Using a quantum dot system to realize perfect state transfer*

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There are some disadvantages to Nikolopoulos *et al.*'s protocol [Nikolopoulos G M, Petrosyan D and Lambropoulos P 2004 *Europhys. Lett.* **65** 297] where a quantum dot system is used to realize quantum communication. To overcome these disadvantages, we propose a protocol that uses a quantum dot array to construct a four-qubit spin chain to realize perfect quantum state transfer (PQST). First, we calculate the interaction relation for PQST in the spin chain. Second, we review the interaction between the quantum dots in the Heitler–London approach. Third, we present a detailed program for designing the proper parameters of a quantum dot array to realize PQST.

Keywords: quantum communication, spin chain, quantum dot

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1. Introduction

In 2004, Nikolopoulos *et al.*[1] first proposed a protocol using a quantum dot array to realize quantum state transfer. They discussed coherent electron wavepacket propagation in an array of coupled quantum dots. Next, Petrosyan and Lambropoulos[2] studied coherent population transfer in coupled quantum dots. Their creative work has significantly improved quantum communication in a solid state system. However, there remain some disadvantages to their protocol. For example, an extra electron is needed to transfer the encoded information, i.e., a subtle injection device is needed to inject the electron. Moreover, the quantum dots are tunnel-coupled by controlling the gate voltage; thus, the communication system need more control lines and the system complexity becomes high. Also, the tunneling interaction must be switched "on" before and "off" after the state transfer: this is inconvenient for high-speed quantum communication. Our objective is to overcome such disadvantages.

In Bose's scheme[3] the spin chain was used to realize PQST and there was no need to turn "on" and "off". Bose's work motivated many studies on the possibility of PQST in spin systems.[4−8] In particular, a general theoretical framework was proposed for PQST[7] and a detailed design of a short spin chain was shown for PQST.[8] We used a quantum dot array to realize Bose's proposal.[3] Quantum dots were first introduced in the study of quantum information processing by Loss and DiVincenzo.[9] They proposed the implementation of a universal set of one- and two-quantum-bit gates for quantum computation using the spin states of coupled single-electron quantum dots. There are now three ways to realize a single spin quantum dot: 1) metal electrodes to partition a two-dimensional electron gas (2DEG);[9] 2) self-assembled technology;[10] or 3) nitrogen-vacancy (N-V) colour centre in diamond.[11,12]

Coupling between single-spin quantum dots was also studied using the Heitler–London approximation of molecular physics,[13] electronic coupling[10] or dipole–dipole interaction.[12] In a system of semiconductor quantum dots, single electron spins were prepared with a well-defined orientation, and their relaxation time reached one second.[14] The state of an individual electron spin can be manipulated through magnetic resonance,[15] indirect electronic control[16] or optical control.[17,18] The state can be read by electrical single-shot measurement technology.[19,20] The state of the coupled electron spins can be manipulated coherently.[21]

The single-spin coherence time has now been extended and the cost of preparing a single-spin quantum dot reduced.[22] More details can be gleaned from

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the review articles[23,24] and references therein. Bose's proposal demands that the system can be initialized, the spin can be manipulated individually, and the interaction between the spins can be controlled effectively. These criteria are elements in current quantum dot technology.

Combining PQST theory in a spin chain and current quantum dot technology, we propose a scheme for short-distance PQST which overcomes the aforementioned disadvantages.[1,2] With the aid of quantum repeater technology, our protocol can realize long-distance quantum communication. The low-energy dynamics of coupled quantum dots can be described by the effective Heisenberg spin Hamiltonian[25] $H(t) = \sum_{i=1}^{N} J_i(t) S_i \cdot S_{i+1}$, where J_i denotes the exchange coupling between the nearest quantum dots,[26] i.e., the energy difference between the triplet and the singlet. Thus, the study of this system is essentially reduced to the study of the exchange energy and how the exchange coupling J can be controlled experimentally. The central idea is that J can be controlled by raising or lowering the tunneling barrier between the dots. We note that increasing (decreasing) the interdot distance is physically equivalent to raising (lowering) the interdot barrier, which can be achieved experimentally.

2. PQST theory in a spin chain

The four-qubit case is longer and has more control parameters than the two- and three-qubit cases, and is one of the best candidates to be integrated for use in quantum information processing. Following the ideas in Refs. [3], [7], and [8], we show how to design a four-qubit spin chain to realize PQST. The reason for this calculation is that the (which will be discussed below) have not been previously published.

