High entanglement generation and high fidelity quantum state transfer in a non-Markovian environment*

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This paper analyses a system of two independent qubits off-resonantly coupled to a common non-Markovian reservoir at zero temperature. Compared with the results in Markovian reservoirs, we find that much higher values of entanglement can be obtained for an initially factorized state of the two-qubit system. The maximal value of the entanglement increases as the detuning grows. Moreover, the entanglement induced by non-Markovian environments is more robust against the asymmetrical couplings between the two qubits and the reservoir. Based on this system, we also show that quantum state transfer can be implemented for arbitrary input states with high fidelity in the non-Markovian regime rather than the Markovian case in which only some particular input states can be successfully transferred.

Keywords: non-Markovian environment, entanglement generation, quantum state transfer

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1. Introduction

Quantum entanglement is of primary importance in quantum information and computation theory.[1] Since real quantum systems unavoidably interact with their environments, the study of the dynamics of open quantum systems is naturally of great interest when it comes to a realistic description of experimental situations.[2] Generally speaking, the system-environment interaction leads to decoherence[3] and entanglement sudden death (ESD),[4−7] which greatly discourages the broad applications of entanglement. However, recent research has shown that the environments not only destroy entanglement but also induce it in open quantum systems.[8−11] This has opened new intriguing perspectives and shed new light on the creation of entanglement in open systems.

It is well known that the dynamics of open quantum systems is critically dependent on the types of environments coupled to the system.[12] Markovian environments are those whose characteristic correlation function time is much shorter than that of the system investigated, i.e., without memory. However, non-Markovian environments appear to be more fundamental in many physical situations, where time scales of the environmental correlation function are orders of magnitude longer than the typical system decoherence time scales, e.g., in photonic band-gap materials,[13,14] high-Q cavity systems,[15] solid state systems,[16] and spin baths.[17,18] The typicality of non-Markovian environments is that they can feed back part of the information into the system due to pronounced memory effects. For this reason, more and more attention has been paid to the investigation of non-Markovian entanglement dynamics.[19−23]

In Refs. [8]–[11] the authors showed that entanglement between two qubits can be induced by Markovian environments if the two qubits interact with a common heat bath in thermal equilibrium, but do not interact directly with each other. Yet, the entanglement is created for a very short time after the interaction with the heat bath is switched on and then disappears rapidly. On the other hand, the value of the created entanglement is small, which severely restricts its use since most of the quantum information tasks are dependent on the degree of entanglement, such as quantum teleportation,[24] quantum dense coding,[25] quantum cryptography,[26] etc. Fortunately, the memory effect of non-Markovian environments has been proved effectively to extend the possible usage time of entanglement.[20−22] Therefore, it is natural that we

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want to know about the memory effect on entanglement induced by non-Markovian environments.

In this paper, we consider two noninteracting two-level systems, off-resonantly coupled to a common non-Markovian reservoir. The results show that for an initially separable two-qubit state containing one excitation, the entanglement induced by a non-Markovian reservoir is much larger than that induced by a Markovian reservoir. The maximal value of entanglement increases with the detuning between the qubits, and the central frequency of the reservoir grows. By choosing the appropriate parameters, the maximally entangled state can be generated with high fidelity. Even considering the asymmetrical couplings between the two qubits and the reservoir, high entanglement still can be created in most situations. Additionally, we discuss the quantum state transfer between these two qubits in the non-Markivian regime. It is interesting to note that unlike in the Markovian case, where the reliable quantum state transfer can be realized only for some particular input states, an arbitrary state encoded in qubit 1 can be transferred to qubit 2 with high fidelity in the non-Markovian case.

The organization of this paper is as follows. In Section 2, the physical model of two qubits coupled to a common structured reservoir is considered. In Section 3, we show that high entanglement can be induced by the non-Markovian reservoir. In Section 4, high-fidelity quantum state transfer is implemented based on this system. Finally, a brief discussion and summary are given in Section 5.

2. Physical model

We study a system of two independent qubits coupled to a common zero-temperature bosonic reservoir. The Hamiltonian of the system and the reservoir is given by ($\hbar = 1$)

$$H = H_0 + H_{\rm int}, \tag{1}$$

$$H_0 = \frac{1}{2}\omega_0(\sigma_z^1 + \sigma_z^2) + \sum_k \omega_k a_k^\dagger a_k, \tag{2}$$

$$H_{\rm int} = (\alpha_1\sigma_+^1 + \alpha_2\sigma_+^2)\sum_k g_k a_k + {\rm H.c.}, \tag{3}$$

where ω_0 is the transition frequency of the two-level system. σ_z^j, σ_+^j, and σ_-^j ($j = 1, 2$) are the inversion, raising, and lowing operators of the j-th qubit ($j = 1, 2$), respectively. $a_k^\dagger$ and a_k are the creation and annihilation operators of the k-th mode of the reservoir with frequency ω_k respectively, and g_k are the coupling constants. The strength of interaction between the j-th qubit and the reservior is measured by the dimensionless constant α_j.

