Long- and short-term average intensity for multi-Gaussian beam with a common axis in turbulence

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With the help of the extended Huygens–Fresnel principle and the short-term mutual coherence function, the analytical formula of short-term average intensity for multi-Gaussian beam (MGB) in the turbulent atmosphere has been derived. The intensity in the absence of turbulence and the long-term average intensity in turbulence can both also be expressed in this formula. As special cases, comparisons among short-term average intensity, long-term average intensity, and the intensity in the absence of turbulence for flat topped beam and annular beam are carried out. The effects of the order of MGB, propagation distance and aperture radius on beam spreading are analysed and discussed in detail.

Keywords: multi-Gaussian beam, turbulence, short-term average intensity

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1. Introduction

With the development of the application of laser beam in atmosphere, research on the control of propagation of laser beam in turbulence has been attracting more and more attention. Adaptive optics (AO) technology is often used to compensate for the wavefront distortions that result from propagation through turbulence. Since the aberration due to wavefront tilt is a substantial part of the total distortion,[1] tilt correction is the most important. To study short-term (tilt-removed) properties of laser beam in turbulence, Yura[2] has derived the short-term mutual coherence function for spherical-wave propagation. Besides this, he[2] also studied the difference between long-term average intensity and short-term average intensity of fundamental Gaussian beam.

In recent years more and more attention is paid to the beam spreading of laser beam through turbulence. Long-term average intensities of various types of beams have been studied.[3−16] But to the best of our knowledge, analytical expressions for short-term average intensities of various types of beams have not been taken into account. As we know, many types of beams can be composed of multi-Gaussian beams (MGBs), such as flat topped beams[8] and annular beams.[3] Namely, the MGB can be regarded as a more general type of beam and can be achieved with the help of beam-combining techniques.[17] Beam combining can be divided into two classes. One is that all arrays share a common axis, and the other is that all arrays are centred at different locations. In the present paper only the first case is considered.

2. General theory

Optical field of the MGB with a common axis can be expressed as[18]

$$u_0(x_0, y_0, 0) = \sum_{n=1}^{N} A_n \exp\left[-\left(\frac{1}{w_n^2} + \frac{\mathrm{i}\,k}{2R_0}\right) \times \left(x_0^2 + y_0^2\right) + \mathrm{i}\,\varphi_n\right], \quad (1)$$

where (x_0, y_0) is the transverse coordinates in the source plane, A_n, w_n and ϕ_n denote the amplitude, the waist width, and the phase of the n-th Gaussian beam respectively, N is the order of the MGB, R_0 is the phase front radius of curvature, and $k = 2\pi/\lambda$ (λ is wavelength) is the wave number. If ϕ_n is a random variable Eq. (1) denotes optical field of incoherently combined beam. If ϕ_n is a constant or a function of n (phase-locked array), Eq. (1) denotes optical field of coherently combined beam. Our previous work showed that diverse intensity profiles could be obtained by choosing A_n, w_n and φ_n.[18]

On the basis of the extended Huygens–Fresnel principle, an expression for short-term average intensity was given in Ref. [2]

$$\langle I_{\mathrm{S}}(x, y, z)\rangle = \left(\frac{k}{2\pi z}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H(p_2, q_2)$$

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$$\times M_{\mathrm{ST}}(p_2, q_2) \times \exp\left[\frac{\mathrm{i}\,k}{z}(xp_2 + yq_2)\right] \mathrm{d}p_2 \mathrm{d}q_2, \quad (2)$$

where $p_1 = (x_{01} + x_{02})/2$, $q_1 = (y_{01} + y_{02})/2$, $p_2 = x_{01} - x_{02}$, $q_2 = y_{01} - y_{02}$, (x_{01}, y_{01}) and (x_{02}, y_{02}) denote the transverse coordinates of two different points in the source plane, (x, y, z) indicate the coordinates at receiver. Besides, the function $H(p_2, q_2)$ and the short-term mutual coherence function for spherical-wave $M_{\mathrm{ST}}(p_2, q_2)$ can be written as[2]

