Preparation of cluster states with trapped electrons on a liquid helium surface*

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(Received 6 January 2011; revised manuscript received 23 April 2011)

We present a scheme for the preparation of one-dimensional (1D) and two-dimensional (2D) cluster states with electrons trapped on a liquid helium surface and driven by a classical laser beam. The two lowest levels of the vertical motion of the electron act as a two-level system, and the quantized vibration of the electron along one of the parallel directions (the x direction) serves as the bosonic mode. The degrees of freedom of the vertical and parallel motions of the trapped electron can be coupled together by a classical laser field. With the proper frequency of the laser field, the cluster states can be realized.

Keywords: cluster states, electrons, liquid helium

PACS: 03.67.–a, 42.50.Dv, 73.20.–r **DOI:** 10.1088/1674-1056/20/10/100303

1. Introduction

In recent years, multi-particle entangled states have played an important role in testing quantum nonlocality[1−3] and implementing quantum information processing. It has been shown that there are two inequivalent classes of tripartite entangled states, i.e. the Greenberger–Horne–Zeilinger (GHZ) state[4] and the W state.[5] The cluster state[6] was introduced by Briegel and Raussendorf; it has a high persistence of entanglement and can be a resource for GHZ states. It is also more immune to decoherence than the GHZ state.[7] It can be used to test nonlocality without inequality.[8] In addition, the cluster state has been shown to constitute a universal resource for one-way quantum computation with the assistance of local measurement.[9]

So far, there have been many theoretical schemes for preparing cluster states with cavity quantum electrodynamics (QED), trapped ions, and so on.[10,11] For example, Ye *et al.*[12] proposed a scheme to generate a four-atom entangled cluster state using the interaction of two two-level atoms with a single-mode nonresonant cavity with the aid of a strong classical driving field. In Refs. [13]–[17], different schemes are proposed to generate entangled states by combining cavity QED with linear optics elements. In Ref. [18], Zheng proposed two schemes for preparing a four-ion cluster state in a trapped-ion system.

Recently, Platzman and Dykman[19] have proposed to implement quantum computation with electrons floating on a liquid helium surface. In this proposal, electrons are trapped on the surface of the liquid helium and controlled by a series of external electric fields. The quantum state of these electrons can be coherently controlled by applying microwave radiation. The system has a relatively long coherence time and can be easily manipulated, so it has attracted much attention in recent years in quantum information processing.[20−23]

In this paper, we propose a scheme to generate one-dimensional (1D) and two-dimensional (2D) cluster states with trapped electrons on a liquid helium surface. The paper is organized as follows: In Section 2 we describe the interaction between trapped electrons and a classical laser beam. Section 3 and Section 4 are devoted to the preparation of a 1D cluster state and 2D cluster state, respectively. A summary is given in Section 5.

*Project supported by the National Natural Science Foundation of China (Grant No. 60978009) and the National Basic Research Program of China (Grant Nos. 2009CB929604 and 2007CB925204).

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2. Model and effective Hamiltonian

The electron is trapped on a liquid helium surface. The two lowest levels of the vertical motion of the electron act as a two-level system, and the quantized vibration of the electron along one of the parallel direction (the x direction) serves as the bosonic mode. The degrees of freedom of vertical and parallel motions of the trapped electron can be coupled together using a classical laser field (see Fig. 1). The Hamiltonian of the system is[24]

$$H = \hbar\nu\left(a^{\dagger}a + \frac{1}{2}\right) + \frac{\hbar\omega_0}{2}\sigma_z + \hbar\Omega(\sigma^- + \sigma^+) \times (\mathrm{e}^{\mathrm{i}\eta(a^{\dagger}+a)-\mathrm{i}\omega_l t+\mathrm{i}\phi_l} + \mathrm{e}^{-\mathrm{i}\eta(a^{\dagger}+a)+\mathrm{i}\omega_l t-\mathrm{i}\phi_l}). \quad (1)$$