Assume that the two qubits are initially prepared in the state

$$|\psi(0)\rangle = |1\rangle_1|0\rangle_2|0\rangle_{\rm R}, \tag{4}$$

where $|1\rangle_1$ and $|0\rangle_2$ denote that qubit 1 is in the excited state and qubit 2 is in the ground state, respectively, while the reservoir is in the vacuum state. From Hamiltonian (1), the state will evolve into

$$\begin{aligned}|\psi(t)\rangle =\ & (c_1(t)|1\rangle_1|0\rangle_2 + c_2(t)|0\rangle_1|1\rangle_2)\,|0\rangle_{\rm R} \\ & + \sum_k c_k(t)|0\rangle_1|0\rangle_2|1_k\rangle_{\rm R},\end{aligned} \tag{5}$$

where the state $|1_k\rangle_{\rm R}$ is the state of the reservoir with only one excitation in the k-th mode.

In what follows, we take the spectral distribution of an electromagnetic field inside an imperfect cavity supporting the mode ω_0 with Lorentzian formation

$$J(\omega) = \frac{1}{2\pi}\frac{\gamma_0\lambda^2}{(\omega_0 - \omega - \Delta)^2 + \lambda^2}, \tag{6}$$

where Δ is the detuning of transition frequency ω_0 and the central frequency of the cavity. The parameter γ_0 is the Markovian decay rate and λ defines the spectral width of the coupling, which is connected to the reservoir correlation time $\tau_{\rm R} = \lambda^{-1}$. Typically, there are two regimes:[27] the weak-coupling regime ($\gamma_0 < \lambda/2$), where the behaviour of the system is Markovian and irreversible decay occurs, and the strong-coupling regime ($\gamma_0 > \lambda/2$), where non-Markovian dynamics occurs accompanied by an oscillatory reversible decay. Here, we focus our attention on the non-Markovian regime and compare the result with that in the Markovian regime. The coefficients can be readily obtained by solving the Schrödinger equation,

$$\begin{aligned}c_1(t) &= r_2^2 + r_1^2\varepsilon(t), \\ c_2(t) &= r_1 r_2\left[\varepsilon(t) - 1\right],\end{aligned} \tag{7}$$

where $\alpha_{\rm T} = \sqrt{\alpha_1^2 + \alpha_2^2}$, $r_1 = \alpha_1/\alpha_{\rm T}$, $r_2 = \alpha_2/\alpha_{\rm T}$,

$$\begin{aligned}\varepsilon(t) =\ & {\rm e}^{-(\lambda - {\rm i}\,\Delta)t/2}\left[\cosh\left(\frac{\Omega t}{2}\right)\right. \\ & \left. + \frac{\lambda - {\rm i}\,\Delta}{\Omega}\sinh\left(\frac{\Omega t}{2}\right)\right],\end{aligned}$$

and $\Omega = \sqrt{(\lambda - {\rm i}\,\Delta)^2 - 2\gamma_0\lambda\alpha_{\rm T}^2}$. In the basis of $\{|1\rangle_1|1\rangle_2, |1\rangle_1|0\rangle_2, |0\rangle_1|1\rangle_2, |0\rangle_1|0\rangle_2\}$, the reduced density matrix of the two qubits is given by

$$\rho(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |c_1(t)|^2 & c_1(t)c_2^*(t) & 0 \\ 0 & c_1^*(t)c_2(t) & |c_2(t)|^2 & 0 \\ 0 & 0 & 0 & 1-|c_1(t)|^2-|c_2(t)|^2 \end{pmatrix}. \quad (8)$$