$$\begin{aligned} H(p_2, q_2) = & \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} t\left(p_1 + \frac{p_2}{2}, q_1 + \frac{q_2}{2}\right) t^* \\ & \times \left(p_1 - \frac{p_2}{2}, q_1 - \frac{q_2}{2}\right) u_0 \\ & \times \left(p_1 + \frac{p_2}{2}, q_1 + \frac{q_2}{2}\right) u_0^* \\ & \times \left(p_1 - \frac{p_2}{2}, q_1 - \frac{q_2}{2}\right) \\ & \times \exp\left[\frac{\mathrm{i}\,k}{z}(p_1 p_2 + q_1 q_2)\right] \mathrm{d}p_1 \mathrm{d}q_1 \quad (3) \end{aligned}$$

and

$$\begin{aligned} M_{\mathrm{ST}}(p_2, q_2) = & \exp\left[-\frac{1}{\rho_0^{5/3}}\left(p_2^2 + q_2^2\right)^{5/6} \right. \\ & \left. + \frac{0.62\delta}{\rho_0^{5/3} D^{1/3}}\left(p_2^2 + q_2^2\right)\right], \quad (4) \end{aligned}$$

respectively, where $t(x_0, y_0)$ is the hard aperture function, D is the diameter of the aperture, ρ_0 is the spherical wave coherence length, and δ is a smoothly varying function equal to 1 for $(x_{01} - x_{02})^2 + (y_{01} - y_{02})^2 >> \lambda z$ and equal to 1/2 for $(x_{01} - x_{02})^2 + (y_{01} - y_{02})^2 << \lambda z$. If we set $\delta = 0$, Eq. (4) reduces into long-term mutual coherence function.

The hard aperture function in Eq. (3) is given by

$$t(x_0, y_0) = \begin{cases} 1, & x_0^2 + y_0^2 \le D^2/4, \\ 0, & \text{otherwise}. \end{cases} \quad (5)$$

To derive analytical formula of short-term average intensity we expand the hard aperture function into a finite sum of complex Gaussian functions,[19] i.e.,

$$t(x_0, y_0) = \sum_{\tau=1}^{N} B_\tau \exp\left[-\frac{4C_\tau}{D^2}\left(x_0^2 + y_0^2\right)\right], \quad (6)$$

where B_τ and C_τ are the expansion coefficients. By substituting Eqs. (1) and (6) into Eq. (3), and performing the integration analytically we obtain

$$\begin{aligned} H(p_2, q_2) = & \sum_{\tau_1=1}^{M}\sum_{\tau_2=1}^{M}\sum_{n_1=1}^{N}\sum_{n_2=1}^{N} \frac{\pi B_{\tau_1} B_{\tau_2}^* A_{n_1} A_{n_2}^*}{g_1 + G_1} \\ & \times \exp[-a\left(p_2^2 + q_2^2\right) \\ & + \mathrm{i}\left(\varphi_{n_1} - \varphi_{n_2}\right)], \quad (7) \end{aligned}$$

where

$$a = \frac{g_1 + G_1}{4}\left[1 - \frac{1}{(g_1 + G_1)^2}\left(g_2 + G_2 - \frac{\mathrm{i}\,k}{z}\sigma_0\right)^2\right], \quad (8)$$

$$g_1 = \frac{4}{D^2}\left(C_{\tau_1} + C_{\tau_2}^*\right), \quad (9)$$

$$g_2 = \frac{4}{D^2}\left(C_{\tau_1} - C_{\tau_2}^*\right), \quad (10)$$

$$G_1 = \frac{1}{w_{n_1}^2} + \frac{1}{w_{n_2}^2}, \quad (11)$$

and

$$G_2 = \frac{1}{w_{n_1}^2} - \frac{1}{w_{n_2}^2}. \quad (12)$$

In Eq. (8) $\sigma_0 = 1 - z/R_0$ can be interpreted as the factor that describes the beam spreading due to geometrical magnification. $\sigma_0 = 0$ and 1 denote the focused beam and the collimated beam, respectively. Substituting Eqs. (4) and (7) into Eq. (2) the short-term average intensity can be rewritten as

