Numerical investigation of multi-beam laser heterodyne measurement with ultra-precision for linear expansion coefficient of metal based on oscillating mirror modulation*

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This paper proposes a novel method of multi-beam laser heterodyne measurement for metal linear expansion coefficient. Based on the Doppler effect and heterodyne technology, the information is loaded of length variation to the frequency difference of the multi-beam laser heterodyne signal by the frequency modulation of the oscillating mirror, this method can obtain many values of length variation caused by temperature variation after the multi-beam laser heterodyne signal demodulation simultaneously. Processing these values by weighted-average, it can obtain length variation accurately, and eventually obtain the value of linear expansion coefficient of metal by the calculation. This novel method is used to simulate measurement for linear expansion coefficient of metal rod under different temperatures by MATLAB, the obtained result shows that the relative measurement error of this method is just 0.4%.

Keywords: linear expansion coefficient, multi-beam laser heterodyne, laser Doppler technique, contactless measurement

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1. Introduction

Usually, we describe linear expansion[1] as a one-dimensional change in the length when solid is heated. The measurement of linear expansion coefficient of materials is not only important in the development of new materials, but also one of the most important standards in choosing materials. Thermal expansion property should be considered in engineering structures designing, machinery and instrument manufacturing, materials processing and other processes. Otherwise, it will impact on the stability of structure and accuracy of instrument. If considering inappropriately, it will even cause damage to the structure, instrument failures, as well as the defects and failures in the processing of welding and so on. Currently, the determinations of linear expansion coefficient of metal are optical lever method, reading microscope method, electrothermal method and laser interferometry method, etc.

There are many parameters of direct measurement and complex operations, for which the system errors and accidental errors of the experiments are too large in using these methods. For example, using the optical lever method to measure the linear expansion coefficient of metal, using approximate formula and complex operation to direct a large system error, and the irrationality of the reading station also introduce large accidental error, the relative error is up to 4.4%.[2−7] Both the accidental errors caused by the visual sense of the reading microscope[8] and the system error caused by actual temperature and sensor delays of the electrothermal method[9] limit their measurement accuracy greatly. As the device of laser interferometry method has the advantages of sharp interference fringes, high resolution, and simple experiment, which greatly reduces the experimental error, achieves accurate measurement of the linear expansion coefficient of the metal, relative error is just 2%.[10−15] However, when this method is used, reading numbers of interference fringes that introduces accidental error caused by visual sense can not satisfy the requirements of current ultra-high-precision measurement.

Among the optical measurements, the domestic and international scholars pay more attention to the laser heterodyne technique, which inherited many advantages of laser heterodynes technology and the

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laser Doppler technique, this is one of the ultra-precision methods of the thickness measurement, the relative error of measurement is just 1%.[16−19] This method has the advantages of high spatial and temporal resolution, high accuracy, good linearity, fast dynamic response, good reproducibility and wide measurement range, which has become one of the representational techniques of modern ultra-precise detections and measuring instruments, and is widely used in ultra-precision measuring, detecting, processing unit, lidar systems, etc.

The principle of traditional heterodyne detection is all dual-beam interference, the spectrum of the heterodyne signal contains only single frequency information,[20−26] one can obtain a single value of parameter under test after demodulation. Based on the laser heterodyne measurement technique, we propose an experimental scheme of multi-beam laser heterodyne measurement for the linear expansion coefficient of metal in this article. Based on this scheme we propose a novel method for multi-beam laser heterodyne measurement, which can improve the accuracy of laser heterodyne measurement. That is, based on the Doppler effect and heterodyne technology, we modulate the frequency of different time incident light by the oscillating mirror in the optical path, and obtain the multi-beam laser heterodyne signal, whose spectrum of signal contains multiple frequencies simultaneously, each frequency contains information of parameter under test, and we can obtain multiple values of parameter which is under test after the heterodyne signal demodulation simultaneously. Processing multiple values of parameter by the weighted-average, we increase the accuracy of parameter which is under test. A detailed theoretical analysis of this method is taken in this article, there is a simulation test at last, and the relative measurement error is just 0.4%.