3. High entanglement induced by a non-Markovian environment

In order to investigate the entanglement induced by the environment, we use the concurrence[28] to measure the created entanglement between the two qubits. This is defined as

$$C_\rho(t) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (9)$$

where λ_i $(i = 1, 2, 3, 4)$ are the eigenvalues, in decreasing order, of the matrix $\tilde{\rho} = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$; σ_y is the Pauli spin matrix and the asterisk denotes complex conjugation. The concurrence varies from $C = 0$ for a separable state to $C = 1$ for a maximally entangled state. For the state described by Eq. (8), the concurrence is

$$C(t) = 2|c_1(t)c_2^*(t)|. \quad (10)$$

The concurrence $C(t)$ as a function of the dimensionless time $\gamma_0 t$ and for various values of detuning Δ is shown in Fig. 1, where we have assumed $\lambda = 0.1\gamma_0$ for the non-Markovian regime and $\lambda = 10\gamma_0$ for the Markovian regime. It is interesting to note that entanglement induced by Markovian environments is much smaller than that induced by non-Markovian environments. Even considering the detuning effect, we argue that this amount of entanglement is not enough to process quantum information tasks. For example, quantum teleportation assisted by this entangled state consequently fails as the fidelity is smaller than 2/3. However, we find that unlike the Markovian case, in the non-Markovian case the entanglement created by non-Markovian dissipation approaches a stationary value with revivals due to the information being exchanged back and forth between the qubits and the non-Markovian reservoir. In particular, when $\Delta \neq 0$, the maximal value of induced entanglement increases as the detuning grows. The physical reason is that irreversible processes such as spontaneous emission are strongly suppressed by the memory effect of the non-Markovian environment and the detunings.[21−23] According to the method proposed in Ref. [29], if we apply two classical driving fields to the two qubits at the proper time, we could expect the entanglement to be preserved for a long time, convenient for processing certain quantum information tasks.

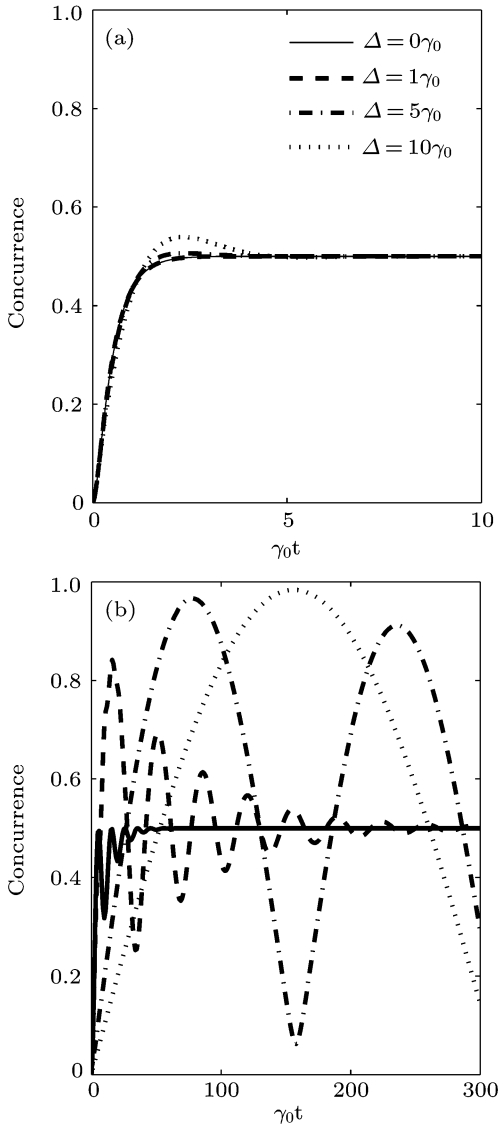


Fig. 1. Entanglement induced by the environment as a function of the dimensionless quantity $\gamma_0 t$ for different values of detuning: (a) in the Markovian environment $\lambda = 10\gamma_0$ and (b) in the non-Markovian environment $\lambda = 0.1\gamma_0$. Other parameters are $\alpha_1 = \alpha_2 = 1$.

In the above discussions, we have assumed that the two qubits are equally coupled to their reservoir. However, equal couplings are very difficult to realize in experiments and the asymmetrical couplings are more

general when we consider the qubit's relative position.

Figure 2 displays the accessible maximal concurrence versus the relative coupling strength r_1. For $r_1 = 0$ and $r_1 = 1$, no entanglement is induced as they both describe the same case in which one of the two qubits is effectively decoupled from the reservoir. For $r_1 = 1/\sqrt{2}$, the two qubits are equivalently coupled with the reservoir, and the accessible maximal entanglements are 0.5 and 0.98 induced by Markovian environment and non-Markovian environment, respectively. Interestingly, from Fig. 2 we find that the optimal maximal entanglement does not occur at the position of equal couplings, but appears at $r_1 = 1/2$ indicating the asymmetrical couplings. The reason is that even though the two qubits are symmetrical in the Hamiltonian, they play different roles in the process of entanglement generation as we have chosen the initial state as $|1\rangle_1|0\rangle_2$ which is directly related to the optimal relative coupling strength r_1. From the perspective of information flow, qubit 1 is initially in the excited state and acts the part of the sender, while qubit 2 is the receiver in this case, and *vice versa* for initial state $|0\rangle_1|1\rangle_2$. Considering the dissipation, we find that equal couplings of qubits 1 and 2 cannot achieve maximal entanglement. For instance, for the initially factorized states $|1\rangle_1|0\rangle_2$ and $|0\rangle_1|1\rangle_2$, it is easy to check that the stationary concurrences (i.e., $t \to \infty$) $C_{\rm s} = 2r_1(1-r_1^2)^{3/2}$ and $C_{\rm s} = 2r_1^3\sqrt{1-r_1^2}$ obtains their maximal values with $r_1 = 1/2$ and $r_1 = \sqrt{3}/2$ (i.e., $r_2 = 1/2$) instead of

