The interaction of nonlinear waves in two-dimensional dust crystals*

Jiang Hong(姜 虹), Yang Xiao-Xia(杨晓霞), Lin Mai-Mai(林麦麦),
Shi Yu-Ren(石玉仁), and Duan Wen-Shan(段文山)†

College of Physics and Electronic Engineering, Northwest Normal University, Lanzhou 730070, *China*

(Received 19 July 2010; revised manuscript received 9 August 2010)

This paper investigates the collision between two nonlinear waves with different propagation directions in two-dimensional dust crystals. Using the extended Poincaré–Lighthill-Kuo perturbation method, two Korteweg–de Vries equations for nonlinear waves in both the ξ and η directions are obtained, respectively, and the analytical phase shifts and trajectories after the collision of two nonlinear waves are derived. Finally, the effects of parameters of the lattice constant a, the arbitrary constant $u_{0\eta}$, the forces $f(r)$, and the colliding angle θ on the phase shifts of both colliding nonlinear waves are examined.

Keywords: nonlinear waves, two-dimensional hexagonal crystal, phase shift
PACS: 94.20.Wj, 52.27.Lw, 05.50.+q **DOI:** 10.1088/1674-1056/20/1/019401

1. Introduction

Dust crystals are strongly coupled dusty plasmas. They can be found typically in plasma discharge experiments due to the strong electrostatic interaction between massive, heavily charged, micron-sized dust particulates (dust grains) injected into the plasma.[1,2] The formation and dynamics of dust crystals have been studied experimentally and theoretically,[2−10] where dust particles were essentially created by injecting artificial microspheres, which subsequently acquired a high electron charge via inherent dynamic charging mechanisms. Such dust crystals typically have a two-dimensional (2D) hexagonal structure, although various two- and three-dimensional (3D) configurations have also been reported.[2] One-dimensional (1D) dust crystals have already been fabricated in experiments by making use of appropriate confinement potentials.[11,12]

Linear and nonlinear waves in a 2D hexagonal crystal system consisting of a monolayer of charged particles have been studied recently. This system can sustain longitudinal waves and two kinds of transverse waves, where the latter can be classified according to whether the particle motion remains in plane or is out of plane. The longitudinal wave is a compressional wave. Its propagation direction is parallel to that of the particle motion. The transverse wave, sometimes called the shear wave, is a wave propagating perpendicular to the particle motion. In the linear wave regime, i.e., the regime where wave amplitude is much smaller than the mean inter-particle spacing, both longitudinal and transverse modes have been observed in several experiments.[13,14] Certain theoretical modes have also been proposed to predict the waves in dusty plasmas.[15−17] Recently, some theoretical and experimental results have shown that longitudinal and transverse waves can propagate in the plasma crystal.[18,19] Duan *et al.*[10] have investigated longitudinal and transverse dust grain vibrations in a 2D hexagonal lattice by considering screened Coulomb interactions (Debye–Hückel or Yukawa system) between charged dust particles, i.e., $u(r) = Q^2 \exp(-r/\lambda_{\rm D})/(4\pi\epsilon_0 r)$, where r is the distance between two dust particles and $\lambda_{\rm D}$ is the Debye radius (ϵ_0 denotes the electric susceptibility of a vacuum). Let us consider a 'dressed' Debye-type interaction potential energy, namely, $u(r) = Q^2(1 - r/\lambda_{\rm D})\exp(-r/\lambda(D))/4\pi\epsilon_0 r$. The electrostatic binary interaction force $f(r)$ exerted onto two dust grains situated at a distance r is derived from a potential function $u(r)$, viz. $f(r) = -\partial u(r)/\partial r$. We may expand the potential energy around equilibrium at $r = a$, namely

$$u(r) = u(a) + (r-a)\frac{\partial u}{\partial r}|_{r=a} + \frac{1}{2}(r-a)^2\frac{\partial^2 u}{\partial r^2}|_{r=a} + \ldots.$$

By defining the 'spring' constant $G = (\partial^2 u/\partial r^2)|_{r=a}$,

*Project supported by the National Natural Science Foundation of China (Grant No. 10875098) and the Scientific and Technical Innovation Foundation of Northwest Normal University (Grant No. NWNU-KJCXGC-0348).
†Corresponding author. E-mail: duanws@ nwnu.edu.cn
© 2011 Chinese Physical Society and IOP Publishing Ltd

and setting the potential energy at equilibrium to zero, we have $u(r) \cong \frac{1}{2}G(r-a)^2$. We have calculated G, for a Yukawa system $u(r)$, the corresponding expressions

$$G = \frac{2Q^2}{4\pi\epsilon_0\lambda_{\rm D}^3}\frac{1+\kappa+\kappa^2/2}{\kappa^3}e^{-\kappa},$$