Here $a^{\dagger}$ and a are the bosonic creation and annihilation operators of the vibrational quanta of the electron's oscillation along the x direction respectively. The $\sigma^+ = |e\rangle\langle f|$, $\sigma^- = |f\rangle\langle e|$, $\sigma_z = (|e\rangle\langle e| - |f\rangle\langle f|)/2$ are Pauli operators. The transition frequency ω_0 is defined by $\omega_0 = (E_e - E_{\mathrm{f}})/\hbar$, E_e and E_f are the electronic energy levels. The $\Omega = \langle f|z|e\rangle eE_z/2\hbar$ is the so-called carrier Rabi frequency describing the strength of the coupling between the applied laser field and the electron, the $\eta = k_l\sqrt{\hbar/2m_e\nu}$ is the Lamb–Dicke (LD) parameter, which describes the coupling strength between the z and x directions of the trapped electron. The k_l, ω_l, and ϕ_l are the wave vector, frequency, and the initial phase for the driving laser beam respectively.

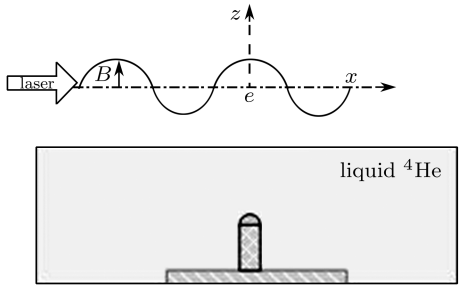


Fig. 1. An electron is trapped on a helium surface and driven by a classical laser propagating along the x direction.

Now the laser beam is tuned to the first lower vibrational sideband $\omega_l = \omega_0 - \nu$. In the interaction picture, in the LD regime ($\eta \ll 1$) and under the rotating-wave approximation, the Hamiltonian of the system can be reduced to[24,25]

$$H_I = \mathrm{i}\eta\hbar\Omega\,\mathrm{e}^{\mathrm{i}\phi_l}\sigma^{\dagger}a + \mathrm{H.c.} \quad (2)$$

3. Implementation of a 1D cluster state

In order to prepare the cluster state, the electronic levels are labeled as $|g\rangle$, $|f\rangle$, $|e\rangle$ (see Fig. 2). We choose the frequency of the laser field in a way such that only $|f\rangle$ and $|e\rangle$ are appropriately affected by the laser field. The transition frequency between the states $|f\rangle$ and $|g\rangle$ is highly detuned from the laser field frequency, thus the state $|g\rangle$ is not affected during the laser–electron interaction. In this case, by choosing $\phi = 0$, the evolution of the system will be described by the unitary operator[24,25]

$$U(t) = \exp[\eta\Omega t(a\sigma^{\dagger} - \mathrm{H.c})]. \quad (3)$$

The states of the system evolve as follows:[25]

$$\begin{aligned} &U(t)\,|f\rangle\,|1\rangle_m \\ &\rightarrow \cos(\eta\Omega t)\,|f\rangle\,|1\rangle_m + \sin(\eta\Omega t)\,|e\rangle\,|0\rangle_m, \\ &U(t)\,|e\rangle\,|0\rangle_m \\ &\rightarrow \cos(\eta\Omega t)\,|e\rangle\,|0\rangle_m - \sin(\eta\Omega t)\,|f\rangle\,|1\rangle_m, \end{aligned} \quad (4)$$

and the state $|f\rangle\,|0\rangle_m$ is unaltered. Here $|0\rangle_m$ ($|1\rangle_m$) denotes a state in which the vibrational mode has no (one) phonon.

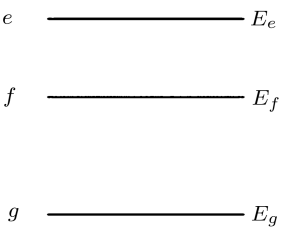


Fig. 2. The electronic energy levels.

