Quantum and classical correlations for a two-qubit X structure density matrix*

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(Received 8 May 2011; revised manuscript received 13 June 2011)

We derive explicit expressions for quantum discord and classical correlation for an X structure density matrix. Based on the characteristics of the expressions, the quantum discord and the classical correlation are easily obtained and compared under different initial conditions using a novel analytical method. We explain the relationships among quantum discord, classical correlation, and entanglement, and further find that the quantum discord is not always larger than the entanglement measured by concurrence in a general two-qubit X state. The new method, which is different from previous approaches, has certain guiding significance for analysing quantum discord and classical correlation of a two-qubit X state, such as a mixed state.

Keywords: quantum and classical mutual information, X structure density matrix, quantum discord, classical correlation, entanglement

PACS: 03.65.Yz, 03.65.Ta, 03.67.–a **DOI:** 10.1088/1674-1056/20/10/100302

1. Introduction

Entanglement[1] as a kind of quantum correlation has been extensively studied both theoretically and experimentally in the past two decades because it is valuable for understanding the fundamental concepts of quantum mechanics and clarifying the accountability of the orthodox interpretation proposed in quantum mechanics.[2] In particular, concurrence as a type of measurement of entanglement has been widely studied since it was proposed.[3−12] Entanglement is studied under different conditions, such as taking into account decoherence and dissipative effects. Although entanglement is extremely important and even indispensible in the field of quantum information processing (QIP),[1] it does not contain all of the quantum correlation and is not a unique measure of quantum correlation; there are other quantum correlations, such as quantum non-locality without entanglement,[13,14] which has been demonstrated by theoretical[15,16] and experimental[17] results, and they have advantages in the field of quantum information, such as improvement of the efficiency of the Carnot engine. Therefore characterizing and qualifying quantum correlations have received much attention.

A bipartite state contains quantum and classical correlations, which are identified by their quantum mutual information.[1] If ρ denotes the density operator of a compound system T ($T = A + B$) and ρ^A (ρ^B) indicates the density operator of system A (B) by partly tracing out the density operator ρ, the quantum mutual information is $I(\rho) = S(\rho^A) + S(\rho^B) - S(\rho)$. Moreover, the quantum mutual information is the maximization of common information between A and B, and it can be written as the sum of quantum and classical correlations by introducing the concept of quantum discord.[18−20] This is also a type of measurement of quantum correlation which is completely different from entanglement, for a mixed state having non-zero quantum discord.

The quantum discord of a general bipartite state is not always larger than the entanglement,[21,22] which means that the quantum discord is not simply the sum of entanglement and some other non-classical correlation. The entanglement of two-qubit states has been characterized and qualified completely for any initial state, while the quantum discord has been identified only for particular cases.[23−25] Recently, some results on quantum discord have been obtained for a certain set of a two-qubit X structure density matrix,[26,27] and it is an independent mea-

*Project supported by the Natural Science Foundation of Hunan Province of China (Grant No. 09JJ6011) and the Natural Science Foundation of Education Department of Hunan Province, China (Grant Nos. 08A055 and 07C528).

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surement of quantum correlation.

According to Ollivier and Zurek,[18] quantum discord is identified by von Neumann-type measurements. $\{B_l\}$ is a measurement which is performed locally on system B; then the quantum state based on the l-th measurement becomes $\rho_l = [(I \otimes B_l)\,\rho\,(I \otimes B_l)]/p_l$ with the probability $p_l = \mathrm{tr}\,[(I \otimes B_l)\,\rho]$,[1] where I is the identity operator for system A. After the l-th measurement, the quantum conditional entropy becomes $S(\rho|\{B_l\}) = \sum_l p_l S(\rho_l)$.[22] Furthermore, the quantum mutual information based on this measurement can be defined as $I(\rho|\{B_l\}) = S(\rho^A) - S(\rho|\{B_l\})$, and the classical correlation is written as $C(\rho) = \max_{\{B_l\}} I(\rho|\{B_l\})$,[21−27] which indicates that the maximization is to be done for all possible von Neumann measurements of system B. Thus the quantum discord is defined as