where we have defined the (dimensionless) lattice parameter $\kappa = a/\lambda_{\rm D}$, the normalized potential energy $u(r)/(Q^2/4\pi\epsilon_0\lambda_{\rm D}^3)$. We should note that studies of the interaction between nonlinear waves have been frequently reported in recent years.[20−22] In a 1D (or quasi-1D) system, there are two distinct interactions. One is the overtaking collision and the other is the head-on collision.[21−23] Until now, however, few theoretical studies have been carried out on a 2D dust crystal. Hence, in this paper, based on a 1D model, we study the nonlinear waves propagating in a 2D hexagonal crystal. In 2D systems, the general case is $0 \leq \theta \leq \pi$; the overtaking collision and head-on collision are only two special cases of $\theta = 0$ and $\theta = \pi$, respectively, where θ is the angle between two propagation directions of two nonlinear waves. In order to investigate the general case, we use the extended Poincaré–Lighthill–Kuo (PLK) perturbation method and obtain two KdV equations to describe nonlinear waves propagating in two different directions respectively. Moreover, we can obtain their phase shifts and trajectories after collision of two nonlinear waves.

The paper is organized as follows. In Section 2, the basic set of fluid equations is presented. We derive the equation of motion for nonlinear waves propagating in two different directions and obtain both the phase shifts and trajectories after interaction of two nonlinear waves. In Section 3, we give a discussion and a summary of our conclusions.

2. Derivation of the KdV equation and phase shift

Let us consider a 2D hexagonal crystal (assumed infinite, for simplicity) consisting of negative dust grains (of constant charge Q and mass M), located at equidistant sites a. For the analysis of the waves in this 2D crystalline monolayer, we use the so-called 'particle string' model, allowing for 2D motion, in the longitudinal (horizontal, along the x axis) and transverse (vertical) directions, the corresponding discrete ordering dust grains being denoted by indices n and m, respectively. Figure 1 shows the monolayer hexagonal lattice in two dimensions.

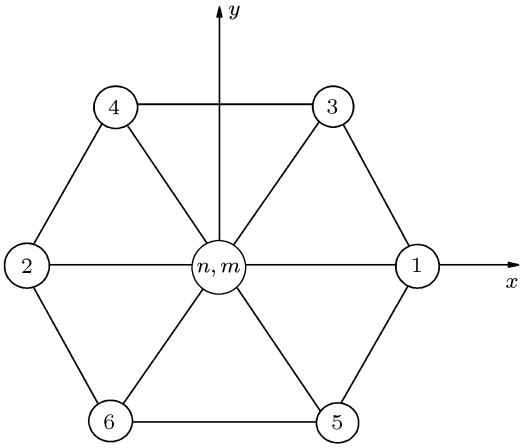


Fig. 1. Monolayer hexagonal lattice in two dimensions.

We now want to find the equation of motion for an arbitrary particle (n,m)th. We consider the forces exerted on the particle (n,m)th produced only by the six nearest particles. Under the assumption of small amplitude and long wavelength limit, for this system the basic equations of motion in the x and y directions are, respectively,[10]

$$\begin{aligned}
M\frac{\partial^2 u}{\partial t^2} &= A_1\frac{\partial^2 u}{\partial x^2} + A_2\frac{\partial^2 u}{\partial y^2} + A_3\frac{\partial^2 v}{\partial x\partial y} + A_4\frac{\partial^4 u}{\partial x^4} + A_5\frac{\partial^4 u}{\partial y^4} + A_6\frac{\partial^4 u}{\partial x^2\partial y^2} + A_7\frac{\partial^4 v}{\partial x^3\partial y} \\
&\quad + A_8\frac{\partial^4 v}{\partial x\partial y^3} + A_9\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + A_{10}\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x^2} + A_{11}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + A_{12}\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial y^2} \\
&\quad + A_{13}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y} + A_{14}\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial x\partial y} + A_{15}\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x^2} + A_{16}\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + A_{17}\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial x^2} \\
&\quad + A_{18}\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2} + A_{19}\frac{\partial u}{\partial x}\frac{\partial^2 v}{\partial x\partial y} + A_{20}\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x\partial y}, \qquad (1) \\
M\frac{\partial^2 v}{\partial t^2} &= B_1\frac{\partial^2 v}{\partial x^2} + B_2\frac{\partial^2 v}{\partial y^2} + B_3\frac{\partial^2 u}{\partial x\partial y} + B_4\frac{\partial^4 v}{\partial x^4} + B_5\frac{\partial^4 v}{\partial y^4} + B_6\frac{\partial^4 v}{\partial x^2\partial y^2} + B_7\frac{\partial^4 u}{\partial x^3\partial y} \\
&\quad + B_8\frac{\partial^4 u}{\partial x\partial y^3} + B_9\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + B_{10}\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2} + B_{11}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x^2} + B_{12}\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial x^2}
\end{aligned}$$

$$+B_{13}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x\partial y}+B_{14}\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x\partial y}+B_{15}\frac{\partial u}{\partial x}\frac{\partial^2 v}{\partial x^2}+B_{16}\frac{\partial u}{\partial x}\frac{\partial^2 v}{\partial y^2}+B_{17}\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x\partial y}$$
$$+B_{18}\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x^2}+B_{19}\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2}+B_{20}\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial x\partial y}. \quad (2)$$