$$\begin{aligned} \langle I_{\mathrm{S}}(r, z)\rangle = & \sum_{\tau_1=1}^{M}\sum_{\tau_2=1}^{M}\sum_{n_1=1}^{N}\sum_{n_2=1}^{N} \frac{k^2 B_{\tau_1} B_{\tau_2}^* A_{n_1} A_{n_2}^*}{2z^2(g_1 + G_1)} \exp\left[\mathrm{i}\left(\varphi_{n_1} - \varphi_{n_2}\right)\right] \\ & \times \int_0^{\infty} \exp\left[-\left(a - \frac{0.62\delta}{D^{1/3}\rho_0^{5/3}}\right)\rho^2 - \frac{\rho^{5/3}}{\rho_0^{5/3}}\right] J_0\left(\frac{kr\rho}{z}\right)\rho \mathrm{d}\rho, \quad (13) \end{aligned}$$

where $r = \sqrt{x^2 + y^2}$ and $\rho = \sqrt{p_2^2 + q_2^2}$. Evaluating the integrals in Eq. (13) numerically, intensity distribution with different parameters can be examined.

In practical applications, an approximate formula for average intensity is always useful for estimating the beam spread. Our previous work showed that the first-order approximation of $\exp\left(-x^{5/3}\right)$ could offer an accurate result to study MGB in turbulent atmosphere.[18] Following Ref. [18] the analytical formula for

short-term average intensity can be derived as

$$\langle I_{\rm S}(r,z)\rangle = \sum_{\tau_1=1}^{M}\sum_{\tau_2=1}^{M}\sum_{n_1=1}^{N}\sum_{n_2=1}^{N}\frac{2B_{\tau_1}B_{\tau_2}^{*}A_{n_1}A_{n_2}^{*}}{W^2(g_1+G_1)}\exp\left[{\rm i}(\varphi_{n_1}-\varphi_{n_2})\right]\exp\left(-\frac{2r^2}{W^2}\right)$$
$$\times\left(1+\frac{W_2^2}{6W^2}\left\{2-\exp\left(\frac{2r^2}{W^2}\right)+\left(1-\frac{2r^2}{W^2}\right)\left[\Gamma_0\left(-\frac{2r^2}{W^2}\right)+\ln\left(-\frac{2r^2}{W^2}\right)-\ln\left(\frac{W^2}{W_2^2}\right)\right]\right\}\right), \quad (14)$$

where $\Gamma_0(x)$ is the first-order incomplete Gamma function, and

$$W^2 = W_1^2 + W_2^2 - W_3^2. \quad (15)$$

Here, $W_1 = \frac{2z}{k}\sqrt{2a}$ can be interpreted as the waist width of MGB in the absence of turbulence, $W_2 = (2\sqrt{2}z)/(k\rho_0)$ is beam spreading due to turbulence, and $W_3 = (2z/k)\sqrt{1.24\delta/D^{1/3}\rho_0^{5/3}}$ denotes the beam spreading due to the random tilt of the wavefront. Comparing W_2 with W_3 it can be found that

$$W_3 = \sqrt{0.62\delta}\left(\frac{\rho_0}{D}\right)^{1/6}W_2. \quad (16)$$

Equation (15) shows that when ρ_0 is much less than D the short-term beam spread is approximately equal to the long-term beam spread. In practice, with the help of acquisition, tracking and pointing (ATP) system, laser beam is often focused on a target. Therefore, only focused beam ($\sigma_0 = 0$) is considered in the following calculation. From Eq. (14) on-axis average intensity can be expressed as

$$\langle I_{\rm S}(0,z)\rangle = \sum_{\tau_1=1}^{M}\sum_{\tau_2=1}^{M}\sum_{n_1=1}^{N}\sum_{n_2=1}^{N}\frac{B_{\tau_1}B_{\tau_2}^{*}A_{n_1}A_{n_2}^{*}}{3W^4(g_1+G_1)}$$
$$\times\exp\left[{\rm i}(\varphi_{n_1}-\varphi_{n_2})\right]$$
$$\times\left\{W_2^2\left[0.423-2\ln\left(\frac{W}{W_2}\right)\right]+6W^2\right\}. \quad (17)$$

In the absence of turbulence ($W_2 = 0$) Eq. (17) can be simplified into

$$I_{\rm F}(0,z) = \sum_{\tau_1=1}^{M}\sum_{\tau_2=1}^{M}\sum_{n_1=1}^{N}\sum_{n_2=1}^{N}\frac{2B_{\tau_1}B_{\tau_2}^{*}A_{n_1}A_{n_2}^{*}}{W^2(g_1+G_1)}$$
$$\times\exp\left[{\rm i}(\varphi_{n_1}-\varphi_{n_2})\right]. \quad (18)$$