2. Experimental scheme setup

The scheme of multi-beam laser heterodynes measurement for linear expansion coefficient of metal is shown in Fig. 1. The scheme mainly consists of heating system and the interferometric system. Heating system consists of digital temperature controller, temperature probe, and electric furnace for heating the metal rod under test. Temperature probe of digital temperature controller obtains the temperature signal through the platinum resistance thermometer sensor. While the setting value of temperature is adjusted by 'setting knob', the two signals go through the selector switch and the A/D converter, measured temperature and setting temperature can be shown in the digitron. Heating the sample uniformly to the setting temperature, we auto shut down the heater circuit by the relays. In the measuring condition, we display the current detected temperature.

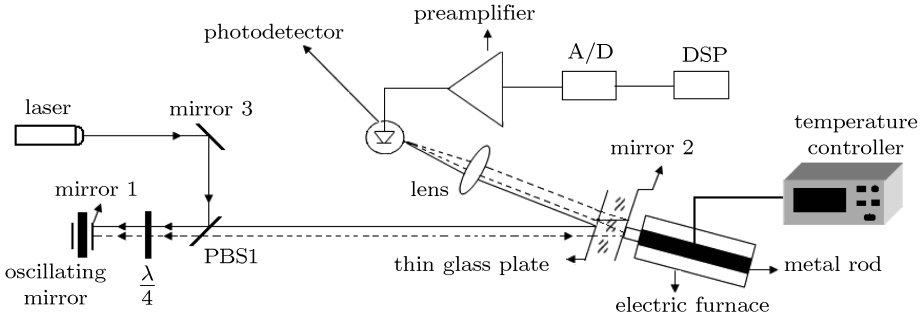


Fig. 1. Schematic diagram of multiple-beam interference to measure metal linear expansion coefficient.

The interferometric system mainly consists of the multi-beam interference device, which consists of holmium (Ho) solid-state laser, quarter-wave plate, mirror, oscillating mirror, mirror 1, polarising beamsplitter PBS1, quarter wave plate, lens, thin glass plate and mirror 2, and the signal processing system, which consists of the photodetector, preamplifier, A/D converter and digital signal processor (DSP). Mirror 1 is agglutinated on the front surface of the oscillating mirror, the oscillating mirror together with the mirror 1 move as uniformly accelerated motion under the action of the signal source, and the advantage of the oscillating mirror is the fact that the laser incidence on the surface of mirror 1 in different moment can be modulated in frequency.

Turning on the laser, we make the linearly polarised beams go through mirror 3, polarising beamsplitter PBS1 and quarter-wave plate, then reach the mirror 1 which agglutinate on the front surface of the oscillating mirror, therefore, the reflected ray in a different moment modulated by the oscillating mirror propagates through the polarising beamsplitter PBS1 and quarter-wave plate oblique incidence at the thin glass plate, the light transmission through the thin glass plate is reflected by mirror 2 and the light reflected by the front surface of the thin glass plate is converged on the sensitive surface of the photodetector by the convex lens together, finally, electrical signal converted by the photodetector propagates through the preamplifier, A/D converter and DSP, and we obtain the parameter under test.

It should be noted that: measuring by this method, we can make sure that the rod is heated uniformly by the electric furnace with mirror 2. At the same time, monitoring the internal temperature of the electric furnace by digital temperature controller, we read and record the displayed temperature T and signal processing value Δl. In which the value of Δl is just equal to Δd, the variation of the distance between thin glass plate and mirror 2, so that the value of Δl can be obtained by recording the variation Δd of the distance between thin glass plate and mirror 2.

3. Measurement principle

3.1. Principle of linear expansion coefficient measurement

Suppose that, the length of metal is l_1 at the temperature T_1, the length of metal is l_2 at the temperature T_2, when the temperature varies little, the elongation of the metal $\Delta l(\Delta l = l_2 - l_1)$ is directly proportional to temperature variation $\Delta T(\Delta T = T_2 - T_1)$ and metal original length l_0,[9] hence

$$\Delta l = \alpha l_0 \Delta T, \tag{1}$$

where α represents the linear expansion coefficient of metal rod

$$\alpha = \frac{\Delta l}{l_0 \Delta T}. \tag{2}$$

Therefore, measuring variation in length of metal rod Δl between T_1, T_2 can find the linear expansion coefficient of metal.