In Eq. (1), the coefficients $A_1, A_2, \ldots, A_{20}$ are given by

$$\begin{pmatrix} A_1 & A_2 & A_3 & A_4 \\ A_5 & A_6 & A_7 & A_8 \\ A_9 & A_{10} & A_{11} & A_{12} \\ A_{13} & A_{14} & A_{15} & A_{16} \\ A_{17} & A_{18} & A_{19} & A_{20} \end{pmatrix} = \begin{pmatrix} \frac{9}{16} & \frac{3}{16} & \frac{3}{8} & \frac{11}{256}a^2 \\ \frac{3}{256}a^2 & \frac{3}{128}a^2 & \frac{1}{64}a^2 & \frac{3}{64}a^2 \\ -\frac{3}{8} & \frac{3}{8} & \frac{3}{8} & -\frac{3}{8} \\ \frac{3}{4} & -\frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} a^2 f'(a) + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{33}{96} & \frac{1}{32} & \frac{1}{32} & \frac{13}{32} \\ \frac{1}{16} & \frac{3}{16} & \frac{1}{16} & \frac{1}{32} \\ \frac{3}{32} & \frac{1}{16} & \frac{1}{32} & \frac{3}{32} \end{pmatrix} a^3 f''(a).$$

In Eq. (2), the coefficients $B_1, B_2, \ldots, B_{20}$ are given by

$$\begin{pmatrix} B_1 & B_2 & B_3 & B_4 \\ B_5 & B_6 & B_7 & B_8 \\ B_9 & B_{10} & B_{11} & B_{12} \\ B_{13} & B_{14} & B_{15} & B_{16} \\ B_{17} & B_{18} & B_{19} & B_{20} \end{pmatrix} = \begin{pmatrix} \frac{3}{16} & \frac{9}{16} & \frac{3}{8} & \frac{1}{256}a^2 \\ \frac{9}{256}a^2 & \frac{9}{128}a^2 & \frac{1}{64}a^2 & \frac{3}{64}a^2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{8} & -\frac{3}{8} \\ -\frac{3}{4} & \frac{3}{8} & -\frac{3}{8} & -\frac{3}{4} \end{pmatrix} a^2 f'(a) + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{3}{32} & \frac{9}{32} & \frac{1}{32} & \frac{3}{32} \\ \frac{1}{16} & \frac{3}{16} & \frac{1}{32} & \frac{3}{32} \\ \frac{3}{16} & \frac{1}{32} & \frac{3}{32} & \frac{3}{16} \end{pmatrix} a^3 f''(a).$$

In the above equations, u and v are the displacements of the particles from their equilibrium positions in the x and y directions, respectively, a is the lattice constants with M the mass of the atom, $f'(a) = \partial f(r)/\partial r\,|_{r=a}$, $f''(a) = \partial^2 f(r)/\partial^2 r\,|_{r=a}$, the electrostatic binary interaction force $f(r)$ exerted onto two dust grains situated at a distance r is derived from a potential function $u(r)$, namely $f(r) = \partial u(r)/\partial r$, and the other notations have their usual meaning.

We assume that there are two nonlinear waves 1 and 2 in the crystal, asymptotically, which are traveling with an arbitrary angle at the initial state. After some time they interact, collide, and then separate. We also assume that nonlinear waves have small amplitudes and the interactions between nonlinear waves are weak. Hence, we expect that the collision will be quasi-elastic, so it will cause phase shifts of the post-collision trajectories (phase shift). In order to analyse the effects of the collision, we employ an extended PLK perturbation method.[23−25] This method is a combination of the standard reductive perturbation method[26,27] and the technique of strained coordinates. The main idea of this perturbation method is as follows. In the limits of the long wavelength approximation, asymptotic expansions for both the flow field variables and the spatial or time coordinates are used. This makes a uniformly valid asymptotic expansion (i.e., eliminates secular terms) and at the same time obtains the changes of the trajectories (i.e., phase shifts) of the nonlinear waves after the collision. According to this method, we introduce the stretched coordinates

$$\xi = \epsilon(k_1 x + l_1 y - c_1 t) + \epsilon^2 P^{(0)}(\eta,\tau) + \epsilon^3 P^{(1)}(\xi,\eta,\tau) + \ldots, \quad (3)$$
$$\eta = \epsilon(k_2 x + l_2 y - c_2 t) + \epsilon^2 Q^{(0)}(\xi,\tau) + \epsilon^3 Q^{(1)}(\xi,\eta,\tau) + \ldots, \quad (4)$$
$$\tau = \epsilon^3 t, \quad (5)$$