The effective Hamiltonian of a general N-spin isotropic "XYZ" chain is

$$H_N = \sum_{i,j} J_{i,j}(t) S_i \cdot S_j + \sum_i \mu_{\rm B} g_i(t) B_i(t) \cdot S_i. \quad (1)$$

For the four-qubit spin chain with nearest interaction $J_{1,2} = J_{3,4} = J_1$ and $J_{2,3} = J_2$, equation (1) can be written as

$$\begin{aligned} H_4 \;=\; & J_1(S_1 \cdot S_2) + J_2(S_2 \cdot S_3) + J_1(S_3 \cdot S_4) \\ & + \sum_i^4 \mu_{\rm B} g_i(t) B_i(t) \cdot S_i, \end{aligned} \quad (2)$$

where the last term on the right-hand side in Eq. (2) is the Zeeman term, g is the effective g factor, and $\mu_{\rm B}$ is the Bohr magneton. The ratio between the Zeeman splitting and the relevant orbital energies is small for $B \ll B_0$ or $B \gg B_0$: indeed, $g\mu_{\rm B}B/\hbar\omega_0 \leq 0.03$ for $B \ll B_0 = (\hbar\omega_0/\mu_{\rm B})(m/m_e) \approx 3.5$ T, and $g\mu_{\rm B}B/\hbar\omega_{\rm L} \leq 0.03$ for $B \gg B_0$, where $\omega_{\rm L} = eB/2mc$ is the Larmor frequency and $\hbar\omega_0 = 3$ meV. Thus, Zeeman splitting can be safely ignored when the effective spin Hamiltonian is discussed. Using $S_i = \frac{\hbar}{2}\sigma_i$ ($i = x, y, z$), one can rewrite Eq. (2) as

$$\begin{aligned} H_4 \;=\; & \frac{\hbar}{2}[J_1(\sigma_{1,x}\sigma_{2,x} + \sigma_{1,y}\sigma_{2,y}) \\ & + J_2(\sigma_{2,x}\sigma_{3,x} + \sigma_{2,y}\sigma_{3,y}) \\ & + J_1(\sigma_{3,x}\sigma_{4,x} + \sigma_{3,y}\sigma_{4,y})]. \end{aligned} \quad (3)$$

We intend to consider whether a quantum state can be transferred from one end of the chain to the other, so we only need the eigenstates that have one spin up and the remaining spins down; such a Hilbert subspace will be spanned by $|1000\rangle$, $|0100\rangle$, $|0010\rangle$, $|0001\rangle$, where $|1\rangle$ denotes the spin up.

For one-qubit state transfer, the key point is to examine whether an initial state $|0001\rangle$ can evolve into $|1000\rangle$. That is to say, the transition matrix element

$$f_{1\to4}(t) = \langle 1000| \exp(-\,{\rm i}\, Ht)|0001\rangle \quad (4)$$

must reach unity at a particular time $t_{\rm p}$. If $f_{1\to4}(t_{\rm p}) = 1$, an arbitrary one-qubit state can transfer from the first spin to the fourth spin. The transition amplitude $f_{1\to4}$ can be written as

$$f_{1\to4} = \sum_{m=1}^{4} \langle 1000|\phi_m\rangle\langle\phi_m|0001\rangle \exp(-\,{\rm i}\, E_m t). \quad (5)$$

The four relevant eigenstates are

$$\begin{aligned} |\phi_1\rangle \;&=\; -|0001\rangle + J_-|0010\rangle - J_-|0100\rangle + |1000\rangle, \\ |\phi_2\rangle \;&=\; -|0001\rangle + J_+|0010\rangle - J_+|0100\rangle + |1000\rangle, \\ |\phi_3\rangle \;&=\; |0001\rangle + J_+|0010\rangle + J_+|0100\rangle + |1000\rangle, \\ |\phi_4\rangle \;&=\; |0001\rangle + J_-|0010\rangle + J_-|0100\rangle + |1000\rangle, \end{aligned} \quad (6)$$

where $J_\pm = (J_2 \pm \sqrt{J_2^2 + 4J_1^2})/(2J_1)$, and the corresponding eigenvalues are

$$\begin{aligned} & E_1 = -J_-J_1, \quad E_2 = -J_+J_1, \\ & E_3 = J_+J_1, \quad E_4 = J_-J_1. \end{aligned} \quad (7)$$

