Band gap control of phononic beam with negative capacitance piezoelectric shunt*

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Periodic arrays of negative capacitance shunted piezoelectric patches are employed to control the band gaps of phononic beams. The location and the extent of induced band gap depend on the mismatch in impedance generated by each patch. The total impedance mismatch is determined by the added mass and stiffness of each patch as well as the shunting electrical impedance. Therefore, the band gap of the shunted phononic beam can be actively tuned by appropriately selecting the value of negative capacitance. The control of the band gap of phononic beam with negative capacitive shunt is demonstrated numerically by employing transfer matrix method. The result reveals that using negative capacitive shunt to tune the band gap is effective.

Keywords: phononic crystal, band gap, negative capacitive shunt, piezoelectric beam
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1. Introduction

Shunted piezoelectric devices have been used, in the last decade, as an effective means for various vibration control and noise applications. Their effectiveness stems form their lightweight, easy use, and good performance. The piezoelectric shunting technique was first introduced by Forward[1] who used piezoelectric elements with inductive shunts to significantly reduce the mechanical response of a membrane mirror. Hagood and Von Flotow[2] provided the first analytical formulation for passive shunt networks. They demonstrated how a piezoelectric patch shunted through a resistive inductive (RL) circuit acted as a vibration absorber tuned at the resonance frequency of the circuit. Since then, more complex shunting circuits have been investigated to extend the effectiveness over broader frequency bands.[3,4]

More recently, negative capacitive shunt networks have received much attention.[5−7] One advantage of a negative capacitive shunt circuit is the ability to suppress multimode structural vibrations. Therefore, the idea introducing the negative capacitive technique into the band gap control of phononic crystal (PC) is attractive. Phononic crystals are of periodic composites, which have elastic or acoustic wave band gaps where the propagation of wave is forbidden. The research on PC materials and structures is driven partly by promising applications such as elastic wave filters, vibrationless environments for high precision systems, as well as pure physical concerns with the Anderson localization of sound and vibration.[8−13] However, the locations and attenuation abilities of band gaps of conventional passive PCs are invariable, once their composing materials and structure forms are given in advance. In order to overcome the above disadvantage of conventional passive PCs, a lot of effort has been made to design intelligent PCs whose band gaps can be actively tuned.[14−17] One promising way is to use periodic arrays of piezoelectric shunts to control the band gaps actively, by varying the circuit parameters.[18−20]

The goal of this research is to develop an analytical model to calculate the band gaps of phononic piezoelectric beam with periodic arrays of negative capacitive shunt networks. Then, the tune ability of band gaps of negative capacitive shunts is analysed. The obtained results suggest that active control of band gaps with periodic arrays of negative capacitive shunts is an effective means to obtain a considerable tune scope. What is more encouraging is that using negative capacitive shunts can obtain very low-frequency bands.

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2. Numerical model of beam with shunted PZT patches

The piezoelectric patches with negative capacitive shunting circuits are periodically surface-bonded to a host beam, forming one-dimensional phononic crystals as shown in Fig. 1. The parts with and without piezoelectric patches are respectively denoted as A and B.

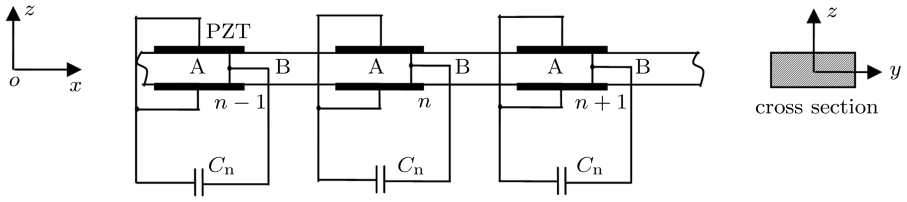


Fig. 1. Phononic beam with negative capacitive circuits.

The polarizing direction of the PZT patches is along the z-axis, and all surfaces of the patch are free of constraint except those in the direction along the x-axis. In the piezoelectric equation, the directions x, y, and z in the coordinate system are denoted as 1, 2 and 3, respectively. Therefore, the piezoelectric equations can be reduced into[16]

$$\begin{bmatrix} S_1 \\ D_3 \end{bmatrix} = \begin{bmatrix} s_{11}^{\mathrm{E}} & d_{31} \\ d_{31} & \varepsilon_{33}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} T_1 \\ E_3 \end{bmatrix}, \tag{1}$$

where S_1 and T_1 are the mechanical strain and the mechanical stress in the x-axis direction, respectively, s_{11}^{E} is the piezoelectric material compliance coefficient at constant electric field intensity, d_{31} is the piezoelectric constant that couples the mechanical and electrical properties of the piezoelectric material, D_3 is the electric displacement on the electrodes, $\varepsilon_{33}^{\mathrm{T}}$ is the dielectric constant of the material, and E_3 is the electric field is in the patch.

