Mechanical effects of light on the Ξ-type three-level atom in a high-finesse optical cavity*

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A theoretical study is carried out for the modification and implication of the effect on the Ξ-type three level atom in a high-finesse optical cavity driven by light field including spontaneous emission and the cavity decay. Analytic expressions for the dipole force, the friction force, the optical potentials and the friction coefficient are obtained. Then the numerical and graphical methods are used to investigate the friction coefficient with the controlling parameters. It is shown that the friction coefficient is strongly dependent on the controlling parameters. The cooling rate can increase by one order of magnitude more than that of a two-level atomic system. The reason can be given using the dressed states and the Sisyphus cooling mechanism, which would stimulate further experimental investigations.

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1. Introduction

A single atom strongly coupled to a high-finesse optical cavity has been investigated experimentally and theoretically.[1−10] New cavity cooling mechanism in the role of the escape of a photon from the cavity has been proposed.[1] An estimate shows that the observed cooling rate is at least five times larger than the cooling rate achieved by free-space cooling methods.[2] Compared with all conventional laser cooling methods,[11−13] cavity cooling avoids several problems of the laser-cooling scheme, such as many consecutive absorption-emission cycles in a closed multilevel system to extract kinetic energy from the atom, photon reabsorption and recoil heating. Moreover, as there is no requirement for a closed multilevel system in cavity cooling, it is an attractive approach to creating ultracold molecules.[14,15] In fact, the atom-cavity and molecules-cavity systems[16−21] are becoming a framework to investigate the foundation of quantum mechanics and their application to the investigation of the Bose–Einstein condensates,[16−18] quantum information processing,[21] etc. This causes a deeper study of how to make the atom and molecule further cooled by cavity cooling, which appears particularly important. As is well known, in the high-finesse optical cavity, the confining of the cavity field has an additional mechanical effect on the atom, on the contrary, the back action of the atom on the intra-cavity field is also notable, which becomes important in the regime of strong coupling.[22,23] Its momentary position of the moving atom can drastically modify the cavity field, which, in turn, exerts forces on the moving atom. This motivates us to investigate these forces in detail. These optical forces induced by the cavity field, acting on the atom, are important and have to be accounted for in the regime of strong coupling. On one hand, these optical forces are quite substantial, which can be viewed as the dipole force and friction force; On the other hand, analytic expressions for the dipole force and the friction force are obtained by using the master equation and the Heisenberg equations.

Usually, an atom is modelled as a two-level atomic system to investigate optical forces in the high-finesse cavity,[3−7,10] the actual experiments are carried out on multilevel atoms. The nature of multilevel atom-cavity field interaction is quantitatively different from that of the two-level one. In general, the involvement of multiple levels enhances the atom-cavity field coupling strength, which leads to a large momentum

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change between the atom and the cavity field. Secondly, the extended level structure provides flexibility, because of which, in some cases, a multilevel atom can be cooled at lower temperature than the two-level atom. Furthermore, there are more parameters that can be varied to control the cavity cooling in the three-level atomic systems, namely, two scattering losses, and two frequency shifts. Because of these reasons, it is of great interest to carry out the investigation on the optical forces acting on the three-level atom in the high-finesse optical cavity. The optical force of a V-type three-level atom was studied recently,[24,25] It is demonstrated that the cooling rate can increase by one order of magnitude more than that of the two-level atomic system. In this paper, we consider a Ξ-type three-level atom strongly coupled to a single mode of the electromagnetic field contained in the high-finesse optical cavity. Several important findings are beyond the scope of previous investigations on cavity cooling using the V-type three-level atom model. First, the value of the friction coefficient is asymmetric with respect to the axial position as compared with that of the V-type three-level atomic model. Moreover, we consider the parameters such as coupling strength and pumping strength, which are not considered in the V-type three-level atomic system. In the present paper, we give first the fully analytical expressions of the dipole force and friction force, and then numerically calculate the friction coefficient and show the calculations graphically as a function of controlling parameters.

The rest of the present paper is organised as follows. In Section 2 we give the fully analytical solutions for the optical forces and friction coefficient by using Heisenberg equations. We study the variations of the friction coefficient with atomic position, atomic detunings and coupling strengths. The friction coefficient is calculated numerically and shown graphically as a function of controlling parameters in Section 3. Some concluding remarks are presented in Section 4.