$$Q(\rho) = I(\rho) - C(\rho). \tag{1}$$

For a two-qubit X state, we choose the direct product space of two qubits with computation basis $\{|1\rangle = |\downarrow\rangle_A|\downarrow\rangle_B,\ |2\rangle = |\downarrow\rangle_A|\uparrow\rangle_B,\ |3\rangle = |\uparrow\rangle_A|\downarrow\rangle_B,\ |4\rangle = |\uparrow\rangle_A|\uparrow\rangle_B\}$. The density matrix is written as

$$\rho = \begin{pmatrix} a & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ w^* & 0 & 0 & d \end{pmatrix}. \tag{2}$$

The main diagonal elements are $a = \rho_{11}$, $b = \rho_{22}$, $c = \rho_{33}$, and $d = \rho_{44}$ satisfying $a + b + c + d = 1$, the vice diagonal elements are $w = \rho_{14}$ and $z = \rho_{23}$, and the other elements are zero. Thus the quantum mutual information is given by

$$I(\rho) = S(\rho^A) + S(\rho^B) + \sum_{j=1}^{4} \lambda_j \log_2 \lambda_j, \tag{3}$$

where $S(\rho^A)$ and $S(\rho^B)$ are the von Neumann entropies of systems A and B, and λ_j $(= 1, 2, 3, 4)$ are the eigen-values of the density matrix ρ obtained easily by linear algebra. In order to obtain the same results for measuring system A, the qualification of quantum discord has been given under some assumptions for these states having X structure.[27−29]

To further understand the relationships of these correlations, we derive two explicit expressions for quantum discord and classical correlation for any two-qubit X state and generalize previous studies. Then the expressions are applied to some examples to explain the relationships among the entanglement, the classical correlation, and the quantum discord. A novel analytical method based on the expressions of quantum discord and classical correlation is developed, which is different from previous approaches, to analyse and verify that quantum discord is indeed different from entanglement.

2. Quantum discord and classical correlation

We present a detailed derivation of the formulas of quantum discord and classical correlation for the above X structure density matrix, which is often utilized in quantum information theory and computing. In the following, we consider the projective measurement on systems B and A separately, and calculate the classical correlation $C(\rho)$; then according to a general procedure the quantum discord is computed.[22]

The von Neumann measurement for the system B is written as $\Pi_l^B(\theta,\varphi) = I \otimes |l\rangle\langle l|$ $(l = 1, 2)$ with the orthogonal states

$$|1\rangle = \cos\left(\frac{\theta}{2}\right)|\uparrow\rangle + \mathrm{e}^{\mathrm{i}\,\varphi}\sin\left(\frac{\theta}{2}\right)|\downarrow\rangle,$$

and

$$|2\rangle = \sin\left(\frac{\theta}{2}\right)|\uparrow\rangle - \mathrm{e}^{\mathrm{i}\,\varphi}\cos\left(\frac{\theta}{2}\right)|\downarrow\rangle.$$

From Eq. (1), the quantum discord should take the minimum value and can be written as

$$Q(\rho) = \min_{\{\theta,\varphi\}}\left\{\sum_{l=1,2} S(\rho^B) - S(\rho^{AB}) + p_l S\left(\frac{\mathrm{tr}_B\left[\Pi_l^B(\theta,\varphi)\,\rho\,\Pi_l^B(\theta,\varphi)\right]}{p_l}\right)\right\}, \tag{4}$$

where $p_l = \mathrm{tr}\left[\Pi_l^B(\theta,\varphi)\,\rho\right]$. Noting that in Eq. (4), the first and second terms on the right-hand side are fixed, we mainly solve the minimum value of the third term. Firstly, substituting $\Pi_l^B(\theta,\varphi) = I \otimes |l\rangle\langle l|$ into Eq. (4), we can obtain the quantum discord expression with two parameters θ and φ. Secondly, taking the partial derivative of the resulting function $Q(\rho)$ with respect to θ and φ separately and making them equal to zero, the extreme value points can be obtained. Finally, we assume that the second partial derivatives of these extreme value points are greater than zero, and that these points have symmetry and infinite numbers; after inserting them into the third term of Eq. (4), it only has two extreme values defined as M and N because of periodic function.