3.2. Principle of multi-beam laser interference

It is shown in Fig. 2, by comparison with the distance between thin glass plate and mirror 2, the thickness of thin glass plate can be ignored, so the reflected light from the back surface of thin glass plate can be ignored, either. Because the beam reflect and refract continuously from thin glass plate and mirror 2, and the reflection and refraction have a contribution to the interference between reflected and transmitted light at the infinite distance or the focal plane of the lens, so when discussing interference, we must consider multiple reflection and refraction effects, hence should discuss the multi-beam laser interference.

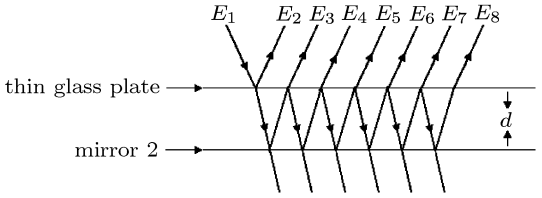


Fig. 2. Multi-beam interference principle.

When the beam makes an oblique incidence at the incident angle θ_0, suppose incident field is $E(t) = E\exp(\mathrm{i}\,\omega_0 t)$. The uniform acceleration motion equation and rate equation of the oscillating mirror are $x(t) = a(t^2/2)$ and $v(t) = at$ respectively. Because of the movement of oscillating mirror, based on Doppler effect, frequency of reflected light changes to $\omega = \omega_0(1 + at/c)$. In this expression, ω_0 stand for angular frequency of laser, a is vibration acceleration, c is light speed. So optical field arrived at the surface of thin glass plate at $t - l/c$ time is

$$E_1(t) = \alpha_1 E_0 \exp\left\{\mathrm{i}\left[\omega_0\left(1 + \frac{a\,(t - l/c)}{c}\right)t + \omega_0 \frac{a(t - l/c)^2/2}{c}\right]\right\}. \tag{3}$$

The beam propagated through the thin glass plate is reflected by mirror 2 repeatedly at different moments, the reflected light can be described as follows:

$$\begin{cases} E_2(t) = \alpha_2 E_0 \exp\left\{ \mathrm{i}\left[\omega_0\left(1 + a\dfrac{t - l/c - (2nd\cos\theta)/c}{c}\right)t + \omega_0\dfrac{\left(a\dfrac{(t - l/c - (2nd\cos\theta)/c)^2}{2} + 2nd\cos\theta\right)}{c}\right]\right\}, \\ \quad \vdots \\ E_m(t) = \alpha_m E_0 \exp\left\{ \mathrm{i}\left[\omega_0\left(1 + a\dfrac{t - \dfrac{l}{c} - \dfrac{2(m-1)nd\cos\theta}{c}}{c}\right)t \right.\right. \\ \qquad \left.\left. + \omega_0\dfrac{\left(a\dfrac{(t - l/c - (2(m-1)nd\cos\theta)/c)^2}{2} + 2(m-1)nd\cos\theta\right)}{c}\right]\right\}, \end{cases} \tag{4}$$

where $\alpha_1 = r$, $\alpha_2 = \beta\beta' r'$,..., $\alpha_m = \beta\beta' r'^{(2m-3)}$, r is the reflectivity of thin glass plate, β is the transmissivity, r' is the reflectivity of mirror 2, β' is the transmissivity when the beam reflected between thin glass plate and mirror 2 propagates out from the thin glass plate, d is the distance between thin glass plate and mirror 2.

