Decoherence-free spin entanglement generation and purification in nanowire double quantum dots*

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We propose a deterministic generation and purification of decoherence-free spin entangled states with singlet–triplet spins in nanowire double quantum dots via resonator-assisted charge manipulation and measurement techniques. Each spin qubit corresponds to two electrons in a double quantum dot in the nanowire, with the singlet and one of the triplets as the decoherence-free qubit states. The logical qubits are immunized against the dominant source of decoherence–dephasing—while the influences of additional errors are shown by numerical simulations. We analyse the performance and stability of all required operations and emphasize that all techniques are feasible in current experimental conditions.

Keywords: decoherence-free, entanglement, nanowire quantum dot, spin qubit

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1. Introduction

Spin entangled states in solid-state systems are a basic resource for quantum information processing, including quantum communication and computation.[1] However, the complex environment of solid-state systems make them significantly more challenging to isolate and operate coherently. Decoherence can spoil the advantage of quantum properties. Decoherence-free encoding has been proposed[2−4] to protect fragile quantum information against the detrimental effects of decoherence. This paper develops a proposal for generation and purification of decoherence-free spin entangled states of a nanowire (NW) double quantum dot (DQD) via resonator-assisted charge manipulation and measurement techniques. Compared to the previous protocol[5] in which the logical Bell states are generated probabilistically via the partial Bell state measurement (BSM) in a post-selected way, our protocol shows a deterministic generation of the decoherence-free Bell states of the singlet–triplet spins in DQDs. Compared with previous proposals,[6−13] which made use of single dots or DQDs defined by a two-dimensional electron gas (2DEG), our proposal is more realistic for implementation. It would be difficult to implement a DQD in a planar resonator with lateral dots, shaped in a 2DEG by surface gates. It is difficult to prevent absorption of microwaves in the 2DEG unless one can make the electric field non-zero only in the DQD region, which is not yet achievable experimentally. In our protocol the state manipulation is done with NWs, which is more likely to couple with the resonator. Very recently, spin dynamics in InAs NW quantum dots coupled to a transmission line via the spin–orbit interaction has been proposed.[14] However, the coupling between the QD spins and the resonator mode is weak in experiments. Whereas the coupling between the electric dipole of charge states and resonator is much stronger, the decoherence of charge states is another obstacle. In this work, combining the advantages of spin and charge states and avoiding the weak points of both, we propose a mechanism to achieve an entangling gate between spin singlets and triplets inside a resonator, namely via resonator-assisted interaction to give an efficient coupling between the resonator photon and the effective electric dipole of a DQD, thus eventually entangle the singlet and triplet spin states, and make a quantum nondemolition measurement of the spin-dependent phase for entanglement purification.

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2. Choice of logical qubits

We consider a system with two electrons located in adjacent QDs inside a semiconductor NW, coupling via tunneling. The idea is to use an effective electric dipole moment of the singlets $|(1,1)S\rangle$ and $|(0,2)S\rangle$ of the double-dot coupling to the oscillating voltage associated with a stripline resonator shown in Fig. 1. With an external magnetic field B_z along the z axis, the spin aligned states $|T_+\rangle = |\uparrow\uparrow\rangle$ and $|T_-\rangle = |\downarrow\downarrow\rangle$, and the spin-anti-aligned states $|(1,1)T_0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ and $|(1,1)S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ have energy gaps due to the Zeeman splitting shown in Fig. 1(a). The notation $(n_{\rm L}, n_{\rm R})$ labels the number of electrons in the left and right quantum dots of a pair. Considering a three-level system, we choose the singlet state and one of the triplet states as our qubit states:

$$|0\rangle \equiv |(1,1)S\rangle; \quad |1\rangle \equiv |(1,1)T_0\rangle, \tag{1}$$

and the doubly occupied state as an ancillary state

$$|a\rangle \equiv |(0,2)S\rangle. \tag{2}$$

In its simplest form, a nonlocal EPR pair of electrons in the state $|(1,1)S\rangle$ or $|(1,1)T_0\rangle$ can be produced by local preparation of a ground singlet state of two electrons in one of the quantum dots, splitting the pair into two adjacent dots and shuttling the electrons to the end nodes.