We can take the minimum value of the third part in Eq. (4) according to the definition of quantum discord. Therefore, as long as we compare the defined quantities M and N, the formula for quantum discord can be easily obtained, and the classical correlation is thus acquired by Eq. (1). Through the above analysis, we can obtain the quantum discord for measuring system B as follows:

$$Q_B(\rho) = \min\{Q_1(\rho), Q_2(\rho)\}, \tag{5}$$

where

$$Q_1(\rho) = S(\rho^B) - S(\rho^{AB}) - M,$$

$$Q_2(\rho) = S(\rho^B) - S(\rho^{AB}) - N.$$

The two quantities M and N are defined by

$$\begin{aligned} M = \sum_{m=1,2} \Bigg\{ & (a+c)\left\{\frac{1}{2}\left(1+(-1)^m\frac{|a-c|}{a+c}\right)\right. \\ & \left.\times \log_2\left[\frac{1}{2}\left(1+(-1)^m\frac{|a-c|}{a+c}\right)\right]\right\} \\ & + (b+d)\left\{\frac{1}{2}\left(1+(-1)^m\frac{|b-d|}{b+d}\right)\right. \\ & \left.\times \log_2\left[\frac{1}{2}\left(1+(-1)^m\frac{|b-d|}{b+d}\right)\right]\right\}\Bigg\}, \end{aligned} \tag{6}$$

$$\begin{aligned} N = \sum_{n=1,2} \Bigg\{ & \frac{1+(-1)^n\sqrt{(a+b-c-d)^2+4(|w|+|z|)^2}}{2} \\ & \times \log_2\left[\frac{1+(-1)^n\sqrt{(a+b-c-d)^2+4(|w|+|z|)^2}}{2}\right]\Bigg\}. \end{aligned} \tag{7}$$

Making use of Eq. (1), the classical correlation is also acquired as follows:

$$C_B(\rho) = \max\{C_1(\rho), C_2(\rho)\}, \tag{8}$$

with $C_1(\rho) = S(\rho^A) + M$ and $C_2(\rho) = S(\rho^A) + N$.

On the other hand, the von Neumann measurement for the system A is also written as $\Pi_l^A(\theta,\varphi) = I \otimes |l\rangle\langle l|$ $(l = 1, 2)$; applying the same analysis, the quantum discord and classical correlation are obtained as follows:

$$\begin{aligned} Q_A(\rho) &= \min\{Q_3(\rho), Q_4(\rho)\}, \\ C_A(\rho) &= \max\{C_3(\rho), C_4(\rho)\}, \end{aligned} \tag{9}$$

where

$$Q_3(\rho) = S(\rho^A) - S(\rho^{AB}) - R,$$

$$Q_4(\rho) = S(\rho^A) - S(\rho^{AB}) - T,$$

and

$$C_3(\rho) = S(\rho^B) + R,$$
$$C_4(\rho) = S(\rho^B) + T.$$

The two quantities R and T are defined by

$$\begin{aligned} R = \sum_{r=1,2} \Bigg\{ & (a+b)\left\{\frac{1}{2}\left(1+(-1)^r\frac{|a-b|}{a+b}\right)\right. \\ & \left.\times \log_2\left[\frac{1}{2}\left(1+(-1)^r\frac{|a-b|}{a+b}\right)\right]\right\} \\ & + (c+d)\left\{\frac{1}{2}\left(1+(-1)^r\frac{|c-d|}{c+d}\right)\right. \\ & \left.\times \log_2\left[\frac{1}{2}\left(1+(-1)^r\frac{|c-d|}{c+d}\right)\right]\right\}\Bigg\}, \end{aligned} \tag{10}$$

$$\begin{aligned} T = \sum_{t=1,2} \Bigg\{ & \frac{1+(-1)^t\sqrt{(a-b+c-d)^2+4(|w|+|z|)^2}}{2} \\ & \times \log_2\left[\frac{1+(-1)^t\sqrt{(a-b+c-d)^2+4(|w|+|z|)^2}}{2}\right]\Bigg\}. \end{aligned} \tag{11}$$