It should be noted that the validity condition of expanding hard aperture function into a finite sum of complex Gaussian functions is that the Fresnel number is small enough.[20]

3. Results and discussion

As an example, we pay our attention to the long-distance propagation of laser beam, such as laser communication and directed energy, so the beam waist δ is selected to be large and set be equal to 1. Other parameters are selected as $\lambda = 3.8\times10^{-6}$ m and $C_n^2 = 10^{-15}$ m$^{-2/3}$ (in weak turbulence). In the following calculation, the intensity in the absence of turbulence is denoted by $I_{\rm F}(r, z)$ and can be calculated from Eq. (14) if we set $W_2 = W_3$=0. The long-term average intensity is denoted by $\langle I_{\rm L}(r,z)\rangle$ and can be calculated from Eq. (14) if we set W_3=0. The normalised intensity is defined as the intensity divided by $I_{\rm F}(0, z)$ and denoted by subscript N. It should be pointed out that the short-term mutual coherence is used in the near field of the effective coherent aperture, i.e., $z \ll kL^2$, where L is given by the smaller value of ρ_0 or D.[2] Because our interest is in the case where ρ_0 is less than D, the analysis is restricted to the case where the propagation distance is much less than $1.39\left(C_n^2k^{7/6}\right)^{-6/11}$.

Case 1: Flat topped beams

If we set $A_n = N!/[n!(N-n)!]$, $\varphi_n = (n-1)\pi$ and $w_n = w_0/\sqrt{n}$ (here w_0 is a constant), Eq. (1) describes the flat topped beam proposed by Li.[21] The intensity profiles with different N values are plotted in Fig. 1 where w_0 is set to be equal to 0.1 m.

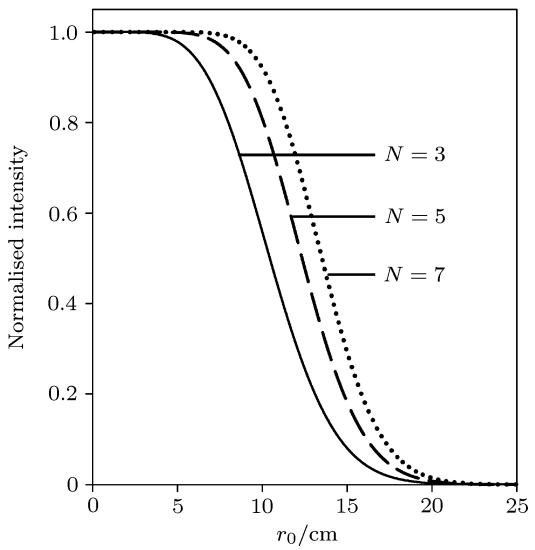


Fig. 1. Intensity profiles of flat topped beams for different N values.

It can be seen that the top of the beam becomes more and more flat with N value increasing.

Intensity distributions for $I_N\ (r,\ z)$, $\langle I_{\rm LN}\,(r,z)\rangle$ and $\langle I_{\rm SN}\,(r,z)\rangle$ at different z-planes for different N values are shown in Figs. 2–4. It can be seen that N value slightly influences the normalised average intensity profile. Under the same condition, both of $\langle I_{\rm LN}\,(0,z)\rangle$ and $\langle I_{\rm SN}\,(0,z)\rangle$ decrease with the increase of N value. As expected, the peak intensity for short exposure is away larger than the corresponding peak intensity for long exposure, and both $\langle I_{\rm LN}\,(0,z)\rangle$ and $\langle I_{\rm SN}\,(0,z)\rangle$ decrease with the increase of propagation distance.