where ξ and η denote the trajectories of the two nonlinear waves, respectively, c_1 and c_2 are the propagation speeds of two nonlinear waves in different directions. We assume that c_1 and c_2 are constants, and the variables $P^{(j)}$ and $Q^{(j)}(j = 0, 1, \ldots)$ are to be determined. Introducing the asymptotic expansions for the quantities u and v as $u = \epsilon u_1 + \epsilon^2 u_2 + \ldots, v = \epsilon v_1 + \epsilon^2 v_2 + \ldots$, then substituting the expansions into Eqs. (1) and (2), and equating the powers of ϵ, we obtain a hierarchy of coupled equations in different orders of ϵ. To leading order, we have

$$\left(c_1^2\frac{\partial^2 u_1}{\partial\xi^2}+2c_1c_2\frac{\partial^2 u_1}{\partial\xi\partial\eta}+c_2^2\frac{\partial^2 u_1}{\partial\eta^2}\right) = \frac{A_1}{M}\left(k_1^2\frac{\partial^2 u_1}{\partial\xi^2}+2k_1k_2\frac{\partial^2 u_1}{\partial\xi\partial\eta}+k_2^2\frac{\partial^2 u_1}{\partial\eta^2}\right) + \frac{A_2}{M}\left(l_1^2\frac{\partial^2 u_1}{\partial\xi^2}+2l_1l_2\frac{\partial^2 u_1}{\partial\xi\partial\eta}+l_2^2\frac{\partial^2 u_1}{\partial\eta^2}\right) + \frac{A_3}{M}\left(k_1l_1\frac{\partial^2 v_1}{\partial\xi^2}+k_1l_2\frac{\partial^2 v_1}{\partial\xi\partial\eta}+k_2l_1\frac{\partial^2 v_1}{\partial\xi\partial\eta}+k_2l_2\frac{\partial^2 v_1}{\partial\eta^2}\right), \quad (6)$$

$$\left(c_1^2\frac{\partial^2 v_1}{\partial\xi^2}+2c_1c_2\frac{\partial^2 v_1}{\partial\xi\partial\eta}+c_2^2\frac{\partial^2 v_1}{\partial\eta^2}\right) = \frac{B_1}{M}\left(k_1^2\frac{\partial^2 v_1}{\partial\xi^2}+2k_1k_2\frac{\partial^2 v_1}{\partial\xi\partial\eta}+k_2^2\frac{\partial^2 v_1}{\partial\eta^2}\right) + \frac{B_2}{M}\left(l_1^2\frac{\partial^2 v_1}{\partial\xi^2}+2l_1l_2\frac{\partial^2 v_1}{\partial\xi\partial\eta}+l_2^2\frac{\partial^2 v_1}{\partial\eta^2}\right) + \frac{B_3}{M}\left(k_1l_1\frac{\partial^2 u_1}{\partial\xi^2}+k_1l_2\frac{\partial^2 u_1}{\partial\xi\partial\eta}+k_2l_1\frac{\partial^2 u_1}{\partial\xi\partial\eta}+k_2l_2\frac{\partial^2 u_1}{\partial\eta^2}\right). \quad (7)$$

The solutions of Eqs. (6) and (7) read

$$u_1 = u_\xi(\xi,\tau)+u_\eta(\eta,\tau), \quad (8)$$

$$v_1 = \frac{B_3}{A_3}u_\xi(\xi,\tau)+\frac{B_3}{A_3}u_\eta(\eta,\tau). \quad (9)$$

Substituting Eqs. (8) and (9) into Eqs. (6) and (7), we obtain $2c_1^2=(A_1+B_1)k_1^2/M+(A_2+B_2)l_1^2/M+(A_3+B_3)k_1l_1/M, 2c_2^2=(A_1+B_1)k_2^2/M+(A_2+B_2)l_2^2/M+(A_3+B_3)k_2l_2/M$.

The unknown functions u_ξ and u_η will be determined at higher order. Equations (8) and (9) imply that, to leading order, we have two nonlinear waves, one of which, $u_\xi(\xi,\tau)$, is travelling in ξ direction, and the other, $u_\eta(\eta,\tau)$, is travelling in η direction. At the next order, the solutions also have the same shape, as follows:

$$u_2 = v_\xi(\xi,\tau)+v_\eta(\eta,\tau), \quad (10)$$

$$v_2 = \frac{B_3}{A_3}v_\xi(\xi,\tau)+\frac{B_3}{A_3}v_\eta(\eta,\tau), \quad (11)$$

where v_ξ and v_η are to be determined.