So the total optical field received by the photodetector is

$$E(t) = E_1(t) + E_2(t) + \cdots + E_m(t). \tag{5}$$

Then the output photocurrent of the photodetector is

$$\begin{aligned} I &= \frac{\eta e}{h\nu}\frac{1}{Z}\iint\limits_S \frac{1}{2}[E_1(t) + E_2(t) + \cdots + E_m(t)][E_1(t) + E_2(t) + \cdots + E_m(t)]^* \mathrm{d}s \\ &= \frac{\eta e}{2h\nu}\frac{1}{Z}\iint\limits_S \left[\sum_{j=1}^{m} E_j^2(t) + \sum_{p=1}^{m-1}\sum_{j=1}^{m-p}(E_j(t)E_{j+p}^*(t) + E_j^*(t)E_{j+p}(t))\right]\mathrm{d}s, \end{aligned} \tag{6}$$

where e is electronic charge, Z is intrinsic impedance of the medium in the surface of photodetector, η is quantum efficiency, S is sensitive surface area of photodetector, h is Planck constant and ν is laser frequency. As the DC component can be filtered after passing through the low-pass filter, therefore, considering AC component only, and call this AC component as intermediate frequency current, we can coordinate and obtain the intermediate frequency current as

$$I_{\mathrm{if}} = \frac{\eta e}{2h\nu}\frac{1}{Z}\iint\limits_S \sum_{p=1}^{m-1}\sum_{j=1}^{m-p}(E_j(t)E_{j+p}^*(t) + E_j^*(t)E_{j+p}(t))\mathrm{d}s. \tag{7}$$

Substitute Eqs. (3) and (4) into the Eq. (7), calculate integral by software and obtain the results as

$$I_{\mathrm{if}} = \frac{\eta e}{h\nu}\frac{\pi}{Z}\sum_{p=1}^{m-1}\sum_{j=1}^{m-p}\alpha_j\alpha_{j+p}E_0^2\cos\left[\frac{4\omega_0 anpd\cos\theta}{c^2}t + \omega_0\frac{\dfrac{an^2(pd\cos\theta)^2}{c^2} - 2npd\cos\theta}{c}\right]. \tag{8}$$

After ignoring the minor term of $1/c^3$ Eq. (8) can be simplified as

$$I_{\mathrm{if}} = \frac{\eta e}{h\nu}\frac{\pi}{Z}\sum_{p=1}^{m-1}\sum_{j=1}^{m-p}\alpha_j\alpha_{j+p}E_0^2\cos\left(\frac{4\omega_0 anpd\cos\theta}{c^2}t - \omega_0\frac{2npd\cos\theta}{c}\right). \tag{9}$$

Equation (9) can be written as

$$\begin{aligned} I_{\mathrm{if}} &= \frac{\eta e}{h\nu}\frac{\pi}{Z}E_0^2\sum_{p=1}^{m-1}\cos[\Omega(p)t + \Phi(p)] \\ &\quad \times\left(\sum_{j=1}^{m-p}\alpha_j\alpha_{j+p}\right), \end{aligned} \tag{10}$$

where

$$\Omega(p) = \frac{4\omega_0 anpd\cos\theta}{c^2}, \tag{11}$$

$$\Phi(p) = \frac{2\omega_0 npd\cos\theta}{c}, \tag{12}$$

here p is defined as positive integer.

By Eq. (10), it can be seen that there is information of the distance d between thin glass plate

and mirror 2 for frequency difference of intermediate frequency (IF) component and phase retardation obtained by the multi-beam heterodyne measurement. Analyse frequency difference of IF component primarily, since the use of fast-Fourier transform (FFT) is easy to implement frequency measurement. According to Eq. (11), the frequency of interference signal can be recorded as

$$f_{\rm p} = \Omega(p) = K_{\rm p} d. \tag{13}$$

According to Eq. (13), the frequency of interference signal is in direct proportion to the distance which is under test, the scale factors are

$$K_{\rm p} = \frac{4\omega_0 anp\cos\theta}{c^2}, \tag{14}$$

which depends on angular frequency of light source ω_0, refractive index n of the medium between thin glass plate and mirror 2, refractive angle θ and vibration acceleration a.

It should be noted that through Eqs. (10) and (14), we can infer that the output photocurrent of detector is composed of different harmonics, each of them corresponds to the positive integer multiple of the frequency, it means that adjacent frequency difference of arrival is a fixed value, after FFT, wave peak of different harmonic frequencies can be seen on the frequency spectrum, the distance d between thin glass plate and mirror 2 can be measured by measuring the different harmonic frequencies. When d changes, the variation Δd of d can be measured according to Eq. (13), and then weighted average to Δd, so that the measurement accuracy of measuring Δd is improved. According to Eq. (2), linear expansion coefficient of sample under test can be calculated when Δd is known.