The expressions of quantum discord and classical correlation are indeed different for measuring the systems A and B. In order to obtain the same results for measuring systems B and A, $a = d$ or $b = c$ is assumed.[22,27] Therefore, the quantum discord and the classical correlation are denoted by $Q(\rho)$ and $C(\rho)$ because of $Q_B(\rho) = Q_A(\rho)$ and $C_B(\rho) = C_A(\rho)$ for this kind of X structure density matrix. Our study

can be extended to a general X structure density matrix and we only take the same value for measuring the systems A and B for any X structure density matrix in our discussion.

A general case

$$\rho = \frac{x}{3}\left|\Psi^+\right\rangle\left\langle\Psi^+\right| + \frac{3-x}{6}\left|10\right\rangle\left\langle 10\right| + \frac{1}{6}\left|11\right\rangle\left\langle 11\right| + \frac{2-x}{6}\left|01\right\rangle\left\langle 01\right|$$

is considered, where $|\Psi^+\rangle = (|1\rangle + |4\rangle)/\sqrt{2}$. Therefore, we can obtain $a = x/6$, $b = (2-x)/6$, $c = (3-x)/6$, $d = (1+x)/6$, $w = x/6$ and $z = 0$, all of which are different. Substituting them into Eqs. (5), (8), and (9), the results for measuring the systems A and B are obtained numerically and shown in Fig. 1. The entanglement measured by concurrence[12,30] is

$$E(\rho) = 2\max\left\{0, \frac{x - \sqrt{(3-x)(2-x)}}{6}\right\}. \quad (12)$$

From Fig. 1, we can obtain the same results for measuring systems A and B in most regimes except from 0.6872 to 1. We only take the regime from 0 to 0.6872 and generalize the previous study to any two-qubit X state. From Eqs. (6) and (9) (or Eq. (10)), these equations are simple and clear and may be regarded as akin to the formulas of entanglement.[30] The quantum discord is computed as $Q_B(\rho)$ and $Q_A(\rho)$ for any X structure density matrix and taken to be the same value between $Q_B(\rho)$ and $Q_A(\rho)$. When parameter $a\,(b)$ is equal to $d\,(c)$, the quantum discord is expressed as $Q(\rho) = \min\{Q_1(\rho),\ Q_2(\rho)\}$.

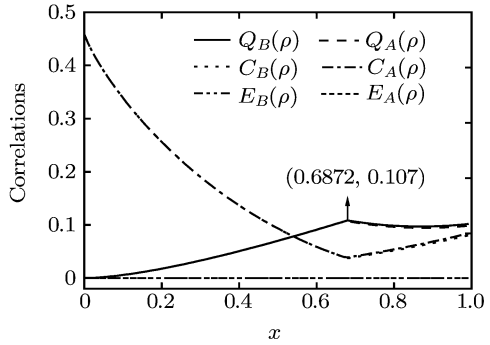


Fig. 1. Evolutions of quantum discord, classical correlation, and entanglement with parameter x for a general density matrix.

3. Examples

Next, we apply Eqs. (6)–(11) to some concrete cases concerning their quantum and classical correlations. The relationships among classical correlation, quantum discord and entanglement are compared in different cases theoretically and numerically.