To the next highest order, after tedious calculations, we can deduce

$$\gamma P\frac{\partial^2 u_\xi}{\partial\xi^2}+\gamma Q_\xi^{(0)}\frac{\partial^2 u_\eta}{\partial\eta^2}+4c_1\frac{\partial^2 u_\xi}{\partial\xi\partial\tau}+4c_2\frac{\partial^2 u_\eta}{\partial\eta\partial\tau}+\beta_1 4c_1\frac{\partial^4 u_\xi}{\partial\xi^4}+\beta_2 4c_2\frac{\partial^4 u_\eta}{\partial\eta^4}+\alpha_1 4c_1\frac{\partial u_\xi}{\partial\xi}\frac{\partial^2 u_\xi}{\partial\xi^2}+\alpha_2 4c_2\frac{\partial u_\eta}{\partial\eta}\frac{\partial^2 u_\eta}{\partial\eta^2}+\lambda_1\frac{\partial u_\eta}{\partial\eta}\frac{\partial^2 u_\xi}{\partial\xi^2}+\lambda_2\frac{\partial u_\xi}{\partial\xi}\frac{\partial^2 u_\eta}{\partial\eta^2}=0. \quad (12)$$

In order to avoid spurious resonances, we must eliminate all secular terms.[27,28] Letting $R=\partial u_\xi/\partial\xi$, $S=\partial u_\eta/\partial\eta$, we have

$$\frac{\partial R}{\partial\tau}+\alpha_1 R\frac{\partial R}{\partial\xi}+\beta_1\frac{\partial^3 R}{\partial\xi^3}=0, \quad (13)$$

$$\frac{\partial S}{\partial\tau}+\alpha_2 S\frac{\partial S}{\partial\eta}+\beta_2\frac{\partial^3 S}{\partial\eta^3}=0, \quad (14)$$

$$\gamma P_\eta^{(0)}+\lambda_1\frac{\partial u_\eta}{\partial\eta}=0, \quad (15)$$

$$\gamma Q_\xi^{(0)}+\lambda_2\frac{\partial u_\xi}{\partial\xi}=0, \quad (16)$$

where

$$\gamma=[2(A_1+B_1)k_1k_2+2(A_2+B_2)l_1l_2+(A_3+B_3)(k_1l_2+k_2l_1)]/M-4c_1c_2,$$

$$\lambda_1=[A_9k_1^2k_2+A_{10}k_1^2k_2+A_{11}k_2l_1^2+A_{12}k_2l_1^2+A_{13}k_1l_1l_2+A_{14}k_1l_1l_2+B_{15}k_1^2k_2+B_{16}k_2l_1^2+B_{17}k_1l_1l_2+B_{18}k_1^2k_2+B_{19}k_2l_1^2+B_{20}k_1l_1l_2]/M,$$

$$\lambda_2=[A_9k_1k_2^2+A_{10}k_1k_2^2+A_{11}k_1l_2^2+A_{12}k_1l_2^2+A_{13}k_2l_1l_2+A_{14}k_2l_1l_2+B_{15}k_1k_2^2+B_{16}k_1l_2^2+B_{17}k_1l_1l_2+B_{18}k_1k_2^2+B_{19}k_1l_2^2+B_{20}k_1l_1l_2]/M,$$

$$\alpha_1=[A_9k_1^3+A_{10}k_1^3+A_{11}k_1l_1^2+A_{12}k_1l_1^2+A_{13}k_1l_1^2+A_{14}k_1l_1^2+B_{15}k_1^3+B_{16}k_1l_1^2+B_{17}k_1l_1^2+B_{18}k_1^3+B_{19}k_1l_1^2+B_{20}k_1l_1^2]/4c_1M,$$

$$\alpha_2=[A_9k_2^3+A_{10}k_2^3+A_{11}k_2l_2^2+A_{12}k_2l_2^2+A_{13}k_2l_2^2+A_{14}k_2l_2^2+B_{15}k_2^3+B_{16}k_2l_2^2+B_{17}k_2l_2^2+B_{18}k_2^3+B_{19}k_2l_2^2+B_{20}k_2l_2^2]/4c_2M,$$

$$\beta_1 = [(A_4+B_4)k_1^4 + (A_5+B_5)l_1^4 + (A_6+B_6)k_1^2l_1^2 + (A_7+B_7)k_1^3l_1 + (A_8+B_8)k_1l_1^3]/4c_1M,$$
$$\beta_2 = [(A_4+B_4)k_2^4 + (A_5+B_5)l_2^4 + (A_6+B_6)k_2^2l_2^2 + (A_7+B_7)k_2^3l_2 + (A_8+B_8)k_2l_2^3]/4c_2M.$$