The complex impedance of shunting circuits can be written as

$$Z = \frac{1}{C_{\mathrm{n}} s}, \tag{2}$$

where s is the Laplace operator, and C_{n} is the negative capacitance of the shunting capacitor.

With the definitions of electric displacement and electric field, Eq. (1) can be solved as

$$S_1 = \left[s_{11}^{\mathrm{E}} - \frac{d_{31}^2 A_{\mathrm{s}}}{h_{\mathrm{p}} s_{11}^{\mathrm{E}} (C_{\mathrm{p}} + C_{\mathrm{n}})} \right] T_1, \tag{3}$$

where A_{s} is the area of electrode, h_{p} is the thickness of piezoelectric patch, and C_{p} is the inherent capacitance of the piezoelectric patch at constant stress, which can be expressed as

$$C_{\mathrm{p}} = \frac{\varepsilon_{33}^{\mathrm{T}} A_{\mathrm{s}}}{h_{\mathrm{p}}}. \tag{4}$$

So the equivalent elastic modules of PZT patch can be written as

$$E_{\mathrm{p}} = \frac{h_{\mathrm{p}} (C_{\mathrm{p}} + C_{\mathrm{n}})}{h_{\mathrm{p}} s_{11}^{\mathrm{E}} (C_{\mathrm{p}} + C_{\mathrm{n}}) - d_{31}^2 A_{\mathrm{s}}}. \tag{5}$$

Equation (5) reveals that the elastic modules of PZT patch can be adjusted by changing the value of negative capacitor, which is the cause of variation of band gaps.

For an Euler–Bernoulli beam, the governing differential equations of transverse vibration can be expressed as

$$\mathrm{EI} \frac{\partial^4 \phi(x,t)}{\partial x^4} + \rho A \frac{\partial^2 \phi(x,t)}{\partial t^2} = 0, \tag{6}$$

where EI, ρ and A are the bending rigidity, density and cross-section area, respectively, at the coordinate x, $\phi(x,t)$ is the displacement of the beam at coordinate x in the z-direction.

Therefore, using periodic boundary conditions and Bloch theorem, the band gaps of the phononic beam shown in Fig. 1 can be calculated by transfer matrix (TM) method.[11,20–22] The band gaps can be described by band structure. For a given frequency f, if the wave vector k is real, the wave can propagate through the phononic beam, corresponding to a pass band; if k is imaginary, the wave cannot propagate, corresponding to a band gap.

3. Results and discussion

In order to validate the theoretical analysis and evaluate the ability of periodic arrays of negative shunts to actively tune the band gaps, as an illustrative example, a phononic beam consisting of epoxy host and PZT-5H patches is chosen to perform the numerical simulation. The geometric properties and the material parameters employed in the calculations are shown in Tables 1 and 2.

Table 1. Geometric and material properties of the base beam.

material	density/(kg/m^3)	elastic modules /GPa	width/m	thickness/m
epoxy	1180	4.35	0.02	0.005

Table 2. Geometric and material properties of PZT patch.

material	density/(kg/m^3)	s_{11}^{E}/(m^3/N)	d_{31}/(C/m^2)	ε_{33}^{T}/(F/m)
PZT-5H	7500	1.65×10^{-11}	-2.74×10^{-10}	3.01×10^{-8}
width/m	thickness/m	length/m	lattice constant/m	
0.02	0.0002	0.04	0.08	

3.1. Implementation of negative capacitance

A negative impedance synthetic circuit, also called gyrator circuit, is used in order to generate the negative capacitance.[7] It consists of a capacitor, resistors and an operational amplifier as shown in Fig. 2.

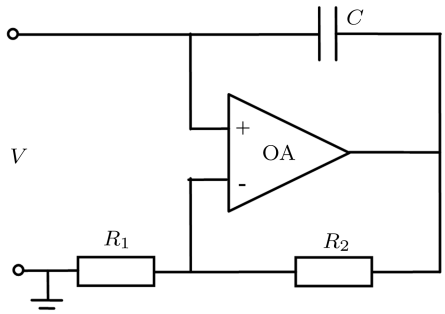


Fig. 2. Synthetic circuit of negative capacitance.

The equivalent impedance of this circuit is given by

$$C_{\mathrm{n}} = -\frac{R_1}{R_2}C. \tag{7}$$

Equation (7) shows that any negative capacitance can be obtained by appropriately selecting the circuit parameters. Moreover, different tunings can be performed by simply varying one resistor of the synthetic circuit.