The four-spin entangled states:

$$\begin{aligned}
|\Phi^+\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle),\\
|\Phi^-\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) = -\frac{1}{\sqrt{2}}(|\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\downarrow\rangle),\\
|\Psi^+\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\uparrow\downarrow\rangle - |\downarrow\uparrow\downarrow\uparrow\rangle),\\
|\Psi^-\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |01\rangle) = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle),
\end{aligned} \tag{3}$$

take full advantages of these properties, suppressing collective phase noise. This combination of subspace choice and exchanging electrons corresponds to a decoherence-free subspace (DFS). The logical qubit decoheres only insofar as the dephasing fails to be collective.

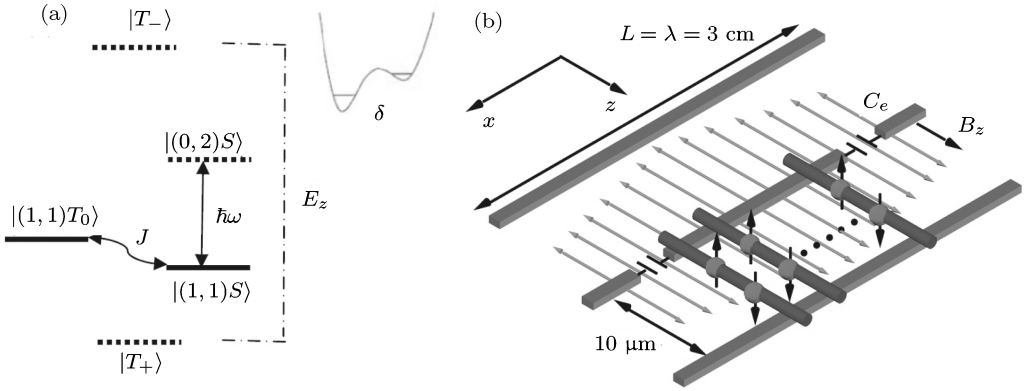


Fig. 1. (a) Energy level diagram showing the $(0,2)$ and $(1,1)$ singlets, the three $(1,1)$ triplets, and the qubit states $|(1,1)S\rangle$ and $|(1,1)T_0\rangle$ with the energy gap J. A schematic diagram is given by the double-well potential with an energy offset δ provided by the external electric field. (b) Schematic diagram of NW DQDs capacitively coupled to the superconducting resonator. The coupling can be switched on and off via the external electric field. The DQD confinement can be achieved by barrier materials or by external gates (not shown).

3. Generation of spin entangled states in DFS

The double-dot system can be described by an extended Hubbard Hamiltonian

$$\begin{aligned}
\hat{H} = {} & (E_{\rm os} + \mu)\sum_{i,\sigma}\hat{n}_{i,\sigma} - T\sum_{\sigma}(\hat{a}^{\dagger}_{L,\sigma}\hat{a}_{{\rm R},\sigma} + {\rm hc})\\
& + U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} + W\sum_{\sigma,\sigma'}\hat{n}_{L,\sigma}\hat{n}_{{\rm R},\sigma'}\\
& + \delta\sum_{\sigma}(\hat{n}_{L,\sigma} - \hat{n}_{{\rm R},\sigma}),
\end{aligned} \tag{4}$$

for $\hat{a}_{i,\sigma}$ ($\hat{a}^{\dagger}_{i,\sigma}$) annihilating (creating) an electron in quantum dot $i \in \{L, R\}$ with spin $\sigma \in \{\uparrow, \downarrow\}$, $\hat{n}_{i,\sigma} = \hat{a}^{\dagger}_{i,\sigma}\hat{a}_{i,\sigma}$ a number operator and δ an energy offset yielded by the external electric field. The first term corresponds to on-site energy $E_{\rm os}$ plus site-dependent field-induced corrections μ. The second term accounts for $i \leftrightarrow j$ electron tunneling at a rate T. The third

term is the on-site charging cost U to put two electrons with opposite spin in the same dot and the fourth term corresponds to inter-site Coulomb repulsion. In the basis $\{|0\rangle, |a\rangle\}$, the Hamiltonian can be reduced to

$$\hat{H}_{\rm d} = -\delta|a\rangle\langle a| + T|0\rangle\langle a| + {\rm hc}. \tag{5}$$

With the energy offset δ, degenerate perturbation theory in the tunneling T reveals an avoided crossing at this balanced point which occurs at the left-most avoided crossing between $|0\rangle$ and $|a\rangle$ with an energy gap $\omega = \sqrt{\delta^2 + 4T^2}$.