If two Bell states are combined with any probability such as

$$\rho = x\left|\Phi^+\right\rangle\left\langle\Phi^+\right| + (1-x)\left|\Psi^+\right\rangle\left\langle\Psi^+\right|$$

with

$$\left|\Phi^\pm\right\rangle = (|2\rangle \pm |3\rangle)/\sqrt{2},$$

we can know $a = d = x/2$ and $Q_A = Q_B$ for this density operator, and quantities Q_1 and Q_2 are

$$Q_1(\rho) = 1 + x\log_2 x + (1-x)\log_2(1-x) - M,$$
$$Q_2(\rho) = 1 + x\log_2 x + (1-x)\log_2(1-x) - N, \quad (13)$$

where the parameters M and N for this case are easily acquired as

$$M = \frac{1+|2x-1|}{2}\log_2\left(\frac{1+|2x-1|}{2}\right) + \frac{1-|2x-1|}{2}\log_2\left(\frac{1-|2x-1|}{2}\right),$$
$$N = 0. \quad (14)$$

Taking minimum values of Q_1 and Q_2, we compare only two parameters M and N, which are shown in Fig. 2. From Fig. 2, quantity M is less than zero and $N = 0$. For classical correlation, quantities $C_1(\rho)$ and $C_2(\rho)$ are $1+M$ and 1; taking the maximum value, the classical correlation is given by $C(\rho) = 1$. The quantum discord can be expressed as $Q(\rho) = Q_2(\rho)$, and it is evident that the latter two terms of $Q_2(\rho)$ are both less than zero, thus the classical correlation is larger than the quantum discord in the regime from 0 to 1. In addition, the entanglement measured by concurrence is expressed as $E(\rho) = \max\{0, 2x-1, 1-2x\}$, which is smaller than the classical correlation and larger than the quantum discord obtained by the relationship of linear and logarithmic functions. In order to understand this more clearly, these three correlations are plotted in Fig. 3, it is clearly seen that the quantum discord and entanglement are symmetric about $x = 1/2$. When the probability varies from 0 to 1/2, the quantum discord and entanglement decrease *vice versa* in a range from 1/2 to 1, while the classical correlation is a constant in the whole regime. As a result, the quantum mutual information follows a similar law to the quantum discord; the analytical results accord with the previous conclusion.[19,20]

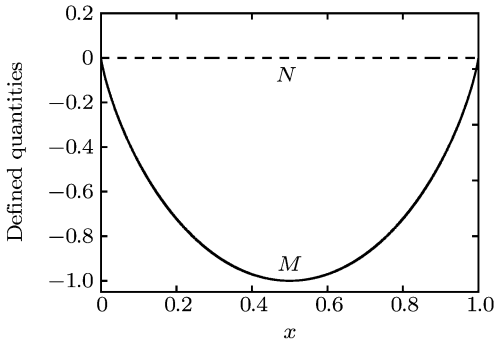


Fig. 2. Evolution of quantity M with the probability x and $N = 0$. M is less than N for any probability apart from two endpoints.

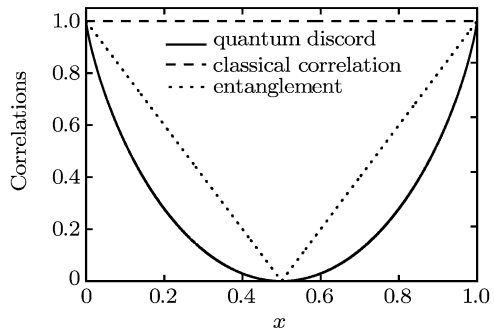


Fig. 3. Evolutions of quantum discord, entanglement, and classical correlation with parameter x for two Bell states combined with any probability, such as $\rho = x\left|\Phi^+\right\rangle\left\langle\Phi^+\right| + (1-x)\left|\Psi^+\right\rangle\left\langle\Psi^+\right|$.

As the second example, we construct an initial state which is defined by

$$\rho = \frac{1-x}{3}\left|10\right\rangle\left\langle 10\right| + \frac{2}{3}\left|\Psi^+\right\rangle\left\langle\Psi^+\right| + \frac{x}{3}\left|01\right\rangle\left\langle 01\right|.$$

We can have that the parameter b is equal to c, and the quantities M and N are computed as

$$\begin{aligned} M =& \frac{1}{3}\log_2\left(\frac{1}{2-x}\right) + \frac{1-x}{3}\log_2\left(\frac{1-x}{2-x}\right) \\ &+ \frac{1}{3}\log_2\left(\frac{1}{1+x}\right) + \frac{x}{3}\log_2\left(\frac{x}{1+x}\right), \\ N =& \sum_{n=1,2}\frac{3+(-1)^n\sqrt{4+(2x-1)^2}}{6} \\ &\times \log_2\left[\frac{3+(-1)^n\sqrt{4+(2x-1)^2}}{6}\right]. \end{aligned} \quad (15)$$