We assume that the first electron is prepared in the state $(|g\rangle_1 + |f\rangle_1)/\sqrt{2}$, and the vibrational mode is in the state $(|0\rangle_m + |1\rangle_m)/\sqrt{2}$, by choosing $\eta\Omega t = \pi$, the state of the system becomes

$$|\varphi\rangle = \frac{1}{2}\left[|0\rangle_m(|g\rangle_1 + |f\rangle_1) + |1\rangle_m(|g\rangle_1 - |f\rangle_1)\right]. \quad (5)$$

Then the first electron is driven by one classical pulse. Choosing an appropriate amplitude and phase, the electron undergoes the transition $|f\rangle_1 \rightarrow (|g\rangle_1 - |f\rangle_1)/\sqrt{2}$ and $|g\rangle_1 \rightarrow (|g\rangle_1 + |f\rangle_1)/\sqrt{2}$, then equation (5) becomes

$$|\varphi\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle_m|g\rangle_1 + |1\rangle_m|f\rangle_1\right]. \quad (6)$$

Now we consider an identical auxiliary trapped electron in the state $|f\rangle_2$; after an interaction time $t = \pi/2\eta\Omega$, the state of the system becomes

$$|\varphi\rangle = \frac{1}{\sqrt{2}}|0\rangle_m \left(|f\rangle_2|g\rangle_1 + |e\rangle_2|f\rangle_1\right). \tag{7}$$

Then a short classical pulse is applied to the second (auxiliary) trapped electron, which induces the transition $|f\rangle_2 \rightarrow \left(|e\rangle_2 + |f\rangle_2\right)/\sqrt{2}$ and $|e\rangle_2 \rightarrow \left(|f\rangle_2 - |e\rangle_2\right)/\sqrt{2}$, so the state of the system becomes

$$\begin{aligned}|\varphi\rangle \;=\; & \frac{1}{2}\big[\,|f\rangle_2 \left(|g\rangle_1 + |f\rangle_1\right) \\ & +\; |e\rangle_2 \left(|g\rangle_1 - |f\rangle_1\right)\big]\,|0\rangle_m .\end{aligned} \tag{8}$$

Now the second trapped electron interacts with the laser field; we choose $t = 3\pi/2\eta\Omega$ and obtain

$$\begin{aligned}|\varphi\rangle \;=\; & \frac{1}{2}\big[\,|0\rangle_m \left(|g\rangle_1 + |f\rangle_1\right) \\ & +\; |1\rangle_m \left(|g\rangle_1 - |f\rangle_1\right)\big]\,|f\rangle_2 ,\end{aligned} \tag{9}$$

which can be rewritten as follows:

$$|\varphi\rangle = \frac{1}{2}\left(|0\rangle_m + |1\rangle_m \sigma_1\right)\left(|g\rangle_1 + |f\rangle_1\right), \tag{10}$$

here $\sigma_i = |g\rangle_i \langle g| - |f\rangle_i \langle f|$, $(i = 1, 2, 3, \ldots)$.

Now we demonstrate how to prepare a one-dimensional cluster state using many trapped electrons on the liquid helium surface. We consider $2(N-1)$ electrons trapped on the surface. The vibrational mode is supposed to be initially prepared in the superposition state $\left(|0\rangle_m + |1\rangle_m\right)/\sqrt{2}$. If the $(2\,i-1)$-th trapped electron performs the same operation as the first trapped electrons and the $(2\,i)$-th electron performs the same operation as the auxiliary trapped electrons, the state of the system is given by

$$\begin{aligned}|\varphi\rangle \;=\; & \frac{1}{\sqrt{2^N}}(|0\rangle_m + |1\rangle_m \sigma_{2N-3})(|g\rangle_{2N-3} \\ & +\; |f\rangle_{2N-3} \sigma_{2N-5}) \otimes \cdots \otimes \left(|g\rangle_3 + |f\rangle_3 \sigma_1\right) \\ & \times \left(|g\rangle_1 + |f\rangle_1\right)|f\rangle_2 |f\rangle_4 \otimes \cdots \otimes |f\rangle_{2N-2} .\end{aligned} \tag{11}$$

So we obtain a multipartite entangled state for the $2(N-1)$ trapped electrons.