The essential idea for entanglement generation is to use an effective electric dipole moment associated with singlet states $|0\rangle$ and $|a\rangle$ of an NW DQD coupled to the oscillating voltage associated with a stripline resonator as shown in Fig. 1(b). Whereas the qubit state $|1\rangle$ is decoupled to the resonator due to the large energy gap J. We consider a stripline resonator with length L, the capacitance per unit length C_0 and the characteristic impedance Z_0. A capacitive coupling $C_{\rm c}$ between the NW DQD and the resonator causes the electron charge state to interact with excitations in the transmission line. We assume that the dot is much smaller than the wavelength of the resonator excitation, so that the interaction strength can be derived from the electrostatic potential energy of the system

$$\hat{H}_{\rm int} = e\hat{V}v|a\rangle\langle a|, \tag{6}$$

where e is the electron charge and

$$\hat{V} = \sum_n \sqrt{\frac{\hbar\omega_n}{LC_0}}(\hat{c}_n + \hat{c}_n^\dagger) \tag{7}$$

is the voltage on the resonator near the right dot, $\hat{c}_n$ and $\hat{c}_n^\dagger$ are the creation and annihilation operators respectively for the mode $k_n = [(n+1)\pi]/L$ of the resonator, $v = C_{\rm c}/C_{\rm tot}$, and $C_{\rm tot}$ is the total capacitance of the DQD. The fundamental mode frequency of the resonator is $\omega_0 = \pi/LZ_0C_0 \approx \omega$. The resonator is coupled to a capacitor $C_{\rm e}$ for writing and reading the signals. Neglecting the higher modes of the resonator and working in a rotating frame with a rotating wave approximation, we obtain an effective interaction Hamiltonian from equation (6)

$$\hat{H}_{\rm eff} = g(\hat{c}|a\rangle\langle 0| + {\rm hc}) \tag{8}$$

with the effective coupling coefficient

$$g = \frac{1}{2}ev\frac{1}{LC_0}\sqrt{\frac{\pi}{Z_0\hbar}}\sin 2\vartheta, \tag{9}$$

where $\vartheta = \frac{1}{2}\tan^{-1}(2T/\delta)$.

The interaction between the resonator and qubit states is switchable via tuning the electric field. In the case of the energy offset $\delta \approx 0$ yielded by the electric field, we obtain the maximum value of the coupling between the resonator and qubit states. For $\delta \gg T$, ϑ tends to 0 and the interaction is switched off.

We consider there are two NW DQDs coupled to the resonator. If both of the DQDs are in the state $|1\rangle$, the incoming pulses in an even coherent state $|\alpha\rangle_- = N_-(|\alpha\rangle - |-\alpha\rangle)$[15] with a normalization constant N_-, which are resonant with the bare resonator mode and performed in the limit with $\tau \gg 1/\kappa$ (here τ is the pulse duration and κ is the resonator decay rate) are resonantly reflected by the bare resonator mode with a flipped global phase of π from standard quantum optics calculation.[16–19] For the other three cases that include the DQDs in the states $|00\rangle$, $|01\rangle$ and $|10\rangle$, the frequency of the dressed resonator mode is significantly detuned from the frequency of the incoming pulses and the frequency shift has a magnitude comparable to g. Thus the resonator functions as a mirror and the shape and global phase of the reflected pulses remain unchanged.