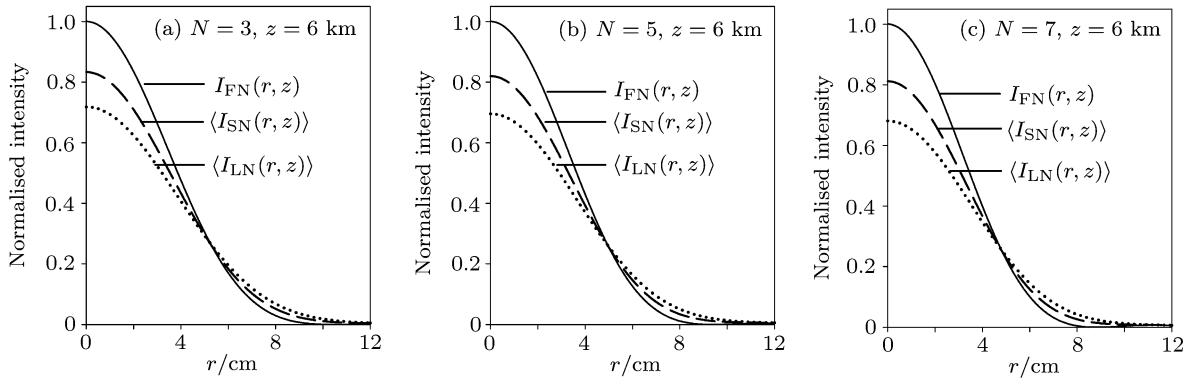


Fig. 2. Intensity profiles of flat topped beams for different N values, where $z = 6$ km and $D = 0.4$ m.

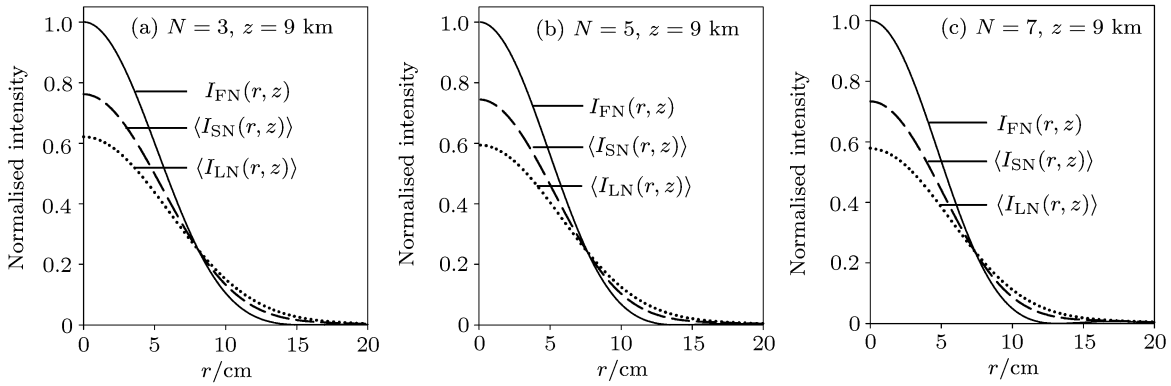


Fig. 3. Intensity profiles of flat topped beams for different N values, where $z = 9$ km and $D = 0.4$ m.

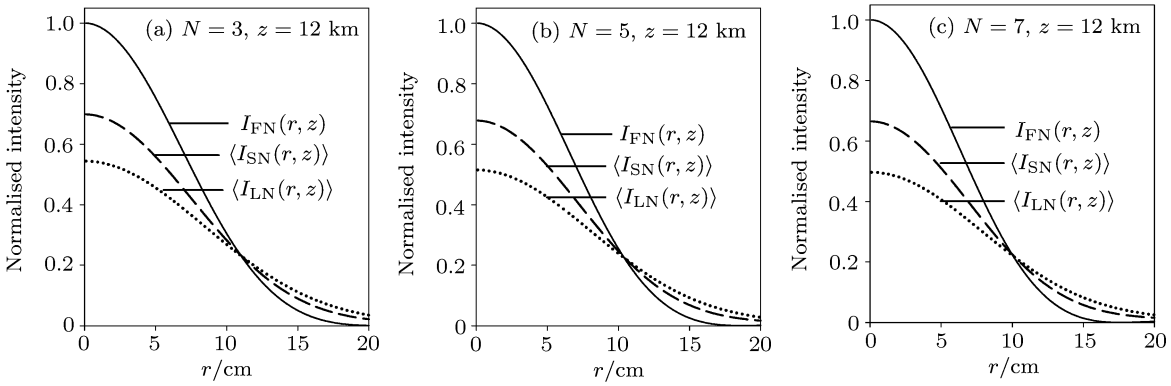


Fig. 4. Intensity profiles of flat topped beams for different N values, where $z = 12$ km and $D = 0.4$ m.