Substituting the eigenvalues and the corresponding eigenstates into Eq. (5), the transition amplitude of the four-qubit spin chain can be obtained as

$$f_{1\to4} \;=\; 2\,{\rm i}\,C_1^2 \sin(E_1 t) + 2\,{\rm i}\,C_2^2 \sin(E_2 t), \quad (8)$$

where $|C_1|^2 = 1/(2+2J_-^2)$, $|C_2|^2 = 1/(2+2J_+^2)$, and $2(|C_1|^2+|C_2|^2) = 1$. Obviously, if condition $\sin(E_1 t) = \sin(E_2 t) = \pm 1$ is satisfied, then $|f_{1\to 4}| = 1$ and PQST is realized. So E_1 and E_2 should satisfy

$$E_1 = \frac{\pi}{2t_{\rm p}}(2n_1 - 1), \quad E_2 = \frac{\pi}{2t_{\rm p}}(4n_2 + 2n_1 - 1), \quad (9)$$

where $n_{1,2}$ are integers. Substituting Eq. (9) into Eq. (7), the relation between J_1 and J_2 for perfect transmission can be obtained as

$$J_2 = \pm\sqrt{\frac{4(2n_2+2n_1-1)^2}{4n_2^2-(2n_2+2n_1-1)^2}}|J_1|, \quad (10)$$

where $n_{1,2}$ are integers, "+" corresponds to $1 \le n_1$ and $\frac{1}{2}(1-2n_1) < n_2 < \frac{1}{4}(1-2n_1)$, "−" corresponds to $1 \le n_1$ and $n_2 < \frac{1}{2}(1-2n_1)$. This explicit solution indicates that there exists an infinite number of possibilities to realize PQST. When we discuss n_1, it is natural to obtain the corresponding value of n_2.

Up to now, we have only shown how to transmit $|1\rangle$ from the first qubit to the fourth qubit. For general communication, the transmission of an arbitrary state $|\alpha\rangle = \cos\frac{\theta}{2}|0\rangle + {\rm e}^{{\rm i}\,\phi}\sin\frac{\theta}{2}|1\rangle$ must be realized. After some simple calculus, the corresponding average fidelity of transmitting $|\alpha\rangle$ can be obtained as

$$F(t) = \frac{1}{2} + \frac{|f_{1\to 4}(t)|^2}{6}, \quad (11)$$

and the average fidelity can reach 2/3 under the condition $|f_{1\to 4}(t_{\rm p})| = 1$. That is to say, our system can be used to transmit an arbitrary one-qubit state.

3. Coupled quantum dot system

Semiconductor quantum dots are small devices in which charge carriers are confined in three dimensions. Here we only consider the confinement realized through electrical gating and etching techniques applied to a two-dimensional electron gas. As in the systems described in Refs. [9] and [13], the quantum dot holding the electron spin is determined by applying negative voltages to metal surface electrodes on top of a semiconductor (GaAs/AlGaAs) heterostructure. In coupled quantum dots, Coulomb blockade effects and magnetization have been observed as well as the formation of a delocalized "molecular state". Spin chains can be achieved by temporarily coupling the two nearest spins $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$, through an exchange coupling J between them, and its Hamiltonian is $H_s = J\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$.

Due to the Coulomb interaction and the Pauli exclusion principle, the ground state of two coupled electrons is a spin singlet, i.e., a highly entangled spin state.[27] Recent experimental work on double quantum dots showed that if two electrons are in the same quantum dot, they also form a singlet. In the case of electrons in separate dots, the two electrons can occupy any spin state. An appropriate pulsing of the exchanged coupling can cause an interaction, thereby creating spin chains. Assuming the Heitler–London approach,[28] the exchange coupling can be written in the following form:[13]

$$J = \frac{\hbar\omega_0}{\sinh[2d^2(2b-1/b)]}\Big[c\sqrt{b}\{\,{\rm e}^{-bd^2}{\rm I}_0(bd^2) - {\rm e}^{d^2(b-1/b)}{\rm I}_0[d^2(b-1/b)]\} + \frac{3}{4b}(1+bd^2)\Big], \quad (12)$$

where $d = a/a_{\rm B}$ is a dimensionless distance, a half the distance between the centres of two dots, $a_{\rm B} = \sqrt{\hbar/m\omega_0}$ the effective Bohr radius of a single isolated harmonic well (which is about 20 nm for $\hbar\omega_0 = 3$ meV), $\rm I_0$ the zeroth-order Bessel function and $b = \omega/\omega_0 = (1+\omega_{\rm L}^2/\omega_0^2)^{1/2}$, where $\omega = (\omega_{\rm L}^2+\omega_0^2)^{1/2}$, $\omega_{\rm L} = eB/(2mc)$ denotes the Larmor frequency. The first and second terms in Eq. (12) are due to the Coulomb interaction C, where the exchange term enters with a minus sign. The parameter $c = 2.43$, for $h\omega_0 = 3$ meV, is the ratio between the Coulomb and confining energies. The last term comes from the confinement potential W. A demonstration of two coupled dots can be seen in Fig. 1.