2. Dipole force and friction force in the Ξ-type three-level atom

Consider the Ξ-type three-level atom strongly coupled to a single mode of the electromagnetic field contained in the high-finesse optical cavity. A schematic sketch of the basic system is given in Fig. 1, the upper level is $|2\rangle$ (energy $\hbar\omega_2$), the middle level is $|0\rangle$ (energy 0) and the lower level is $|1\rangle$ (energy $-\hbar\omega_1$). The transition between levels $|2\rangle$ and $|0\rangle$ (levels $|0\rangle$ and $|1\rangle$) is mediated by cavity mode of frequency ω_2 (ω_1). The coupling strengths are denoted as $g_1(\boldsymbol{r})$ and $g_2(\boldsymbol{r})$, respectively. The transition between levels $|2\rangle$ and $|1\rangle$ is forbidden. The level $|2\rangle$ decays radiatively into level $|0\rangle$ (level $|0\rangle$ decays radiatively into level $|1\rangle$) at rate Γ_2 (Γ_1). In general, the system is driven by a monochromatic filed of frequency $\omega_{\rm p}$ and effective amplitude η, and the cavity photons decay with rate κ. The master equation describing the model in Fig. 1 is given in the rotating-wave approximation and in a frame rotating with the pump frequency $\omega_{\rm p}$ as[6]

$$\dot{\rho} = -\frac{\mathrm{i}}{\hbar}[H_{\rm ac}, \rho] - \frac{\mathrm{i}}{\hbar}[H_{\rm p}, \rho] + L_\kappa\rho + L_\gamma\rho, \qquad (1)$$

here,

$$\begin{aligned} H_{\rm ac} &= \hbar\Delta_1 A_{11} - \hbar\Delta_2 A_{22} - \hbar\Delta_{\rm c} a^\dagger a \\ &\quad + \hbar g_1(\boldsymbol{r})(a^\dagger A_{10} + A_{01} a) \\ &\quad + \hbar g_2(\boldsymbol{r})(a^\dagger A_{02} + A_{20} a), \qquad (2a) \\ H_{\rm p} &= \mathrm{i}\hbar\eta(a^\dagger - a), \qquad (2b) \\ L_\kappa\rho &= \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a), \qquad (2c) \\ L_\gamma\rho &= \Gamma_1(2A_{10}\rho A_{01} - \rho A_{00} - A_{00}\rho) \\ &\quad + \Gamma_2(2A_{02}\rho A_{20} - \rho A_{22} - A_{22}\rho), \qquad (2d) \end{aligned}$$

with atomic detunings $\Delta_1 = \omega_{\rm p} - \omega_1$ and $\Delta_2 = \omega_{\rm p} - \omega_2$, and cavity detuning $\Delta_{\rm c} = \omega_{\rm p} - \omega_{\rm c}$. The term $H_{\rm ac}$ represents the energy of the atomic system and the interaction of the atom with the cavity field, and the term $H_{\rm p}$ is the pump Hamiltonian. $L_\kappa\rho$ and $L_\gamma\rho$ describe the decay of the resonator mode and the atomic damping to background modes, respectively. Instead of solving the master equation one can equivalently deal with the Heisenberg equations for atomic and cavity filed mode operators.

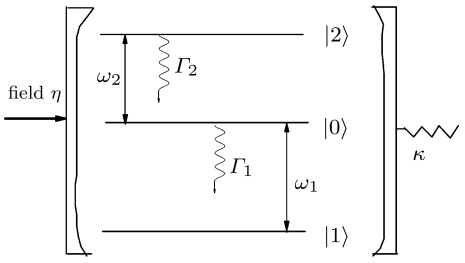


Fig. 1. Configuration of a Ξ-type three-level atom coupled to a single-mode cavity and driven by a laser field.