If we encode $|g\rangle_i \rightarrow |0\rangle$ and $|f\rangle_i \rightarrow |1\rangle$ thus we can obtain the state of the system

$$\begin{aligned}|\varphi\rangle \;=\; & \frac{1}{\sqrt{2^N}}(|0\rangle_m + |1\rangle_m \sigma_{2N-3})(|0\rangle_{2N-3} \\ & +\; |1\rangle_{2N-3} \sigma_{2N-5}) \otimes \cdots \otimes \left(|0\rangle_3 + |1\rangle_3 \sigma_1\right) \\ & \times \left(|0\rangle_1 + |1\rangle_1\right).\end{aligned} \tag{12}$$

In this way, we present a scheme to prepare a 1D cluster state by using trapped electrons on a liquid helium surface.

4. Preparing 2D cluster states

In this section, we generate a 2D cluster state by fusing two 1D cluster states. According to Ref. [26], two 1D cluster states can be written as

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|f\rangle_i |\varphi_{i1}\rangle + |g\rangle_i |\varphi_{i2}\rangle\right), i \in (a, b), \tag{13}$$

where $|\varphi_{i1}\rangle$ and $|\varphi_{i2}\rangle$ are the arbitrary normalized cluster state of the subsystems and $\langle\varphi_{i1}|\varphi_{i2}\rangle = 0$, the i-th trapped electron is entangled with the subsystems. We use an electron labeled by $|g\rangle$, $|f\rangle$, $|e\rangle$. The states of $|f\rangle$ and $|e\rangle$ follow the transformation Eq. (4), the state $|g\rangle$ is not affected during the laser–electron interaction. In this system, $|1\rangle$ denotes a state of the quantized vibrational mode of the electron along one of the parallel directions. If we perform the series of unitary operators $U_a(t = \pi/2\eta\Omega)$, $U_b(t = \pi/2\eta\Omega)$, and the transition $|g\rangle_b \rightarrow |e\rangle_b$, the initial system state

$$\frac{1}{2}\left(|f\rangle_a |\varphi_{a1}\rangle + |g\rangle_a |\varphi_{a2}\rangle\right)\left(|f\rangle_b |\varphi_{b1}\rangle + |g\rangle_b |\varphi_{b2}\rangle\right)|1\rangle \tag{14}$$

will evolve to

$$\begin{aligned}\frac{1}{2}\big(\, & |e\rangle_a |\varphi_{a1}\rangle |\varphi_{b1}\rangle |f\rangle_b |0\rangle + |g\rangle_a |\varphi_{a2}\rangle |\varphi_{b1}\rangle |e\rangle_b |0\rangle \\ & +\; |e\rangle_a |\varphi_{a1}\rangle |\varphi_{b2}\rangle |e\rangle_b |0\rangle \\ & +\; |g\rangle_a |\varphi_{a2}\rangle |\varphi_{b2}\rangle |e\rangle_b |1\rangle \,\big).\end{aligned} \tag{15}$$

Then we measure the state of the electron b in the basis $|e\rangle_b |0\rangle$, the system state becomes

$$\frac{1}{2}\left(|g\rangle_a |\varphi_{a2}\rangle |\varphi_{b1}\rangle + |e\rangle_a |\varphi_{a1}\rangle |\varphi_{b2}\rangle\right). \tag{16}$$

In this way, we obtain a 2D cluster state[26,27] using trapped electrons on a liquid helium surface.

5. Summary

In this system, we take into account two important factors: the decoherence of the electron and the value of the introduced LD parameter. Decoherence is always a challenge in various quantum systems. In the present system, the main source of decoherence is the so-called ripplons, i.e., the thermally excited surface waves of liquid helium.[21] The coherence time due to this fluctuation is estimated to be 10^{-4} s.[19] If the amplitude of the applied laser field is set as the typical value $E_z = 10^2$ V/m[19] the LD parameter is $\eta = 1.2 \times 10^{-4} \ll 1$ in the above system. Thus, the usual LD approximation is valid. Our numerical estimations show that the duration of a $\pi/2$ pulse is

$t = \pi/2\eta\Omega < 10^{-4}$ s, so it still works within the typical coherence time 10^{-4} s.

In summary, we have proposed a scheme for generating 1D and 2D cluster states using trapped electrons on a liquid helium surface, which constitutes a universal resource for quantum computation.

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