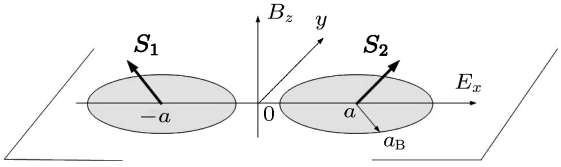


Fig. 1. Two coupled dots confined in the xy plane, where the spins of the electrons in dots 1, 2 are denoted by S_1 and S_2 respectively; the magnetic field B is along the z axis; the electric field is along the x axis.

For clarity, we plot the diagram of $J(d,b)$ in Fig. 2.

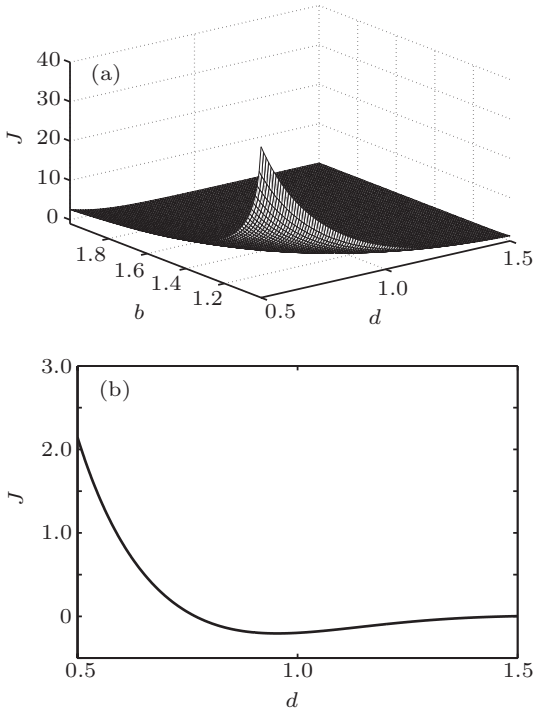


Fig. 2. (a) Diagram of $J(d,b)$, where $d = a/a_{\rm B}$, a is half the distance between the centres of two dots, $a_{\rm B} = \sqrt{\hbar/m\omega_0}$ the effective Bohr radius of a single isolated harmonic well and $b = \sqrt{1+(eB)^2/(2mc\omega_0)^2}$; (b) the diagram of $J(d,b)$, where b=1.

4. Using a quantum dot system to construct a spin chain to realize PQST

With the aid of Eqs. (10) and (12), we design the concrete parameters of the quantum dot system for realizing PQST. There are four choices: I) $J_{1,2} > 0$; II) $J_{1,2} < 0$; III) $J_1 > 0, J_2 < 0$; and IV) $J_1 < 0, J_2 > 0$. For convenience, we list some values of J_2 for small $n_{1,2}$ in Table 1.

Table 1. Values of J_2 for small $n_{1,2}$; $J_{1,2}$ in units of meV are scaled by $\pi/(2t_{\rm p})$.

n_1	n_2	$J_2(\lvert J_1\rvert) > 0$	n_1	n_2	$J_2(\lvert J_1\rvert) < 0$
2	−1	$2/\sqrt{3}$	1	−1	$-2/\sqrt{3}$
3	−2	$2/\sqrt{15}$	1	−2	$-6/\sqrt{7}$
4	−2	$6/\sqrt{7}$	1	−3	$-10/\sqrt{11}$
4	−3	$2/\sqrt{35}$	2	−2	$-2/\sqrt{15}$
5	−3	$2/\sqrt{3}$	2	−3	$-2/\sqrt{3}$
5	−4	$2/3\sqrt{7}$	2	−4	$-10/\sqrt{39}$

In fact, with current technology, we can only realize case I) $J_{1,2} > 0$, because $J < 0$ requires hundreds of Tesla. In the subsequent discussion only case $J_{1,2} > 0$ will be considered.