We now turn to the detailed description of the resonator-assisted interaction. The resonator output $\hat{c}_{\rm out}$ is connected with the resonator mode $\hat{c}$ via the following relationships

$$\dot{\hat{c}}(t) = -{\rm i}\left[\hat{c}(t), \hat{H}_{\rm sys}\right] - \left({\rm i}\,\Delta + \frac{\kappa}{2}\right)\hat{c}(t) - \sqrt{\kappa}\hat{c}_{\rm in}(t), \tag{10}$$

and

$$\hat{c}_{\rm out}(t) = \hat{c}_{\rm in}(t) + \sqrt{\kappa}\hat{c}(t). \tag{11}$$

Here Δ denotes the detuning of the resonator field mode $\hat{c}(t)$ from the input pulse $\hat{c}_{\rm in}(t)$ with the standard communication relationship $\left[\hat{c}_{\rm in}(t), \hat{c}_{\rm in}(t')\right] = \delta(t-t')$ and $\hat{H}_{\rm sys} = g\sum_{i=1,2}(\hat{c}|a\rangle_i\langle 0| + {\rm hc})$. We obtain the following results. If both of the DQDs are in the state $|1\rangle$, the Hamiltonian $\hat{H}_{\rm sys}$ is not active. Based on Eqs. (10) and (11), we obtain $\hat{c}_{\rm out}^{11}(t) = -\hat{c}_{\rm in}^{11}(t)$ if resonant interaction satisfies $\Delta = 0$ and the input pulse shape changes slowly with time t as compared with the cavity decay rate κ. That means if the state of the DQDs is in $|11\rangle$, the output field acquires the phase π after the interaction. In other input cases, however, there are effective detunings of the dressed cavity mode from the input pulse $\pm\sqrt{2}g$ for $|00\rangle$ and $\pm g$ for $|01\rangle$ ($|10\rangle$) and the input–output equations $\hat{c}_{\rm out}^{mn}(t) = \xi_{mn}\hat{c}_{\rm in}^{mn}(t)$ $(m,n = 0,1)$ where $\xi_{11} = -1$,

$\xi_{00} = (8s-1)/(8s+1)$ and $\xi_{01} = \xi_{10} = (4s-1)/(4s+1)$ with $s = g^2 T_1/\kappa$ (T_1 is charge relaxation time). If $s \gg 1$ is satisfied, ξ_{00}, ξ_{01}, and ξ_{10} tend to 1. Thus for an arbitrary initial state, after the resonator-assisted interaction and tracing out the resonator field, we implement a controlled phase flip gate

$$U_{\rm cpf} = {\rm e}^{{\rm i}\,\pi|11\rangle\langle 11|} \tag{12}$$

on the spin states of the two DQDs and the gate time is about $t_{\rm cpf} \sim \tau$.

In order to generate the logical entangled states, a single qubit rotation is required, which can be implemented by applying a static magnetic field gradient between the two dots of each DQD ${\rm d}B = g^*\mu_{\rm B}(B_z^{\rm L} - B_z^{\rm R})$ to mix the singlet and triplet states.[20] With the static magnetic field gradient the Hamiltonian for the three-state system (5) changes into

$$\hat{H}'_{\rm d} = -\delta|a\rangle\langle a| + T|0\rangle\langle a| + {\rm d}B|0\rangle\langle 1| + {\rm hc}. \tag{13}$$

If ${\rm d}B \gg T$ is satisfied, the evolution of the Hamiltonian (13) can be used to implement a logical single qubit rotation about the x axis $R^x(\beta) = {\rm e}^{{\rm i}\,\beta\sigma_x}$ with the Pauli operator $\sigma_x = |0\rangle\langle 1| + {\rm hc}$. The logical-controlled phase flip gate applied on the pairs of DQDs followed by a logical x rotation $R^x(\pi/4)$ on the second logical qubit can eventually generate a logical Bell state shown in equation (3)

$$|\Phi^+\rangle = R_2^x\left(\frac{\pi}{4}\right)U_{\rm cpf}R_1^x\left(\frac{\pi}{4}\right)\otimes R_2^x\left(\frac{\pi}{4}\right)|0\rangle_1|0\rangle_2. \tag{14}$$

The other three logical Bell states can be easily generated from $|\Phi^+\rangle$ by applying a certain single qubit rotation $R^x(\pi/2)$ about the x axis or $R^z(\pi/2)$ about the z axis. The latter rotation can be implemented with a similar method to the two-qubit controlled phase flip gate. If only one DQD is coupled to the resonator, with the same incoming pulse, after the resonator-assisted interaction and tracing out the resonator field, we should implement $R^z(\pi/2) = {\rm e}^{{\rm i}\,\pi/2\sigma_z}$ with the Pauli operator $\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$.