Equations (13) and (14) are the two side traveling wave KdV equations in the reference frames of ξ and η, respectively. Their corresponding solutions are

$$R = r_{m\xi}\mathrm{sech}^2\frac{\xi - u_{0\xi}\tau}{w_\xi} \quad \text{and} \quad S = s_{m\eta}\mathrm{sech}^2\frac{\eta - u_{0\eta}\tau}{w_\eta},$$

respectively, where

$$r_{m\xi} = \frac{3u_{0\xi}}{\alpha_1}, \quad s_{m\eta} = \frac{3u_{0\eta}}{\alpha_2}$$

and

$$w_\xi = \left(\frac{4\beta_1}{u_{0\xi}}\right)^{1/2}, \quad w_\eta = \left(\frac{4\beta_2}{u_{0\eta}}\right)^{1/2}$$

are the amplitudes and widths of two nonlinear waves, respectively. The $u_{0\xi}$ and $u_{0\eta}$ are two arbitrary constants. As a result, we obtain

$$u_\xi = r_{m\xi}w_\xi\tanh\frac{\xi - u_{0\xi}\tau}{w_\xi}, \tag{17}$$

$$u_\eta = s_{m\eta}w_\eta\tanh\frac{\eta - u_{0\eta}\tau}{w_\eta}. \tag{18}$$

We denote u_ξ for the nonlinear wave 1, and u_η for the nonlinear wave 2.

Using Eqs. (13)–(18), we can obtain the phase functions of $P^{(0)}$ and $Q^{(0)}$ as follows:

$$P^{(0)}(\eta,\tau) = -\frac{\lambda_1 s_{m\eta}w_\eta}{\gamma}\tanh\frac{\eta - u_{0\eta}\tau}{w_\eta}, \tag{19}$$

$$Q^{(0)}(\xi,\tau) = -\frac{\lambda_2 r_{m\xi}w_\xi}{\gamma}\tanh\frac{\xi - u_{0\xi}\tau}{w_\xi}. \tag{20}$$

Substituting the above equations into Eqs. (3) and (4), we can obtain the trajectories of two nonlinear waves up to $O(\epsilon^2)$, which are, respectively, given by

$$\xi = \epsilon(k_1x + l_1y - c_1t) - \epsilon^2\frac{\lambda_1 s_{m\eta}w_\eta}{\gamma}\tanh\frac{\eta - u_{0\eta}\tau}{w_\eta} + O(\epsilon^3), \tag{21}$$

$$\eta = \epsilon(k_2x + l_2y - c_2t) - \epsilon^2\frac{\lambda_2 r_{m\xi}w_\xi}{\gamma}\tanh\frac{\xi - u_{0\xi}\tau}{w_\xi} + O(\epsilon^3). \tag{22}$$

At the same time, we can obtain the magnitude of $u(v)$

$$u(v) = \epsilon\left[r_{m\xi}w_\xi\tanh\frac{\xi - u_{0\xi}\tau}{w_\xi} + s_{m\eta}w_\eta\tanh\frac{\eta - u_{0\eta}\tau}{w_\eta}\right] + O(\epsilon^2) + \ldots. \tag{23}$$

In order to obtain the phase shifts after the collision of two nonlinear waves, we let $P_1 = P^{(0)}(\eta,\tau)\ |_{(\eta - u_{0\eta}\tau)\to-\infty}$, which is the phase value of nonlinear wave 1 before the collision, and $P_2 = P^{(0)}(\eta,\tau)\ |_{(\eta - u_{0\eta}\tau)\to\infty}$, which correspond to that of nonlinear wave 1 after the collision. Similarly, we define $Q_1 = Q^{(0)}(\xi,\tau)\ |_{(\xi - u_{0\xi}\tau)\to-\infty}$, the phase value of nonlinear wave 2 before the interaction, and $Q_2 = Q^{(0)}(\xi,\tau)\ |_{(\xi - u_{0\xi}\tau)\to\infty}$, corresponding to that of nonlinear wave 2 after the interaction, using Eqs. (19) and (20) we can obtain their corresponding phase shifts ΔP and ΔQ as follows:

$$\Delta P = P_2 - P_1 = -\frac{2\lambda_1 s_{m\eta}w_\eta}{\gamma}, \tag{24}$$

$$\Delta Q = Q_2 - Q_1 = -\frac{2\lambda_2 r_{m\xi}w_\xi}{\gamma}. \tag{25}$$

3. Discussion

The present study describes the propagation and interaction of two small but finite amplitude wavs in a 2D dust crystal using the extended PLK perturbation method. The effects of parameters of the lattice constant a, the arbitrary constant $u_{0\eta}$, the forces $f(r)$, and the colliding angle θ on the phase shifts of both colliding nonlinear waves are examined as follows.