First we must know how to choose magnetic field B. The principle is not complicated: 1) B is homogeneous for $J_{1,2}$ (this is convenient for experiment); 2) B must be small enough to neglect the Zeeman effect, i.e. $B \ll (\hbar\omega_0/\mu_{\rm B})(m/m_{\rm e}) \approx 3.5$ T, so $B < 3.5\ {\rm T} \times 10\% = 0.35$ T is used. If $B < 0.35$ T, b in $J(d,b)$ will be 1, so $J_{1,2}$ will depend only on the interdot distance d.

Second, we must know how to choose $J_{1,2}$. Larger J_1 will correspond to smaller perfect transfer time $t_{\rm p}$, so for J_1 the bigger the better. In theory, $J_1(d_1)$ can be any positive value. Our protocol is based on neglecting the next-nearest interaction $J(d_1+d_2)$, so $J(d_1+d_2) \ll \min\{J_1(d_1), J_2(d_2)\}$ must be satisfied. Generally speaking, if $J(d_1+d_2) < \min\{J_1(d_1), J_2(d_2)\} \times 10\%$, the next-nearest interaction can be neglected. The numerical results show that only some d_1 under the condition $n_1 = 2$, and $n_2 = -1$ can satisfy $J(d_1+d_2) < \min\{J_1(d_1), J_2(d_2)\} \times 10\%$. Table 2 lists some corresponding parameters that can realize PQST.

Table 2. Values of $J_{1,2}$ ($J_2 = (2/\sqrt{3})J_1$) that can realize PQST, where $d_{1,2} = a_{1,2}/a_{\rm B}$, $a_{1,2}$ corresponds to half the interdot distance. $J_{1,2}$ is in unit of meV.

$J_1(d_1)$	$J_2(d_2)$	$J(d_1+d_2)$	$J/\min\{J_1, J_2\}$
4.83(4.40)	5.58(0.38)	0.44(0.78)	9.1%
3.43(0.45)	3.96(0.43)	0.26(0.88)	7.6%
2.47(0.50)	2.85(0.52)	0.16(1.02)	7.5%
1.79(0.55)	2.07(0.53)	0.14(1.08)	7.8%
1.30(0.60)	1.50(0.58)	0.11(1.18)	8.5%

At least when $0.40 < d_1 < 0.60$, the nearest-neighbour interaction model is effective. To illustrate this point we describe the complete process of designing the proper parameters of a quantum dot system for realizing PQST. The design process is summarized in the following three steps: 1) taking the possible d_1 and using Eq. (12) to calculate the corresponding J_1; 2) substituting J_1 into Eq. (10) ($n_1 = 2$, $n_2 = -1$) and calculating J_2 and d_2; and 3) using Eq. (1) to calculate $J(d_1+d_2)$ and judging whether $J(d_1+d_2)/J_1(d_1) < 10\%$; if the answer is affitmative, d_1 and d_2 will be effective, otherwise jump to step 1). It is worth noting that the initialization and manipulation of our four-qubit system can be realized with current technology.[15−18]

5. Conclusion

In this paper, we proposed how to construct a four-qubit quantum dot system to realize PQST. Our method can be generalized to the cases of more than four qubits, as long as the interaction for PQST[5,7,8] is known, though our discussion is not suitable for a quantum dot created by self-assembled technology and nitrogen-vacancy (N-V) colour centre in diamond, where the interactions are different from those by controlling interdot distance. In our protocol, we suppose that the interaction J_1 (between qubits 1 and 2) is equal to the interaction J_3 (between qubits 3 and 4) and we only consider the nearest-neighbour interaction. Using the same method as in Ref. [8], we obtain the condition (Eq. (10)) of realizing PQST. In theory, there exist infinite choices of designing a spin chain to realize PQST. However, with current technology, only the case $J_{1,2} > 0$ can be chosen. Some potential parameters for a practical quantum dot system are listed in Table 2. We have summarized the general method of finding the proper parameters for quantum dot system to realize PQST at the end of Section 4.

Compared with previous protocol,[1,2] our protocol does not need an extra electron and does not need to be turned "on" and "off" during communication. The exchange coupling J between two quantum dots is a function of interdot distance and magnetic field B; thus, more control lines are not needed. In ending this discussion we note that there remain some open problems. The first is about the next-nearest interaction; even its minimum value will reach 7.5% of the nearest-neighbour interaction. If the next-nearest interaction is neglected, then the interaction must be very small (with larger interdot distance) and the perfect transfer time $t_{\rm p}$ will be very large, which is not economical for communication. If the next-nearest interaction is considered, then the interaction must be very large (with smaller interdot distance) and $t_{\rm p}$ will be very small, while smaller interdot distance will correspond to smaller transportation distance, which is also not economical for communication. Second, we are not sure how to use a quantum dot system to realize a quantum repeater to join our communication unit. Third, the sensitivity and the decoherence analysis of the two coupled quantum dots used in Ref. [9] cannot be generalized in our case. Further consideration will be given to this in our future work.