Now we analyse the feasibility of the proposal with the elongated QDs oriented along the NW. Quantum dots have been realized within NWs in various material systems.[21] A realization of DQDs defined using local gates to electrostatically deplete InAs NWs grown by chemical beam epitaxy was reported.[22] The quantum-mechanical tunneling T between the two quantum dots is about 0 μeV–150 μeV.[22] Thus at the optimal point $\delta \approx 0$ where the coupling is strongest, the energy gap between the singlets is about $\omega \sim 2T \simeq 0$ GHz–72 GHz. The stripline resonator can be fabricated with existing lithography techniques.[23] The dots can be placed within the resonator formed by the transmission line to strongly suppress the spontaneous emission. A small-diameter ($d \sim 65$ nm), long-length ($l \sim 270$ nm) and $g^* = -13$[24] InAs NW is positioned perpendicularly to the transmission line containing DQDs that are elongated along the NW shown in Fig. 1(b). To prevent current flow, the NW and transmission line need to be separated by some insulating coating material obtained, for example, by atomic layer deposition. We assume that the 1D resonator is 3-cm long and 10-μm wide, $Z_0 = 50\ \Omega$ which implies for the fundamental mode $\omega_0 = 2\pi \times 10$ GHz. The external magnetic field along the z axis is about $B_z = 1$ T to make sure the energy splitting $E_z = g^*\mu_{\rm B}B_z$ between the two triplet states $|T_\pm\rangle$ is larger than ω. In practice, careful fabrication permits a strong coupling capacitance, with $v \approx 0.2$,[22] so that the coupling coefficient $g \sim 2\pi \times 120$ MHz is achievable due to the numerical estimations in Eq. (9). The frequency ω_0 and coupling coefficient g can be tuned via LC_0. In order to implement a gate quickly, which is about pulse duration τ and satisfies $\tau \ll 1/\kappa$, the system works in a weak coupling regime. In the bad cavity limit, we have $g \sim \kappa$ where $\kappa = \omega_0/Q \approx 2\pi \times 100$ MHz with the quality factor of the resonator $Q = 100$.[25] Consider the effect of photon-assisted tunneling (PAT) in our system, which is harmful because it destroys the qubit by lifting the spin-blockade. To avoid this, one needs to make tunneling barriers close enough to the leads.

We address now the issue of relaxation and decoherence of our system. There are three types of contributions to the relaxation processes, one arising from the finite decay rate of the resonator, one from the intrinsic decoherence of the spin states and the third one from the charge-based dephasing and relaxation which occur during gate operation involving the electric dipole between $|a\rangle$ and $|0\rangle$.

For the charge relaxation time T_1, the decay is caused by coupling qubits to a phonon bath. With the spin-boson model, the perturbation theory gives an overall error rate from the relaxation and incoherent excitation, with which one can estimate the relaxation time $T_1 \sim 1$ μs,[10] which is studied in great detail for the GaAs QDs in 2DEG and a similar rate is expected for NW QDs.

The charge dephasing T_2 arises from variations of the energy offset and is about $T_2 \sim \omega T_b^2$ near the optimal point $\delta = 0$, where the bare dephasing time $T_b \sim 1$ ns was observed in Ref. [26]. Then charge dephasing is about $T_2 \sim 10$ ns–100 ns. Using quantum control techniques, such as better high- and low-frequency filtering of electronic noise, T_b exceeding 1 μs was observed in 2DEG[6] (we assume a similar result for the present case), which suppresses the charge dephasing.

The hyperfine interactions with the host nuclei cause nuclear spin-related dephasing T_2^*. The hyperfine field can be treated as a static quantity, because the evolution of the random hyperfine field is several orders slower than the electron spin dephasing. From an operating viewpoint, the most important decoherence due to hyperfine field is the dephasing between the singlet state $|0\rangle$ and one of the triplet states $|1\rangle$. By suppressing nuclear spin fluctuation, the spin dephasing time can be obtained by quasi-static approximation as $T_2^* = 1/g^* \mu_{\mathrm{B}} \langle \Delta B_n^z \rangle_{\mathrm{rms}}$, where ΔB_n^z is the nuclear hyperfine gradient field between two coupled dots and rms means a root-mean-square time-ensemble average. A measurement of the spin dephasing time $T_2^* \sim 4$ ns was demonstrated in Ref. [4] and we expect coherently driving the qubit will prolong the T_2^* time up to 1 μs and with echo up to 10 μs.[6]