Considering the spontaneous emission and cavity decay, the Heisenberg equations about the atomic coupling to the cavity field can be obtained:[26]

$$\dot{a} = \mathrm{i}\Delta_{\rm c} a - \mathrm{i}g_1(\boldsymbol{r})A_{10} - \mathrm{i}g_2(\boldsymbol{r})A_{02} - \kappa a + \eta, \qquad (3a)$$

$$\dot{A_{10}} = \mathrm{i}\Delta_1 A_{10} + \mathrm{i}g_1(\boldsymbol{r})(A_{00} - A_{11})a - \Gamma_1 A_{10}, \qquad (3b)$$

$$\dot{A_{02}} = \mathrm{i}\Delta_2 A_{02} + \mathrm{i}g_2(\boldsymbol{r})(A_{22} - A_{00})a - \Gamma_2 A_{02}, \quad (3c)$$

where the atomic operator $A_{ij} = |i\rangle\langle j|$ satisfies the commutator relation $[A_{ij}, A_{kl}] = A_{il}\delta_{jk} - A_{kj}\delta_{il}$. Finding an exact solution of Eqs. (3) is by no means an easy job. Therefore, as an approximation, we assume that the cavity is weakly driven, then the weak atomic saturation and the weak excitation are fulfilled, i.e., either there is one photon in the cavity and the atom is in the ground state or there is no photon in the cavity and the atom is in the excited state. Therefor, the Hilbert-space is restricted to the 4 states, which are $\langle 1,0|$ (atom in the ground state $\langle 1|$, no photon in the cavity mode), $\langle 1,1|$ (atom in the ground state $\langle 1|$, one photon in the cavity mode), $\langle 0,1|$ (atom in the middle state $\langle 0|$, one photon in the cavity mode), and $\langle 2,0|$ (atom in the excited state $\langle 2|$, no photon in the cavity mode). It follows immediately that we can eliminate the cavity field dynamics from the atomic system, all expectation values are factorised, e.g. $\langle A_{ii}a\rangle = \langle A_{ii}\rangle\langle a\rangle$,[6,8,9]

$$\langle (A_{22} - A_{00})a\rangle = -(1-F)\langle a\rangle, \quad (4a)$$

$$\langle (A_{00} - A_{11})a\rangle = -(2F-1)\langle a\rangle. \quad (4b)$$

For simplicity, we have neglected the population of the upper excited state for the low saturation and assumed $\langle A_{11}\rangle = F$ and $\langle A_{00}\rangle = 1-F$. Thus the time evolutions of the expectation values may be written as

$$\begin{aligned}\langle \dot{a}\rangle &= \mathrm{i}\Delta_\mathrm{c}\langle a\rangle - \mathrm{i}g_1(\boldsymbol{r})\langle A_{10}\rangle - \mathrm{i}g_2(\boldsymbol{r})\langle A_{02}\rangle \\ &\quad - \kappa\langle a\rangle + \eta, \quad (5a)\\ \langle \dot{A_{10}}\rangle &= \mathrm{i}\Delta_1\langle A_{10}\rangle - \mathrm{i}(2F-1)g_1(\boldsymbol{r})\langle a\rangle \\ &\quad - \Gamma_1\langle A_{10}\rangle, \quad (5b)\\ \langle \dot{A_{02}}\rangle &= \mathrm{i}\Delta_2\langle A_{02}\rangle - \mathrm{i}(1-F)g_2(\boldsymbol{r})\langle a\rangle \\ &\quad - \Gamma_2\langle A_{02}\rangle. \quad (5c)\end{aligned}$$

With the definitions

$$\boldsymbol{Y} = \begin{pmatrix} a \\ A_{10} \\ A_{02}\end{pmatrix},$$

$$\boldsymbol{Z} = \begin{pmatrix} \mathrm{i}\Delta_\mathrm{c} - \kappa & -\mathrm{i}g_1(\boldsymbol{r}) & -\mathrm{i}g_2(\boldsymbol{r}) \\ -\mathrm{i}(2F-1)g_1(\boldsymbol{r}) & \mathrm{i}\Delta_1 - \Gamma_1 & 0 \\ -\mathrm{i}(1-F)g_2(\boldsymbol{r}) & 0 & \mathrm{i}\Delta_2 - \Gamma_2\end{pmatrix},$$

$$\boldsymbol{I}_\eta = \begin{pmatrix} \eta \\ 0 \\ 0\end{pmatrix},$$

the system of linear differential equations (5) can be written in a compact matrix notation

$$\langle \dot{\boldsymbol{Y}}\rangle = \boldsymbol{Z}\langle \boldsymbol{Y}\rangle + \boldsymbol{I}_\eta. \quad (6)$$