The two curves are calculated by numerical simulation as shown in Fig. 4, which depicts the tendency of M and N varying with x respectively. It is found that the quantity M remains less than N for any x ranging from 0 to 1. According to Eqs. (6) and (9), the quantum discord and classical correlation are obtained as follows:

$$\begin{aligned} Q(\rho) =& -\frac{2-x}{3}\log_2\left(\frac{2-x}{3}\right) \\ &- \frac{1+x}{3}\log_2\left(\frac{1+x}{3}\right) + \frac{2}{3}\log_2\frac{2}{3} \\ &+ \sum_{j=1,2}\frac{1+(-1)^j\left|2x-1\right|}{6} \\ &\times \log_2\left[\frac{1+(-1)^j\left|2x-1\right|}{6}\right] - N, \\ C(\rho) =& -\frac{2-x}{3}\log_2\left(\frac{2-x}{3}\right) \\ &- \frac{1+x}{3}\log_2\left(\frac{1+x}{3}\right) + N. \end{aligned} \quad (16)$$

The concurrence can be expressed by

$$E(\rho) = \max\left\{0, \frac{1-\sqrt{x(1-x)}}{3}\right\}.$$

For this initial state, the relationships among quantum discord, classical correlation, and entanglement are similar to those for a state

$$\rho = \frac{2}{3}\left|\Phi^+\right\rangle\left\langle\Phi^+\right| + \frac{1-x}{3}\left|00\right\rangle\left\langle 00\right| + \frac{x}{3}\left|11\right\rangle\left\langle 11\right|$$

by numerical simulation,[27] but our analytical method is convenient and completely different.

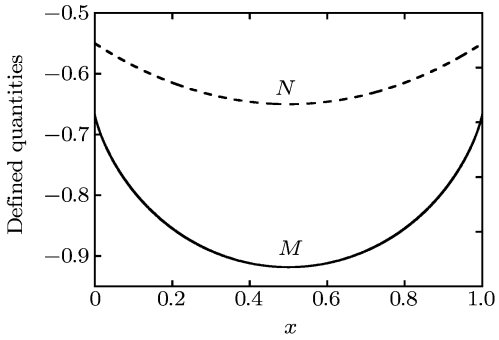


Fig. 4. Variations of quantities M and N with variable x. M is smaller than N for any variable x corresponding to any probability distribution.

The final example is

$$\begin{aligned} \rho =& \frac{1}{3}\left|\Phi^+\right\rangle\left\langle\Phi^+\right| + \frac{1+x}{6}\left|01\right\rangle\left\langle 01\right| \\ &+ \frac{3-x}{6}\left|11\right\rangle\left\langle 11\right|. \end{aligned} \quad (17)$$

For this case, all the main diagonal elements are different, thus we need to compute $Q_B(\rho)$ and $Q_A(\rho)$. First for measuring system B, we can only compare quantities M and N, and then for system A, quantities R and T are compared; the four defined parameters can be calculated as follows:

$$M = \sum_{m=1,2}\frac{5}{12}\left[1+(-1)^m\frac{\left|2x-1\right|}{5}\right]$$

$$\times \log_2 \left[\frac{1}{2}\left(1+(-1)^m \frac{|2x-1|}{5}\right)\right],$$

$$N = \sum_{n=1,2} \frac{1}{2}\left[1+(-1)^n \frac{\sqrt{1+(x-1)^2}}{3}\right] \times \log_2 \left\{\frac{1}{2}\left[1+(-1)^n \frac{\sqrt{1+(x-1)^2}}{3}\right]\right\}, \quad (18)$$

and

$$R = \sum_{r=1,2} \frac{4-x}{12}\left[1+(-1)^r \frac{2-x}{4-x}\right] \times \log_2 \left[\frac{1}{2}\left(1+(-1)^r \frac{2-x}{4-x}\right)\right],$$

$$T = \sum_{t=1,2} \frac{1}{2}\left[1+(-1)^t \frac{\sqrt{5}}{3}\right] \times \log_2 \left\{\frac{1}{2}\left[1+(-1)^t \frac{\sqrt{5}}{3}\right]\right\}. \quad (19)$$