3.2. Variation of band gaps with different negative capacitances

The ratio between the absolute values of C_{n} and C_{p} is defined as

$$\lambda = \left|\frac{C_{\mathrm{n}}}{C_{\mathrm{p}}}\right|. \tag{8}$$

Figure 3 shows the variation of equivalent elastic modulus of PZT patch with λ. When the value of λ increases from 0.9 to 1.1, E_{p} will increase from –120 GPa to 24 GPa. Therefore, the impedance between cells of the phononic beam can be tuned by changing the value of C_{n}, which leads to the band gaps variation.

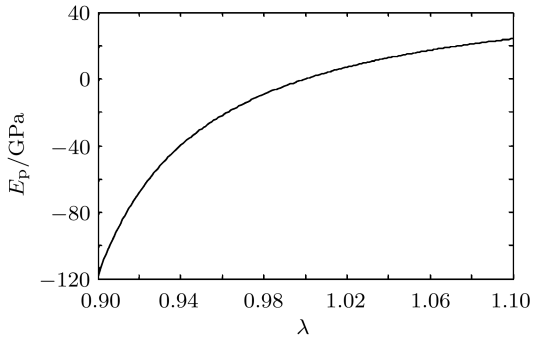


Fig. 3. Variation of equivalent elastic modulus of PZT patch with λ.

The band gaps within the range from 0 to 8 kHz are shown as the shadowed regions in Fig. 4. Figure 4(a) shows the band gaps when $\lambda = 0$, i.e., the shunting circuits are open. The impedance mismatch between the sub-cell with PZT patch surface-bonded and that with pure epoxy produces Bragg scattering, which induces band gaps in the phononic beam. Figure 4(b) shows the band gaps when $\lambda = 0.9$. Comparing the two diagrams, one can find that the width of the two band gaps is enlarged greatly by negative capacitive shunts. However, when $\lambda = 1$ or $\lambda = 1.1$, the width of the band gaps almost decreases to zero. Moreover, comparing the four figures, one can find that the locations of band gaps are also distinctly changed. Therefore, both the location and the width of band gaps can be actively tuned by selecting the value of negative capacitance.

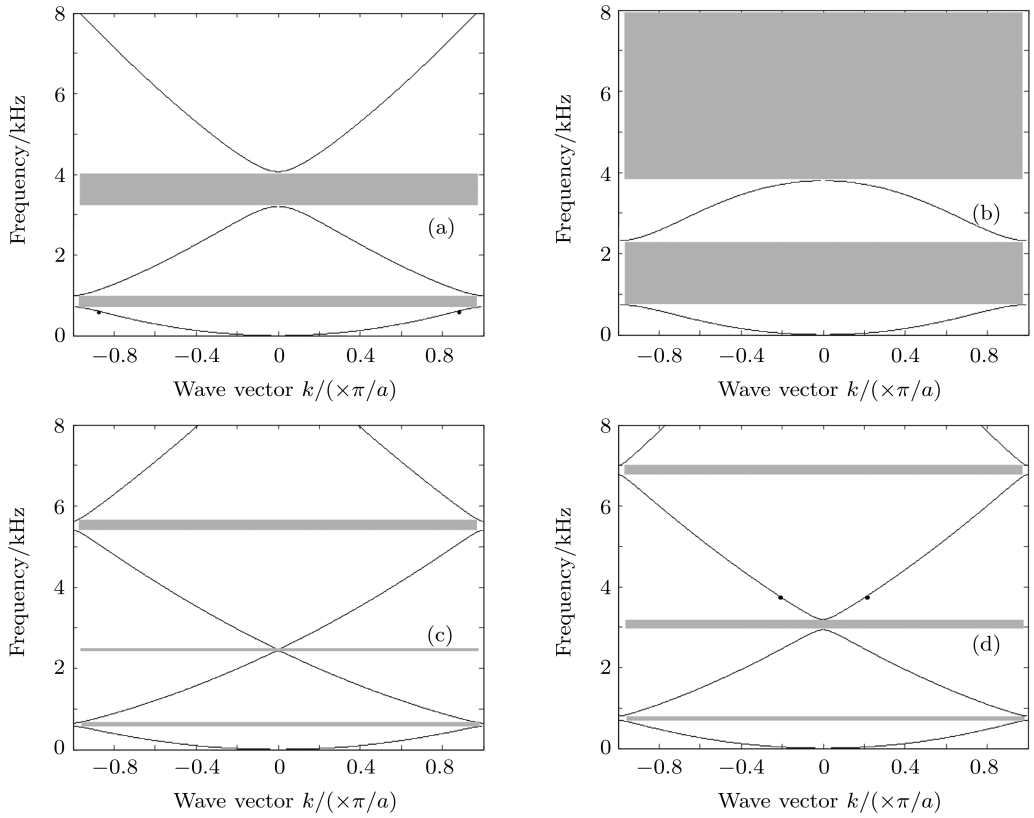


Fig. 4. Band structures of the phononic beam with negative capacitive shunts for different values of λ: (a)$\lambda = 0$ (shunting circuits are open), (b) $\lambda = 0.9$, (c) $\lambda = 1$, and (d) $\lambda = 1.1$.