Case 2: Annular beams

If we set $A_n = 1$, $w_n = w_0/\sqrt{n}$, and $\varphi_n = (n-1)\,\pi/2$ (here $N = 4,\ 8,\ 12,\ \ldots$), Eq. (1) describes annular beams. The intensity distributions with different N values are plotted in Fig. 5, where $w_0 = 0.1$ m. From Fig. 5, it can be seen that the differences among these annular beams with different N values are very slight. With the increase of N the outer and the inner radius of the annular beam decrease. For simplicity only $N = 4$ is considered in the following calculation.

The intensity profiles of the annular beams with different radii of the hard aperture in different z-planes are plotted in Figs. 6–8. It can be seen from these figures that the normalised peak intensities for the long and the short exposure are large and the beam sizes are small when their propagation distances are shorter. With the increase of propagation distance, both of the normalised peak intensities decrease and their beam sizes become large.

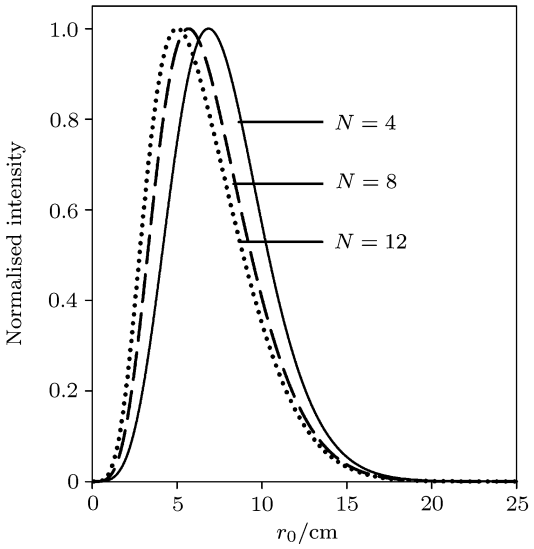


Fig. 5. Intensity profiles of the annular beams for different N values.

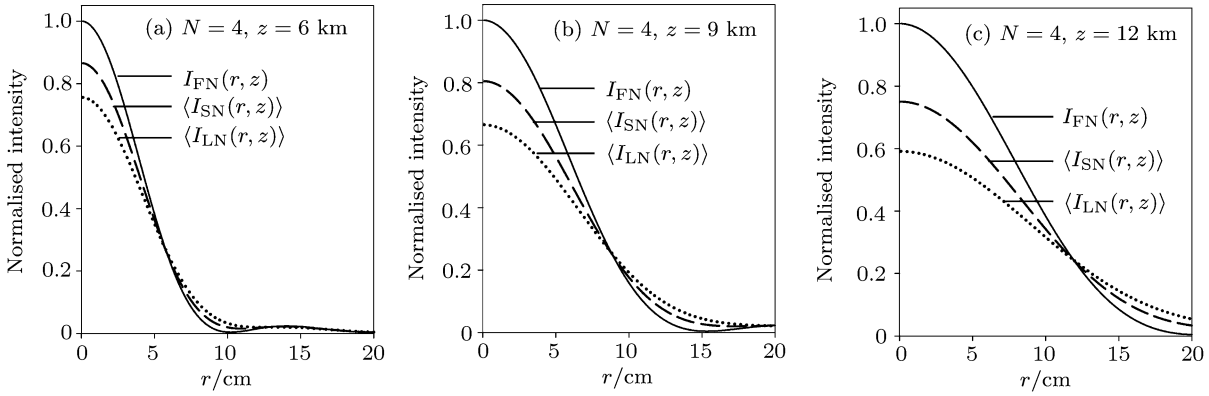


Fig. 6. Intensity profiles of annular beams with different z values, where $N = 4$ and $D = 0.3$ m.