We can obtain $M > N$ and $R > T$ numerically as shown in Fig. 5. The quantum discord can be expressed as $Q_B(\rho) = S(\rho^B) - S(\rho^{AB}) - M$ and $Q_A(\rho) = S(\rho^A) - S(\rho^{AB}) - R$; we only take the same value between $Q_B(\rho)$ and $Q_A(\rho)$, and the entanglement is computed as $E(\rho) = 1/3$. The results of quantum discord, classical correlation, and entanglement changing with variable x are depicted in Fig. 6, from which we surprisingly find that the quantum discord $Q_B(\rho)\,(C_B(\rho))$ is completely consistent with $Q_A(\rho)\,(C_A(\rho))$. The classical correlation is smaller than the quantum discord in a range from 0 to 0.6, which means that the quantum correlation is larger than the classical correlation *vice versa* in a range from 0.6 to 1; the quantum mutual information increases continuously. From the above discussion, quantum discord and entanglement are two different methods of measurement in describing quantum correlation for an X structure state.

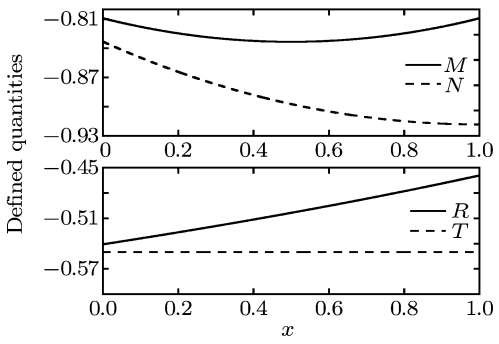


Fig. 5. Evolution of the four defined quantities M, N, R, and T with variable x for initial condition Eq. (17).

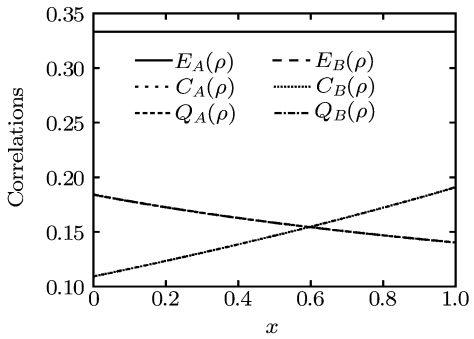


Fig. 6. Evolution of quantum discord, classical correlation, and entanglement with variable x for initial condition Eq. (17).

4. Conclusions

We have given general expressions for quantum discord and classical correlation for an X structure density matrix, which are similar to the formula for entanglement. These expressions are extended to any X structure density matrix containing seven parameters.[22,27] Based on the features of quantum discord and classical correlation expressions, the relationships among quantum discord, entanglement, and classical correlation are investigated by another analytical method. The calculations of quantum discord and classical correlation depend mainly on two quantities $M(R)$ and $N(T)$ under the assumption $a = d$ or $b = c$. When all the main diagonal elements are different, the same value between $Q_A(\rho)$ and $Q_B(\rho)$ is adopted. The quantum discord $Q_B(\rho)$ corresponds to a smaller value for M and N, while $Q_A(\rho)$ corresponds to a smaller value for R and T. For classical correlation, the method is similar and the only difference is in the adoption of the maximum value. Our results show that the quantum discord is not always larger than the entanglement measured by concurrence in a general two-qubit X state. In addition, these formulas may be applied to any two coupled or uncoupled physical systems having an X structure density matrix, such as a system initially prepared in a mixed state. Thus the expressions for quantum discord and classical correlation will become more universal. Further applications of our expressions involving good analytic approximations will enable more detailed comparisons to be made.

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