4. Numerical simulation and analysis

The MATLAB is used to simulate the measurement of the linear expansion coefficient of the brass rod, whose length is 150 mm, diameter is Φ18 mm, and the feasibility of the multi-beam laser heterodyne method is verified. Model number of temperature controller is the TED200C digital display temperature controller. We use the Ho solid-state laser with the wavelength of 2050 nm, the laser is safe to human eyes, and usually take the refractive index of medium between mirror 2 and thin glass plate as $n = 1$, aperture of photosensitivity of detector $D = 1$ mm, sensitivity of the detector is 1 A/W, acceleration of vibration of the oscillating mirror $a = 4 \times 10^6$ m/s^2.

According to the simulation, when the rod is at 15 °C room temperature, spectrum of FFT of the signal of multi-beam laser heterodyne obtained by signal processing is shown in Fig. 3, solid line is spectrum of FFT of the signal of multi-beam laser heterodyne corresponding to the measuring rod length variation Δl at room temperature 15 °C and the oblique incidence case. Dashed line is spectrum of FFT of the signal of multi-beam laser heterodyne corresponding to the measuring rod length variation Δl under room temperature 15 °C and the normal incidence.

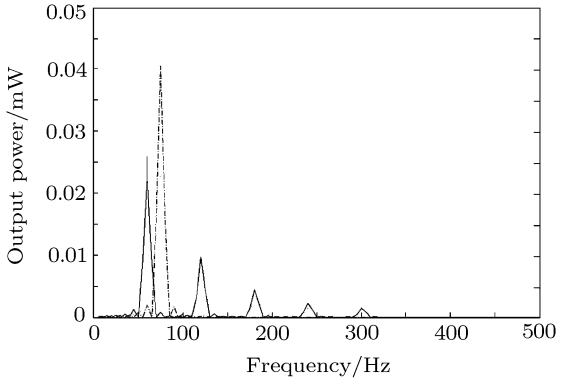


Fig. 3. Spectrum of FFT of the signal of multi-beam laser heterodyne at 15 °C.

The solid line in Fig. 3 shows spectrum distribution of the signal of multi-beam laser heterodyne, the spectrum distribution is equally spaced, corresponding to the previous theoretical analysis. Also, in Fig. 3, the theoretical curve in case of normal incidence is given in the simulation experiment (as the dashed line in the figure), the purpose is that, in the spectrum of the signal of multi-beam laser heterodyne, the central frequency value of the central frequency of first peak of the multi-beam with oblique incidence and the value of the central frequency of the theoretical curves of normal incidence can be obtained at the same time, so, the ratio of their central frequency is

$$\zeta = \cos\theta. \tag{15}$$

After obtaining the central frequency, we can calculate the refractive angle θ of the laser which propagates through the thin glass plate by Eq. (15), then obtain the value of θ_0 by the law of refraction. Therefore, we can obtain the value of $K_{\rm p}$ according to Eq. (14), ultimately, obtain the variation Δd of the distance between thin glass plate and mirror 2 according to Eq. (13). Because $\Delta d = \Delta l$, we can calculate linear

expansion coefficient of metal at arbitrary angle of incidence according to Eq. (2).