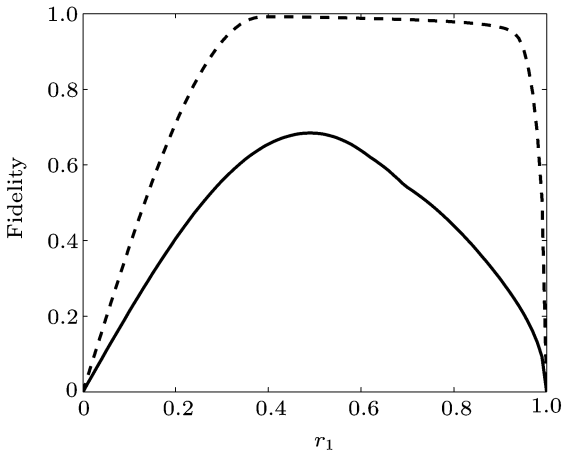


Fig. 2. Maximal concurrence as a function of relative coupling strength r_1 in the non-Markovian regime $\lambda = 0.1\gamma_0$ (dashed line) and Markovian regime $\lambda = 10\gamma_0$ (solid line) with $\Delta = 10\gamma_0$.

$r_1 = 1/\sqrt{2}$, respectively. Another aspect is that high values of entanglement can be induced in non-Markovian environments as long as $r_1 \in (0.3, 0.9)$. This gives an advantage over the Markovian case and also greatly lowers the experimental requirements.

As mentioned above, we find that a high value of entanglement can be induced by a non-Markovian environment when the detuning is non-zero. Knowledge of the characteristic of the entangled state is helpful for its applications. For $\Delta = 10\gamma_0$ and $\alpha_1 = \alpha_2 = 1$, we find that the maximally entangled state $(|1\rangle_1|0\rangle_2 - {\rm i}\,|0\rangle_1|1\rangle_2)/\sqrt{2}$ can be generated with high fidelity (more than 0.99) in the non-Markovian regime. After a single-qubit operation, this state can be transformed to a Bell state which is widely used in quantum information and computation.

4. High-fidelity quantum state transfer

The transfer of quantum states from one location (A) to another (B) is an important feature in many quantum information processing systems. Depending on current technology, this task can be accomplished in a number of ways, such as optical lattices,[30] ion traps,[31] spin chains,[32] and arrays of quantum dots.[33] The essence of these systems is that permanently coupled systems can be used for the transfer of quantum information. In the following, we demonstrate that high-fidelity quantum state transfer is also available in non-Markovian environments for open quantum systems. Initially, suppose that the first qubit is in the state $\alpha|0\rangle_1 + \beta|1\rangle_1$, $(\alpha^2 + |\beta|^2 = 1$, where we have assumed that α is a real number without loss of generality), while the second qubit is in the state $|0\rangle_2$ and the reservoir is in the vacuum state $|0\rangle_{\rm R}$. The goal of quantum state transfer is to accomplish the process

$$(\alpha|0\rangle_1 + \beta|1\rangle_1)|0\rangle_2 \to |0\rangle_1(\alpha|0\rangle_2 + \beta|1\rangle_2). \qquad (11)$$

From the Hamiltonian (1), the initial state will evolve into

$$\begin{aligned}|\psi(t)\rangle \;=\; & \alpha|00\rangle_{12}|0\rangle_{\rm R} + \beta[c_1(t)|10\rangle_{12}|0\rangle_{\rm R} \\ & + c_2(t)|01\rangle_{12}|0\rangle_{\rm R} \\ & + \sum_k c_k(t)|00\rangle_{12}|1_k\rangle_{\rm R}]. \qquad (12)\end{aligned}$$

The target state of the process of quantum state transfer is $|\varphi\rangle = |0\rangle_1(\alpha|0\rangle_2 + \beta|1\rangle_2)|0\rangle_{\rm R}$. Based on this model, we should add an operation σ_z on qubit 2 to obtain the target state. Then the fidelity of quantum state transfer is

$$F = |\langle\varphi|\sigma_z^{(2)}|\psi(t)|^2 = |\alpha^2 - |\beta|^2 c_2(t)|^2. \qquad (13)$$

In this section, let $r_1 = r_2 = 1/\sqrt{2}$.