The quality factor Q of the superconducting resonator in the microwave domain can be achieved 10^6.[23] In practice, the local external magnetic field $B_z = 1$ T reduces the limit of the quality factor.[25] However, in our proposal the low Q resonator is good for implementing an entangling gate with a short operation time. The dissipation of the resonator κ takes the decay time to about 10 ns, which also leads to photon loss together with the decay of the charge qubit. Due to photon loss, the amplitude of the output field α_{mn} for $m, n = 0, 1$ representing the different initial states of the DQDs is probably different from the input amplitude α, usually $|\alpha_{mn}| < |\alpha|$. The fundamental source of photon loss in the resonator can be qualified by the photon loss parameter $\eta = 1 - \min\{|\alpha_{mn}/\alpha|^2\} \propto \kappa/g^2 T_1$.

We obtain a high fidelity up to 0.99 under typical experimental configurations with the realistic parameters $(g, \kappa, 1/T_1)/2\pi = (120, 100, 1)$ MHz and the coherent input pulse with a remarkable amplitude $\alpha \sim 20$. Furthermore, we can estimate the time scale of the controlled phase flip gate operation $t_{\mathrm{cpf}} \sim \tau = 100$ ns with high fidelity, which is shorter than the decoherence time.

4. Logical qubits in DFS

There is a substantial amount known about the protection of encoded qubits with the $\{|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle\}$ code, beyond the passive DFS level, e.g. even how to actively suppress errors that are not due to collective dephasing. One of the proposed remedies is the method of "dynamical decoupling", or 'Bang–Bang' pulses,[27] in which strong and sufficiently fast pulses are applied to the system. In this manner one can either eliminate or symmetrize the system-bath Hamiltonian so that the system and bath are effectively decoupled.

We briefly review the decoupling technique[27] as it pertains to our proposal. Low frequency noise can be simply corrected. To be specific, we assume a phase noise term $\varepsilon(t)$ acting on the internal states of the DQDs, characterized by a power spectrum $S(\omega)$ of integrated power $(T_2)^2$ with a high frequency cutoff at $\omega_h \ll 1/T_2$. The action of $\varepsilon(t)$ can be represented by a stochastic evolution operator $U_x(t) = \mathrm{e}^{-\mathrm{i}\int_0^t \varepsilon(t')\,\mathrm{d}t'\sigma_x}$, where σ_x is a Pauli operator for the encoded subspace, which can be implemented simply by swapping the two qubits. The pulse sequence $[\Delta t, U_x, \Delta t, U_x]$ gives a reduced power spectrum $S_{\mathrm{DFS}}(\omega) \propto S(\omega) \sin^4(\Delta t\omega/2) / (\Delta t\omega)^2$, where Δt is free evolution time (cycle time). For frequencies below $1/\Delta t$, the bath-induced error rate is reduced by a factor proportional to $(\Delta t\omega)^2$. If the dominant noise mechanism has only low frequency components, the suppression can be dramatic.

The DFS also reduces phase errors during transport of DQDs with a separation time $\tau_{\mathrm{T}} \sim 2d/c$ (d is the distance between the two doubles of a pair). During transport, the spatial variations of the magnetic field strength along the transport path produce a phase noise term $\varepsilon(x, t)$. We set

$$\langle \varepsilon(x,t)\,\varepsilon(x',t') \rangle = N(|x - x'|) \int_{-\infty}^{\infty} S(\omega)\, \mathrm{e}^{\mathrm{i}\,\omega(t-t')}\,\mathrm{d}\omega$$

for transport into or out of the cavity, where $N(x) = \mathrm{e}^{-x^2/2r^2}$, r is the lateral radius of each dot. The resulting spectral function is

$$S_{\tau_{\mathrm{T}}}(\omega) = \int_{-\infty}^{\infty} S(\omega - \nu) \sin^2[(\omega - \nu)\tau_{\mathrm{T}}/2] \times \frac{\mathrm{e}^{-\tau_{\mathrm{T}}^2 \nu^2/2}}{\sqrt{2\pi\tau_{\mathrm{T}}^2}}\,\mathrm{d}\nu,$$

which has a suppression of noise with frequencies $\ll 1/\tau_{\rm T}$ by $(\tau_{\rm T}\omega)^2$.