At the same time, Fig. 4 shows the simulation result of the spectrum of FFT of the signal of multi-beam laser heterodyne corresponding to measurement of the variation of rod in length by the multi-beam laser heterodyne measurement in different temperature. Figure 4 shows that, as the temperature increases, the relative position of spectrum moves towards the low-frequency, namely, as the temperature increases the frequency decreases. The reason is that, if the linear expansion coefficient of the metal rod is fixed, the variation of temperature is proportional to the variation of length of the metal rod, when the temperature increases the length of the metal rod increases, namely, the variation of the distance between thin glass plate and mirror 2 decreases, as the relationship between frequency $f_{\rm p}$ and the distance d between thin glass plate and mirror 2 is $f_{\rm p} = K_{\rm p}d$, when $K_{\rm p}$ is fixed, the frequency $f_{\rm p}$ and d present linear relation. Therefore, when the variation of the distance between thin glass plate and mirror 2 decreases, frequency decreases, namely, as the temperature increases, the frequency decreases, the relative position of spectrum moves towards the low-frequency. Figure 4 illustrates the correctness of the previous theoretical analysis well. It must be explained that, in Fig. 4 illustrating the frequency variation with temperature only gives a single peak spectrum of the multi-beam heterodyne after FFT, while spreading spectrum of Fig. 4 can obtain a spectrum which is similar from multi-peak FFT spectrum in Fig. 3. At the same time, since heterodyne detection is a detection method with near-diffraction limit, sensitivity of detection is extremely high, so the signal-to-noise ratio (SNR) of the heterodyne signal is extremely high in Figs. 3 and 4, as well.

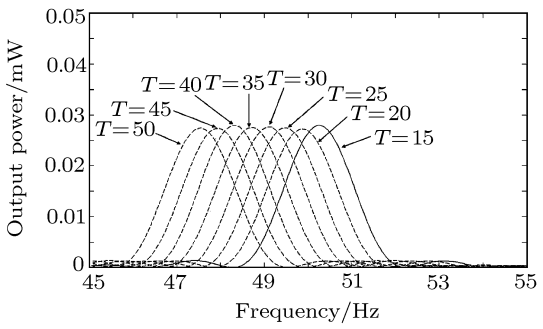


Fig. 4. Frequency spectrum of the measurement of the metal rod variation at different temperatures.

In the theoretical derivation process, we ignore the thickness of the thin glass plate that does not consider the effect of the reflected light of the back surface of the thin glass plate in heterodyne signal, but in fact, the thickness of thin glass plate is generally less than 1 mm, to overcome this effect, according to Eq. (13) we can see that the frequency of the multi-beam heterodyne signal of the reflected light of the back surface of thin glass plate are all distributed in the area which is around the zero frequency, designing a digital filter in the signal process so that we can filter the interfere of low frequency heterodyne signal. Using multi-beam laser heterodyne measurement, we obtain the simulation result of the metal rod variation in different temperatures when the angle of incidence is 15^o, the results are shown in Table 1.

Table 1. Simulation data of the metal rod variation at different temperatures.

temperature/°C	15.0	20.0	25.0	30.0
$\Delta l_i/\times 10^{-2}$ mm	0.00	1.59	2.98	4.58
temperature/°C	35.0	40.0	45.0	50.0
$\Delta l_i/\times 10^{-2}$ mm	6.17	7.76	9.15	10.74

It should be noted that 15 °C in Table 1 is room temperature in the laboratory, using the simulation data in Table 1, we can calculate average measured value of the linear expansion coefficient of the metal rod according to Eq. (2), which is 2.050216×10^{-5}/°C, while the theoretical value of the linear expansion coefficient of the metal rod[27] is 2.060000×10^{-5}/°C, so that the relative error is 0.4%, we can see that the accuracy of this method is extremely high. At the same time, it can be seen from analysing data, when heating uniformly, the accuracy of the temperature controller is 0.001 °C, therefore, the system error made by environment and reading error can be ignored when simulating, the error of simulation experiment mainly comes from trueness error after FFT and round-off error in the processing of calculation.

5. Conclusion

In this paper, by using oscillating mirror in the optical patch, we add a light frequency to the signals of different time incident light, so that the interference conditions of the reflected light transmitted through the thin glass plate and the light reflected by the mirror several times are met, resulting in the multi-beam laser heterodyne signal, thus loaded the under test information to the frequency difference of intermediate frequency heterodyne signal based on Doppler effect.

In the process of simulation of measuring sample linear expansion coefficient, this method can obtain multiple frequencies which contain the information of the length variation of metal simultaneously in frequency domain, and obtain multiple length variation after signal demodulation. Finally, we can obtain the accurate length variation versus temperature by the weighted-average. Brass, for example in order to simulate, the relative error of measurement for linear expansion coefficient is just 0.4%, this improves measurement accuracy observably. This method can be widely used in many areas such as engineering design, etc.

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