The fidelities as a function of scale time $\gamma_0 t$ and α^2 are plotted in Fig. 3 for both the Markovian and non-Markovian cases. It can be noted that in the Markovian regime, only when $\alpha^2 \to 1$ can reliable quantum state transfer be implemented. Otherwise, the fidelity is very small and the state transfer fails. Compared with the result in the Markovian regime, the arbitrary state encoded in qubit 1 can be transferred to qubit 2 with high fidelity in the non-Markovian regime. By controlling the interaction time, quantum state transfer can be realized with sufficiently high fidelity (more than 0.96) for all values of α^2, which means that the fidelity is robust against the decrease of α^2 in non-Markovian environments.

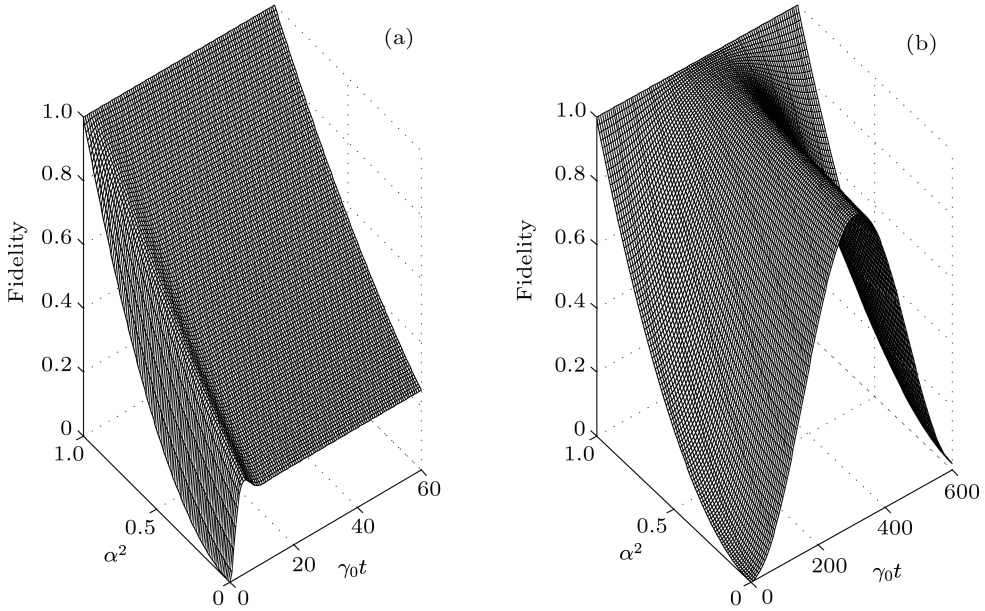


Fig. 3. Fidelities of quantum state transfer as a function of the dimensionless quantities $\gamma_0 t$ and α^2: (a) Markovian environment $\lambda = 10\gamma_0$; (b) non-Markovian environment $\lambda = 0.1\gamma_0$ with $\Delta = 10\gamma_0$.

5. Conclusions and discussions

We have considered a system of two qubits coupled to a common zero-temperature non-Markovian reservoir. The results show that high entanglement between the qubits can be induced when the qubits off-resonantly interact with the non-Markovian reservoir. The maximal value of entanglement is much larger than in the Markovian case. In particular, even considering the asymmetrical couplings between the two qubits and the reservoir, high entanglement still can be created in most situations. Moreover, we have shown that quantum state transfer can be implemented with high fidelity based on this model. In contrast with the Markovian case where the fidelity is highly dependent on the input states, in the non-Markovian case the fidelity is larger and robust against the input states. In conclusion, in the non-Markovian regime, we can increase the detuning appropriately to obtain high entanglement and increase the fidelity of the quantum state transfer.

The model examined in this paper, as a basic model of quantum optics, is particularly relevant to current experimental capabilities. For example, in cavity quantum electrodynamics experimental configurations, the strong coupling between trapped atoms or ions and high finesse optical cavities has already been demonstrated and their quantum states can be fully controlled.[34,35] Our work on the assessment of the entanglement induced by environments, especially in a non-Markovian regime, is of great importance for preparing entanglement in an open quantum system and using it to implement some quantum information tasks.

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