The steady state solutions of this system of linear differential equations are given by

$$\langle A_{10}\rangle_0 = \frac{\mathrm{i}g_1(\boldsymbol{r})(2F-1)\langle a\rangle}{(\mathrm{i}\Delta_1 - \Gamma_1)}, \quad (7a)$$

$$\langle A_{02}\rangle_0 = \frac{\mathrm{i}g_2(\boldsymbol{r})(1-F)\langle a\rangle}{(\mathrm{i}\Delta_2 - \Gamma_2)}. \quad (7b)$$

Inserting Eqs. (7) into Eq. (5a) for the amplitude of the field we find

$$\begin{aligned}\langle \dot{a}\rangle &= (\mathrm{i}\Delta_\mathrm{c} - \kappa - \gamma_1(\boldsymbol{r}) - \mathrm{i}U_1(\boldsymbol{r}) - \gamma_2(\boldsymbol{r}) \\ &\quad - \mathrm{i}U_2(\boldsymbol{r}))\langle a\rangle + \eta, \quad (8)\end{aligned}$$

where

$$\begin{aligned}U_1(\boldsymbol{r}) &= \Delta_1\frac{g_1^2(\boldsymbol{r})(4F^2-1)}{\Delta_1^2 + \Gamma_1^2},\\ U_2(\boldsymbol{r}) &= \Delta_2\frac{g_2^2(\boldsymbol{r})(1-F^2)}{\Delta_2^2 + \Gamma_2^2},\\ \gamma_1(\boldsymbol{r}) &= \Gamma_1\frac{g_1^2(\boldsymbol{r})(2F-1)^2}{\Delta_1^2 + \Gamma_1^2},\\ \gamma_2(\boldsymbol{r}) &= \Gamma_2\frac{g_2^2(\boldsymbol{r})(1-F)^2}{\Delta_2^2 + \Gamma_2^2}. \quad (9)\end{aligned}$$

The dipole force acting on the Ξ-type three-level atom at rest is obtained as

$$\begin{aligned}\boldsymbol{F}(\boldsymbol{r}) &= \dot{\boldsymbol{P}}\\ &= \frac{\mathrm{i}}{\hbar}[H, \boldsymbol{P}]\\ &= -\hbar\nabla g_1(\boldsymbol{r})(a^\dagger A_{10} + A_{01}a)\\ &\quad - \hbar\nabla g_2(\boldsymbol{r})(a^\dagger A_{02} + A_{20}a). \quad (10)\end{aligned}$$

Inserting Eqs. (7) and (8) into Eq. (10) yields

$$\begin{aligned}&\langle \boldsymbol{F}(\boldsymbol{r})\rangle_0\\ &= -\hbar\eta^2(\nabla g_1(\boldsymbol{r}))\frac{2\Delta_1 g_1(\boldsymbol{r})(2F-1)(\Delta_2^2 + \Gamma_2^2)}{|\det(\boldsymbol{Z})|^2}\\ &\quad - \hbar\eta^2(\nabla g_2(\boldsymbol{r}))\frac{2\Delta_2 g_2(\boldsymbol{r})(1-F)(\Delta_1^2 + \Gamma_1^2)}{|\det(\boldsymbol{Z})|^2}, \quad (11)\end{aligned}$$

where $\det(\boldsymbol{Z})$ is the determinant of $\boldsymbol{Z}$, and can be given by

$$\begin{aligned}&\det(\boldsymbol{Z})\\ &= (\mathrm{i}\Delta_\mathrm{c} - \kappa)(\mathrm{i}\Delta_1 - \Gamma_1)(\mathrm{i}\Delta_2 - \Gamma_2)\\ &\quad + g_1^2(\boldsymbol{r})(2F-1)(\mathrm{i}\Delta_2 - \Gamma_2)\\ &\quad + g_2^2(\boldsymbol{r})(1-F)(\mathrm{i}\Delta_1 - \Gamma_1).\end{aligned}$$

It is straightforward to find the expression for the friction force, the linear velocity dependence of the force for small velocities ($kv << \kappa$) due to the cavity dynamics. We use Eq. (8) in power of v to obtain the operator expectation values to first order,[6]

$$\langle a\rangle_1 = \frac{v\cdot\nabla\langle a\rangle_0}{\mathrm{i}\Delta_\mathrm{c} - \mathrm{i}\Delta_1\dfrac{g_1^2(\boldsymbol{r})(4F^2-1)}{\Delta_1^2+\Gamma_1^2} - \mathrm{i}\Delta_2\dfrac{g_2^2(\boldsymbol{r})(1-F^2)}{\Delta_2^2+\Gamma_2^2} - \kappa - \Gamma_1\dfrac{g_1^2(\boldsymbol{r})(2F-1)^2}{\Delta_1^2+\Gamma_1^2} - \Gamma_2\dfrac{g_2^2(\boldsymbol{r})(1-F)^2}{\Delta_2^2+\Gamma_2^2}}. \quad (12)$$