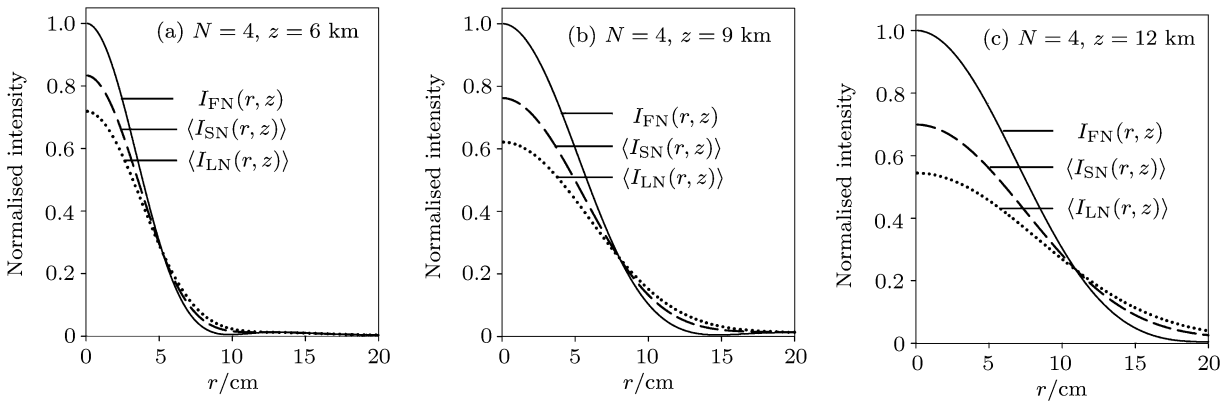


Fig. 7. Intensity profiles of annular beams with different z values, where $N = 4$ and $D = 0.4$ m.

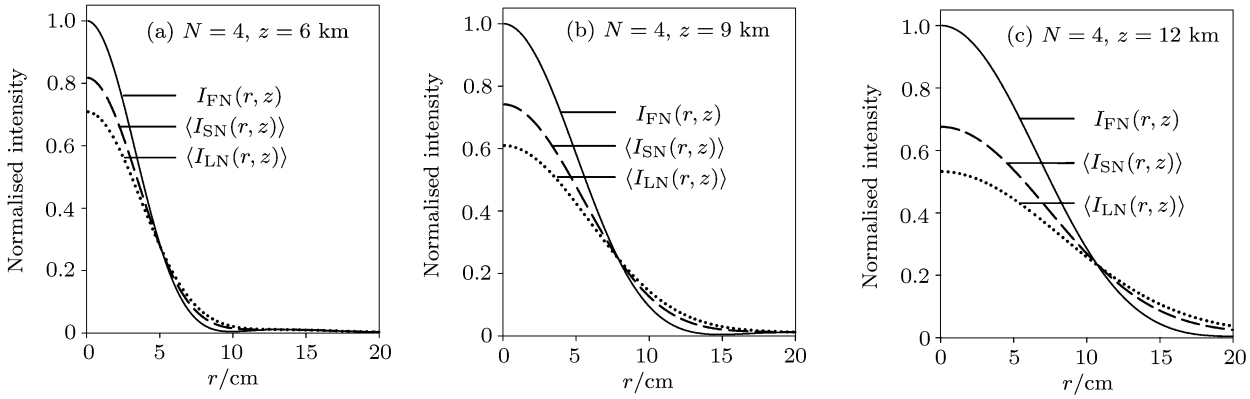


Fig. 8. Intensity profiles of annular beams with different z values, where $N = 4$ and $D = 0.5$ m.

Comparison among these figures for different aperture radii shows that the normalised peak intensities for the long and the short exposure are larger with a smaller aperture radii than those with larger aperture radii when other parameters are the same. The reason is that the turbulence-induced wavefront variance is a function of the diameter of aperture. With the use of Zernike polynomials, the total aberration (wavefront variance) of Kolmogoroff turbulence is $1.02(D/r_0)^{5/3}$, where r_0 is Fried's coherence length.[1] With the increase of wavefront variance, Strehl ratio (normalised peak intensity) decreases.

4. Conclusion

The MGB adopted in the present paper can be achieved with the help of beam-combining techniques. It has many potential applications in the propagation of beam in atmosphere because various beam shapes can be composed of the MGBs. In general, tilt-induced aberration of wavefront can be removed by using ATP system, so high quality beam can be obtained at target. In the present paper the analytical formula for short-term average intensity is derived with the help of the short-term mutual coherence function. As an example beam spreadings for flat topped beam and annular beam are analysed and discussed in detail.

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