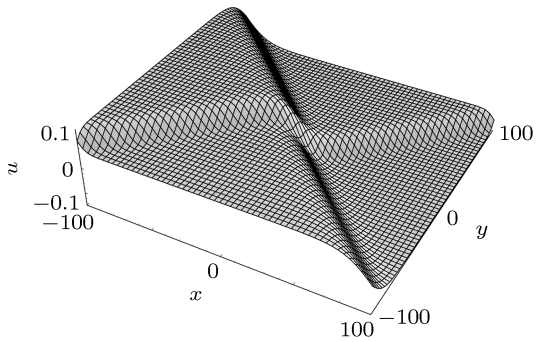


Fig. 2. The profile of interaction between two shock waves with a given colliding angle $\theta = \pi/2$.

It is noted from Eqs. (3) and (4) that two nonlinear waves travel in two different directions $r_1 = (k_1, l_1)$ and $r_2 = (k_2, l_2)$, respectively. The angle θ between propagation directions of the two waves satisfies $\cos\theta = (k_1k_2 + l_1l_2)/[(k_1^2 + l_1^2)(k_2^2 + l_2^2)]^{1/2}$. From Eqs. (6) and (7) we note $k_1^2 = l_1^2$ and $k_2^2 = l_2^2$, we can obtain $\cos\theta = 0$ and $\cos\theta = 1$. Here we know that

$\theta = 0$ is not valid for our perturbation method in this paper; there is one condition $\theta = \pi/2$ in these systems. Figure 2 shows the profile of interaction between two shock waves with a given colliding angle $\theta = \pi/2$.

Since shock wave 1 is traveling along the ξ direction and shock wave 2 is traveling along the η direction, we see from Eqs. (24) and (25) that due to the collision each soliton has a negative phase shift. The magnitudes of the phase shifts are related to the physical parameters i.e., the arbitrary constants $u_{0\xi}$, $u_{0\eta}$, the lattice constant a, and the forces $f(r)$. In order to know how these parameters influence the solitons, we give the results as shown in Figs. 3–5. In these figures, all physical quantities are dimensionless and $k_1 = 0.5, l_1 = -0.5, k_2 = 0.5, l_2 = 0.5, M = 1$.

Figure 3 shows the variation of the phase shift ΔP of the shock wave 1 against arbitrary constant $u_{0\eta}$ for different values of Debye screening length $\lambda_{\rm D}$ $(= 2, 3, 4)$. We note that the magnitude of phase shift increases with $u_{0\eta}$ and decreases with $\lambda_{\rm D}$, the phase shift increases sharply when $\lambda_{\rm D}$ is small, the change is weaker when $\lambda_{\rm D} = 3, 4$. The curves of $\lambda_{\rm D} = 3$ and $\lambda_{\rm D} = 4$ are fairly close to each other, but the curve of $\lambda_{\rm D} = 2$ is far away from both of them.

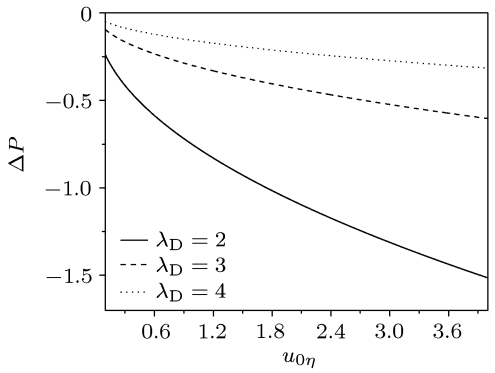


Fig. 3. Variation of the phase shift ΔP of the shock wave 1 against arbitrary constant $(u_{0\eta})$ for different values of Debye screening length $\lambda_{\rm D}$ $(= 2, 3, 4)$.

Figure 4 represents the variation of the phase shift ΔP of shock wave 1 with respect to lattice constant a for different values of Debye screening length $\lambda_{\rm D} = 4, 5, 6$. From this figure we note that, the magnitude of phase shift increases with a and decreases with $\lambda_{\rm D}$, for a given value of parameter $\lambda_{\rm D}$, the phase shift increases steeply with a when $a > 0.5$. For given a, the phase shift decreases with $\lambda_{\rm D}$, but the change is weaker.

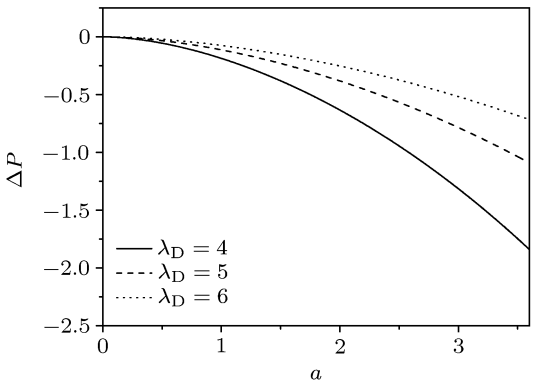


Fig. 4. Variation of the phase shift ΔP of shock wave 1 with respect to lattice constant a for different values of Debye screening length $\lambda_{\rm D} = 4, 5, 6$.

Figure 5 shows the variation of the phase shifts of shock wave 1 against the forces $f(r)$. We note that phase shift decreases as the forces $f(r)$ increases.