References

[1] Nikolopoulos G M, Petrosyan D and Lambropoulos P 2004 *Europhys. Lett.* **65** 297

[2] Petrosyan D and Lambropoulos P 2006 *Opt. Commun.* **264** 419

[3] Bose S 2003 *Phys. Rev. Lett.* **91** 207901

[4] Li Y, Shi T, Chen B, Song Z and Sun C P 2005 *Phys. Rev.* A **71** 022301

[5] Christandl M, Datta N, Ekert A and Landahl A J 2004 *Phys. Rev. Lett.* **92** 187902

[6] Osborne T J and Linden N 2004 *Phys. Rev.* A **69** 052315

[7] Kostak V, Nikolopoulos G M and Jex I 2007 *Phys. Rev.* A **75** 042319

[8] Xi X Q, Gong J B, Zhang T, Yue R H and Liu W M 2008 *Eur. Phys. J.* D **50** 193

[9] Loss D and DiVincenzo D P 1998 *Phys. Rev.* A **57** 120

[10] Bayer M, Hawrylak P, Hinzer K, Fafard S, Korkusinski M, Wasilewski Z R, Stern O and Forchel A 2001 *Science* **291** 451

[11] Santori C, Tamarat P, Neumann P, Wrachtrup J, Fattal D, Beausoleil R G, Rabeau J, Olivero P, Greentree A D, Prawer S, Jelezko F and Hemmer P 2006 *Phys. Rev. Lett.* **97** 247401

[12] Neumann P, Kolesov R, Naydenov B, Beck J, Rempp F, Steiner M, Jacques V, Balasubramanian G, Markham M L, Twitchen D J, Pezzagna S, Meijer J, Twamley J, Jelezko F and Wrachtrup J 2010 *Nature Phys.* **6** 249

[13] Burkard G and Loss D 1999 *Phys. Rev.* B **59** 2070

[14] Amasha S, MacLean K, Radu Iuliana P, Zumbühl D M, Kastner M A, Hanson M P and Gossard A C 2008 *Phys. Rev. Lett.* **100** 046803

[15] Koppens F H L, Buizert C, Tielrooij K J, Vink I T, Nowack K C, Meunier T, Kouwenhoven L P and Vandersypen L M K 2005 *Nature* **442** 766

[16] Nowack K C, Koppens F H L, Nazarov Y V and Vandersypen L M K 2007 *Science* **318** 1430

[17] Mikkelsen M H, Berezovsky J, Stoltz N G, Coldren L A and Awschalom D D 2007 *Nature Phys.* **3** 770

[18] Press D, Ladd T D, Zhang B Y and Yamamoto Y 2008 *Nature* **456** 218

[19] Elzerman J M, Hanson R, Willems van Beveren L H, Witkamp B, Vandersypen L M K and Kouwenhoven L P 2004 *Nature* **430** 431

[20] Hanson R, Willems van Beveren L H, Vink I T, Elzerman J M, Naber W J M, Koppens F H L, Kouwenhoven L P and Vandersypen L M K 2005 *Phys. Rev. Lett.* **94** 196802

[21] Petta J R, Johnson A C, Taylor J M, Laird E A, Yacoby A, Lukin M D, Marcus C M, Hanson M P and Gossard A C 2005 *Science* **309** 2180

[22] Sanderson K 2009 *Nature* **459** 760

[23] Hanson R and Awschalom David D 2008 *Nature* **453** 1043

[24] Hanson R, Kouwenhoven L P, Petta J R, Tarucha S and Vandersypen L M K 2008 *Rev. Mod. Phys.* **79** 1217

[25] Mattis D C 1965 *The Theory of Magnetism* (New York: Harper and Row)

[26] Bennett C H, DiVincenzo D P, Bacon D, Burkard G and Whaley K B 2000 *Nature* **408** 339

[27] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters K 1993 *Phys. Rev. Lett.* **70** 1895

[28] Herring C 1962 *Rev. Mod. Phys.* **34** 4