Considering experimental parameters, we set $\omega_h \simeq 1$ kHz, $T_2 \simeq 100$ ns and use $S(\omega) = \exp(-\omega^2/2\omega_h^2)/(T_2\sqrt{2\pi\omega_h^2})$. Even for a cycle time $\Delta t \simeq 100$ ns and transport time $\tau_{\rm T} \simeq 400$ ns approaching T_2, phase errors due to low frequency terms occur at rates much less than milliseconds, indicating a suppression of more than 10^5. Thus the DFS and dynamical decoupling technique provides a powerful quantum memory and low-error transport channel, limited by errors in the logical x rotation.

5. Purification of logical Bell states

We now introduce a purification protocol[5,28] for logical Bell states based on the partial BSM and logical single qubit rotations. First we can exploit the NW double-dot system to make a partial BSM with the resonator-assisted interaction. As shown in figure 2, if only a DQD in an arbitrary state $a|0\rangle + b|1\rangle$ ($|a|^2 + |b|^2 = 1$) is coupled to the resonator and the incoming pulse is in a coherent state $|\alpha\rangle$, after the interaction between the DQDs and the resonator, the joint state of the DQD and the output pulse is $a|0\rangle|\alpha\rangle + b|1\rangle|-\alpha\rangle$. After a homodyne detection of the output pulse which measures the relative phase, we obtain the project measurement $P_1 = |0\rangle\langle 0|$ and $P_2 = |1\rangle\langle 1|$. Performing the project measurement $\{P_1, P_2\}$ on both DQDs of the logical Bell state allows one to distinguish the subspace spanned by $\{|\Phi^+\rangle, |\Phi^-\rangle\}$ and $\{|\Psi^+\rangle, |\Psi^-\rangle\}$. The measurement outcomes correspond to $P_{\{|\Phi^+\rangle,|\Phi^-\rangle\}} = |\Phi^+\rangle\langle\Phi^+| + |\Phi^-\rangle\langle\Phi^-|$ and $P_{\{|\Psi^+\rangle,|\Psi^-\rangle\}} = |\Psi^+\rangle\langle\Psi^+| + |\Psi^-\rangle\langle\Psi^-|$, respectively.

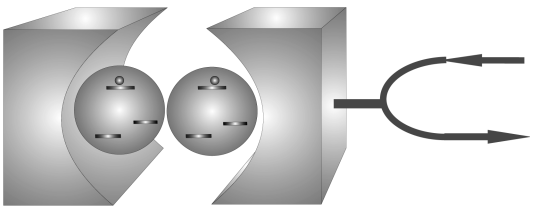


Fig. 2. Schematic setup for implementation of an entangling gate on two DQDs through resonator-assisted interaction.

Errors during generation, transport and storage processes can lead to (i) errors within the logical subspace $\{|0\rangle, |1\rangle\}$ and (ii) population of states $\{|2\rangle, |3\rangle\}$ outside the DFS where $|2\rangle = 1/\sqrt{2}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle), |3\rangle = 1/\sqrt{2}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)$. Both kinds of errors reduce the fidelity of the logical entangled states $|\Phi^+\rangle$ and need to be corrected. Our purification protocol completely corrects arbitrary strength errors of type (ii) and corrects for errors of type (i) that occur with a probability of less than 1/2.

Consider a mixed state ρ_{AB} resulting from an imperfect distribution of $|\Phi^+\rangle$. We decompose ρ into three terms, $\rho = \widehat{\rho} + \rho_{\rm od} + \rho_R$, and express the density operator $\widehat{\rho}$ in the logical Bell basis $\{|\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle, |\Psi^-\rangle\}$ and denote the diagonal elements in that basis by $\{A, B, C, D\}$. All off-diagonal elements in the Bell basis ($\rho_{\rm od}$) and terms containing $\{|2\rangle, |3\rangle\}$ (ρ_R) are made irrelevant by the protocol. Note that the first diagonal element $A = \langle\Phi^+|\rho|\Phi^+\rangle$, which is the definition of the fidelity.