Thus the expectation value of the force operator in the first order of the velocity v,

$$\langle \boldsymbol{F}(\boldsymbol{r})\rangle_1 = -(\langle a^\dagger\rangle_0\langle a\rangle_1 + \langle a^\dagger\rangle_1\langle a\rangle_0 + \langle a\rangle_1\langle a^\dagger\rangle_0 + \langle a\rangle_0\langle a^\dagger\rangle_1)\nabla(U_1(\boldsymbol{r}) + U_2(\boldsymbol{r})) = -\beta v, \quad (13)$$

where β is called the friction coefficient. Inserting Eq. (12) into an expansion of the force equation (13), finally yields the friction force in the high-finesse optical cavity,

$$\begin{aligned}\langle \boldsymbol{F}(\boldsymbol{r})\rangle_1 &= -\beta v\\ &= v\cdot\frac{16\eta^2}{(C^2+D^2)(A^2+B^2)^2}\cdot g_1^2(\boldsymbol{r})(\nabla g_1(\boldsymbol{r}))^2\Delta_1(4F^2-1)(2F-1)(\Delta_2^2+\Gamma_2^2)(\Delta_2 N-\Gamma_2 M)\\ &\quad +g_2^2(\boldsymbol{r})(\nabla g_2(\boldsymbol{r}))^2\Delta_2(1-F^2)(1-F)(\Delta_1^2+\Gamma_1^2)(\Delta_1 N-\Gamma_1 M)\\ &\quad +g_1(\boldsymbol{r})g_2(\boldsymbol{r})\nabla g_1(\boldsymbol{r})\nabla g_2(\boldsymbol{r})(\Delta_2(1-F^2))(2F-1)(\Delta_1^2+\Gamma_1^2)(\Delta_2 N-\Gamma_2 M)\\ &\quad +\Delta_1(4F^2-1)(1-F)(\Delta_2^2+\Gamma_2^2)(\Delta_1 N-\Gamma_1 M))), \end{aligned} \quad (14)$$

where

$$\begin{aligned} A &= \Delta_C(\Delta_1\Gamma_2+\Delta_2\Gamma_1) - \kappa(\Gamma_1\Gamma_2-\Delta_1\Delta_2) - \Gamma_2 g_1^2(\boldsymbol{r})(2F-1) - \Gamma_1 g_2^2(\boldsymbol{r})(1-F),\\ B &= \kappa(\Delta_1\Gamma_2+\Delta_2\Gamma_1) - \Delta_C(\Gamma_1\Gamma_2-\Delta_1\Delta_2) - \Delta_2 g_1^2(\boldsymbol{r})(2F-1) - \Delta_1 g_2^2(\boldsymbol{r})(1-F),\\ C &= -\kappa - \Gamma_1\frac{g_1^2(\boldsymbol{r})(2F-1)^2}{\Delta_1^2+\Gamma_1^2} - \Gamma_2\frac{g_2^2(\boldsymbol{r})(1-F)^2}{\Delta_2^2+\Gamma_2^2},\\ D &= \Delta_\mathrm{c} - \Delta_1\frac{g_1^2(\boldsymbol{r})(4F^2-1)}{\Delta_1^2+\Gamma_1^2} - \Delta_2\frac{g_2^2(\boldsymbol{r})(1-F^2)}{\Delta_2^2+\Gamma_2^2},\\ M &= AC-BD,\quad N = AD+BC. \end{aligned} \quad (15)$$

The total force can be written as

$$\langle \boldsymbol{F}(\boldsymbol{r})\rangle = \langle \boldsymbol{F}(\boldsymbol{r})\rangle_0 + \langle \boldsymbol{F}(\boldsymbol{r})\rangle_1 = \langle \boldsymbol{F}(\boldsymbol{r})\rangle_0 - \beta v, \quad (16)$$

which describes the total force acting on the Ξ-type three-level atom within the high-finesse cavity. The first term on the right-hand side of Eq. (16) is the dipole force, which can be found from the steady state quantum average of the atom-cavity field interaction. The second term is the friction force. When the friction coefficient is positive, the friction corresponds to cooling, and when the friction coefficient is negative, the friction leads to heating. The value of the friction coefficient can significantly modify the cooling process, so it is necessary to investigate the friction coefficient.