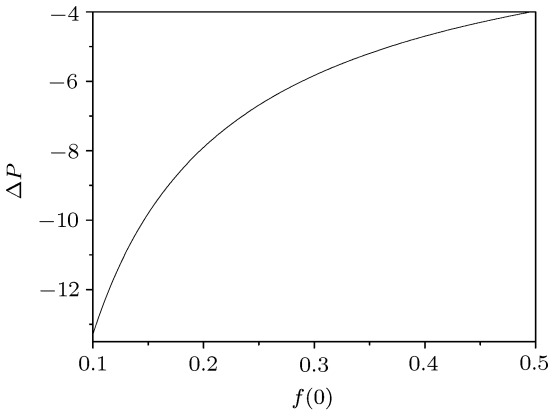


Fig. 5. The variation of the phase shifts of shock wave 1 against the forces $f(r)$.

In summary, using the extended PLK perturbation method, we find that this system admits a solution with two solitary waves: one is moving in the ξ direction and the other is moving in the η direction. Moreover, the angle between them is $\theta = \pi/2$, the leading-order analytical phase shifts of collision between two shock waves in a 2D dust crystal are derived explicitly, the collision of two shock waves is elastic and the waves preserve their original properties after collision. Due to the collision each wave has a negative phase shift in its direction of travel. The effects of parameters of the lattice constant a, the arbitrary constant $u_{0\eta}$, the forces $f(r)$, and the colliding angle θ on the phase shifts of both colliding nonlinear waves are studied. We expect that the present investigation will provide insight into and understanding of the phenomena related to collision waves.

References

[1] Shukla P K 2001 *Phys. Plasmas* **8** 1791

[2] Hou L J and Wang Y N 2003 *Acta Phys. Sin.* **52** 434 (in Chinese)

[3] Duan P, Liu J Y, Gong Y, Zhang Y, Liu Y and Wang X G 2007 *Acta Phys. Sin.* **56** 7090 (in Chinese)

[4] Samsonov D, Zhdanov S and Morfill G 2005 *Phys. Rev.* E **71** 026410

[5] Yang X X, Duan W S, Li S C and Han J N 2008 *Chin. Phys.* B **17** 2989

[6] Nunomura S, Samsonov D and Goree J 2000 *Phys. Rev. Lett.* **84** 5141

[7] Zhdanov S K, Samsonov D and Morfill G E 2002 *Phys. Rev.* E **66** 026411

[8] Chen J H 2009 *Chin. Phys.* B **18** 2121

[9] Liu Y H, Liu B, Chen Y P, Yang S Z, Wang L and Wang X G 2003 *Phys. Rev.* E **67** 066408

[10] Duan W S, Wan G X, Wang X Y and Lin M M 2004 *Phys. Plasmas* **11** 4408

[11] Huang G X 2001 *Chin. Phys.* **10** 523

[12] Misawa T, Ohno N, Asano K, Sawai M, Takamura S and Kaw P K 2001 *Phys. Rev. Lett.* **86** 1219

[13] Wang X G, Bhattacharjee A and Hu S 2001 *Phys. Rev. Lett.* **86** 2569

[14] Wang X G, Liu Y, Bhattacharjee A and Hu S 2001 *Thin Solid Films* **390** 228

[15] Nunomura S, Goree J, Hu S, Wang X, Bhattacharjee A and Avinash K 2002 *Phys. Rev. Lett.* **89** 035001

[16] Duan W S and Lin M M 2006 *Chaos, Solitons and Fractals* **28** 1212

[17] Kalman G, Rosenberg M and DeWitt H E 2000 *Phys. Rev. Lett.* **84** 6030

[18] Duan W S, Parkes J and Lin M M 2005 *Phys. Plasmas* **12** 022106

[19] Nunomura S, Goree J, Hu S, Wang X and Bhattacharjee A 2002 *Phys. Rev.* E **65** 066402

[20] Li S C, Wu L H, Lin M M and Duan W S 2007 *Chin. Phys. Lett.* **24** 2312

[21] Huang G X, Velarde M G and Makarov V A 2001 *Phys. Rev.* A **64** 013617

[22] Han J N, Li S C, Yang X X and Duan W S 2008 *Eur. Phys. J.* D **47** 197

[23] Su C H and Mirie R M 1980 *J. Fluid. Mech.* **98** 509

[24] Jeffery A and Kawahawa T 1982 *Asymptotic Methods in Nonlinear Wave Theory* (London: Pitman) p9

[25] Huang G X and Velarde M G 1996 *Phys. Rev.* E **53** 2988

[26] Taniuti T and Wei C C 1968 *J. Phys. Soc. Jpn.* **24** 941

[27] Kakutani T, Ono H, Taniuti T and Wei C C 1968 *J. Phys. Soc. Jpn.* **24** 1159