 2007 *Rev. Mod. Phys.* **79** 135

[17] Myers C R and Laflamme R *arXiv: quant-ph*/0512104

[18] Bouwmeester D, Pan J W, Daniell M, Weinfurter H and Zeilinger A 1999 *Phys. Rev. Lett.* **82** 1345

[19] Pan J W, Bouwmeester D, Daniell M, Weinfurter H and Zeilinger A 2000 *Nature* (London) **403** 515

[20] Pan J W, Daniell M, Gasparoni S, Weihs G and Zeilinger A 2001 *Phys. Rev. Lett.* **86** 4435

[21] Kok P and Braunstein S L 2000 *Phys. Rev.* A **62** 064301

[22] Jin G S, Lin Y and Wu B 2007 *Phys. Rev.* A **75** 054302

[23] Nemoto K and Munro W J 2004 *Phys. Rev. Lett.* **93** 250502

[24] Milburn G J and Walls D F 1984 *Phys. Rev.* A **30** 56

[25] Imoto N, Haus H A and Yamamoto Y 1985 *Phys. Rev.* A **32** 2287

[26] Grangier P, Levenson J A and Poizat J P 1998 *Nature* (London) **396** 537

[27] Barrett S D, Kok P, Nemoto K, Beausoleil R G, Munro W J and Spiller T P 2005 *Phys. Rev.* A **71** 060302(R)

[28] Prevedel R, Walther P, Tiefenbacher F, Böhi P, Kaltenbaek R, Jennewein T D and Zeilinger A 2007 *Nature* (London) **445** 65

[29] Ladd T D, Loock P V, Nemoto K, Munro W J and Yamamoto Y 2006 *New J. Phys.* **8** 184

[30] Loock P V, Ladd T D, Sanaka K, Yamaguchi F, Nemoto K, Munro W J and Yamamoto Y 2006 *Phys. Rev. Lett.* **96** 240501