The variations of band gaps are shown as the shadowed region in Fig. 5. When λ increases from 0.90 to 0.97, the width of band gap also greatly increases. Especially, when the value of λ is between 0.96 and 0.97, there will be almost a complete band gap at all frequencies. However, when the value of λ is between 0.97 and 1.1, the widths of band gaps decrease a lot and the frequency locations of band gaps increase with the increase of λ.

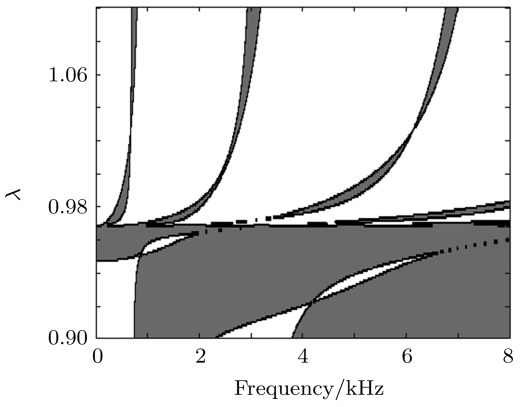


Fig. 5. Variations of band gap width with λ.

For the finite beam with N cell, the transmission property can give an intuitive representation of the propagation property, instead of the dispersion relation of an infinite system. Since the frequency response function (FRF) of vibration can be used to describe the gaps effectively, the FRF of the beam with finite periods is calculated.

The transmissions properties of three and six periods are shown in Fig. 6. As an example, the curves are respectively calculated at $\lambda = 0$ (Fig. 6(a)) and $\lambda = 0.9$ (Fig. 6(b)). Comparing Figs. 6(a) and 4(a), one can observe that there is a little difference between them, which both show the band gaps when shunting circuits are open. By comparison between Figs. 6(b) and 4(b), those curves all prove that the band gaps of the beam can be actively tuned by appropriately selecting the negative capacitance.

The numerical simulation proves the idea that using negative capacitive shunting circuits to tune the band gaps in phononic beam is effective. Both locations and widths of the band gaps can be tuned by simply varying the value of negative capacitance of the shunting networks without needing to modify the configuration of the structure.

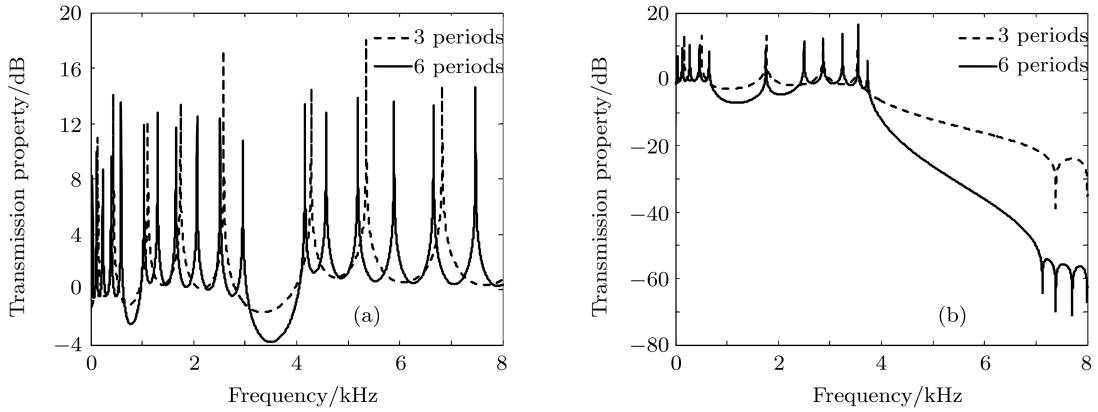


Fig. 6. Transmission properties of finite periods at (a) $\lambda = 0$ and (b) $\lambda = 0.9$.

4. Conclusions

In this work, a beam with periodically surface-bonded PZT patches is designed, and each PZT patch is shunted with a negative capacitive circuit. The band gaps of the piezoelectric phononic beam are calculated by transfer matrix method. The calculation result is encouraging in the sense that using negative capacitive shunts to tune the band gaps is not only feasible but also effective. The proposed approach gives a promising way to design intelligent PCs whose band gaps can be tuned without needing to modify the configuration of the structure.

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