Given two mixed states ρ_{AB} and $\rho_{A'B'}$, described by $\{A, B, C, D\}$ and $\{A', B', C', D'\}$ (and the irrelevant $\rho_{\rm od} + \rho_R$) respectively, the following sequence of local operations obtains with a certain probability a state with higher fidelity and hence purifies the state: (i) application of $\sigma_{x,A} \otimes \sigma_{x,B}$ or I with probability 1/2 to ρ_{AB} and similar to $\rho_{A'B'}$; (ii) the partial BSM on both ρ_{AB} and $\rho_{A'B'}$, i.e., $\sum_{i=1,2}\sum_{j=1,2} M_i^{AA'} M_j^{BB'} \rho_{AB}\rho_{A'B'}$, where $M_1 = P_{\{|\Phi^+\rangle,|\Phi^-\rangle\}}$ and $M_2 = P_{\{|\Psi^+\rangle,|\Psi^-\rangle\}}$. We only keep the state ρ_{AB} if $i = j$, i.e. the results in the final measurement coincide in both states.

The effect of (i) is to erase off-diagonal terms of the form $|\Phi^\pm\rangle\langle\Psi^\pm|$ which may contribute to the protocol. The operation in (ii) realizes—in addition to the projection into the $\{|0\rangle, |1\rangle\}$ logical subspace in A and B (also A' and B') which erases all terms ρ_R, $\rho_{R'}$—a purification map. In particular, we find that the remaining off-diagonal elements do not contribute to the operation and the action of the protocol can be described by the nonlinear mapping of corresponding vectors $\boldsymbol{x} = (A, B, C, D)$, $\boldsymbol{x}' = (A', B', C', D')$. The resulting state is of the form $\hat{\rho}' + \tilde{\rho}_{\rm od}$, and $\hat{\rho}'$ on average has diagonal elements given by

$$\widetilde{A} = (AA' + CC')/N, \quad \widetilde{B} = (BB' + DD')/N,$$
$$\widetilde{C} = (BD' + DB')/N, \quad \widetilde{D} = (AC' + CA')/N, \quad (15)$$

and $N = (A + D)(A' + D') + (B + C)(B' + C')$ is the probability of success of the protocol. This map is equivalent to the purification map described in Ref. [28] for non-encoded Bell states. It follows that an iteration of the map—which corresponds to iteratively applying the purification procedure [(i) and (ii)] to two identical copies of states resulting from previous

successful purification rounds—leads to a logical maximally entangled state. That is, the map has $\{1,0,0,0\}$ as an attracting fixed point whenever $A > B+C+D$. We emphasize that all errors leading to out of the logical subspace and independent of their probability of occurrence, can be corrected. The method is capable of purifying a collection of pairs in any state $\widehat{\rho}$, whose average fidelity with respect to at least one maximally entangled state is with probability greater than 1/2.

Here we remark that the methods such as nested purification can also be applied via logical BSMs, which significantly reduces the required number nodes.[29] The above scheme of entanglement purification can also be extended to perform logical BSMs, in principle, between two logical qubits represented by remote DQDs trapped in different cavities at an arbitrary distance.

6. Conclusion

In summary, we proposed a deterministic generation and purification of decoherence-free spin entangled states of singlet–triplet spins in NW DQDs via resonator-assisted charge manipulation and measurement techniques. We encode the quantum information in spin states with a long coherent lifetime and the state manipulation is implemented via the charge dipole transition driven by a resonator with strong coupling. Our work shows how an experiment can be performed under existing conditions to demonstrate the first entanglement generation and purification for spin qubits in QDs in the laboratory. The decoherence-free subspace immunizes the logical qubits against the dominant source of decoherence–dephasing—while the influences of additional errors are shown by numerical simulations. We can implement single-qubit rotations and a partial BSM only via an interaction with the resonator mode. This leads to an extremely simple implementation of the entanglement purification. Because of the switchable coupling between the double-dot pairs and the resonator, we can apply this entangling gate to any two qubits without affecting others, which is not trivial for implementing scalable quantum computing and generating large entangled states. We analyse the performance and stability of all required operations and emphasize that all techniques are feasible in current experimental conditions.

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