3. Numerical calculation and the interpretation of friction coefficient

Using the theory described in the previous section, we give a detailed analysis of the friction coefficient of the Ξ-type three-level atom in the high-finesse optical cavity.

First, we demonstrate the dependence of the friction coefficient on atomic position. The atom is in the Fabry–Perot cavity with the spatial modefunction $\Phi(\boldsymbol{r}) = \Phi_0\cos(\boldsymbol{k}z)\exp(-(x^2+y^2)/\omega_0^2)$. Here Φ_0 is a constant, and in the following discussion, we assume that $\Phi_0 = 1$, $\boldsymbol{k}$ is the wave vector, and ω_0 is the waist of the cavity mode. The friction coefficient is plotted as a function of atomic position in Fig. 2. One can easily observe that the friction coefficient vanishes near the node of the cavity field, while it reaches a maximum 35 MHz between the nodes and antinodes, which increases by one order of magnitude more than that of the two-level atom.[6,10] Hence eventually the Ξ-type three-level atom in the high-finesse cavity can be cooled more rapidly than the two-level case. Moreover, one can find that the value of the friction coefficient is asymmetric with respect to the axial position, which is an obvious characteristic compared with that in the case of the V-type three-level atom in a driven high-Q cavity.[24] This is because the Ξ-type three-

level atom has one upper state, one middle state and one lower state, while the V-type three-level atom has two excited states and one lower state, and we neglect the population of the excited state. Hence, the population of the V-type three-level atom is in the lower state, while the population of the Ξ-type three-level atom satisfies the relation $\langle A_{11}\rangle + \langle A_{00}\rangle = 1$, which leads to the asymmetry value of the friction coefficient.

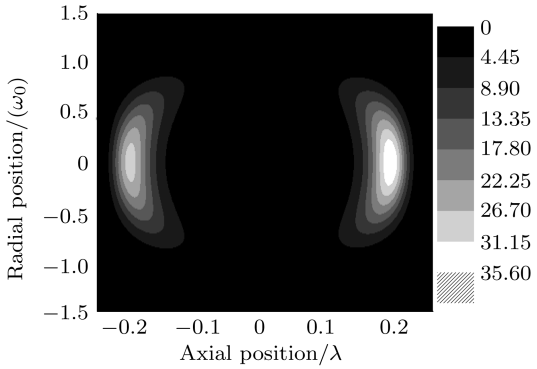


Fig. 2. Friction coefficient as a function of the axial and radial position of the atom. The friction coefficient vanishes near the node and reaches its maximum between nodes and antinodes. The parameters are set to be $(g_1 = g_2, \gamma_1 = \gamma_2, \kappa) = 2\pi \times (16, 3, 1.4)$ MHz.

Moreover, we illustrate the dependence of the friction coefficient with cavity detuning on coupling strength. To select detuning which is experimentally advantageous for cavity cooling of the three-level atom, it is necessary to investigate the average of friction coefficient that the three-level atom will experience in the high-finesse cavity. The simplest approach is to spatially average the friction coefficient. Figure 3 shows the average friction coefficients along the cavity axis over a length of $\lambda/2$ versus atomic detunings Δ_1 and Δ_2 for different coupling strengths: (a) $g_1 = g_2 = 2\pi \times 16$ MHz, (b) $g_1 = g_2 = 2\pi \times 12$ MHz and (c) $g_1 = g_2 = 2\pi \times 8$ MHz. The maximal friction coefficient is 8.3 MHz, and the other parameters are set to $(\gamma_1 = \gamma_2, \kappa, \Delta_c) = 2\pi \times (3, 1.4, 0)$ MHz. We focus on Fig. 3(a), where heating, and cooling regions are clearly indicated. When the friction coefficient is positive, the region is corresponding to cooling, while the friction coefficient is negative, the region is corresponding to heating. It is clearly found that the regions for cooling and heating are strongly dependent on the atomic detunings. Furthermore, we consider the dependence of the average of the friction coefficient on the coupling strength, which can be seen from Figs. 3(a)–3(c). It can found that the average friction coefficient is enhanced with increase of the coupling strength.

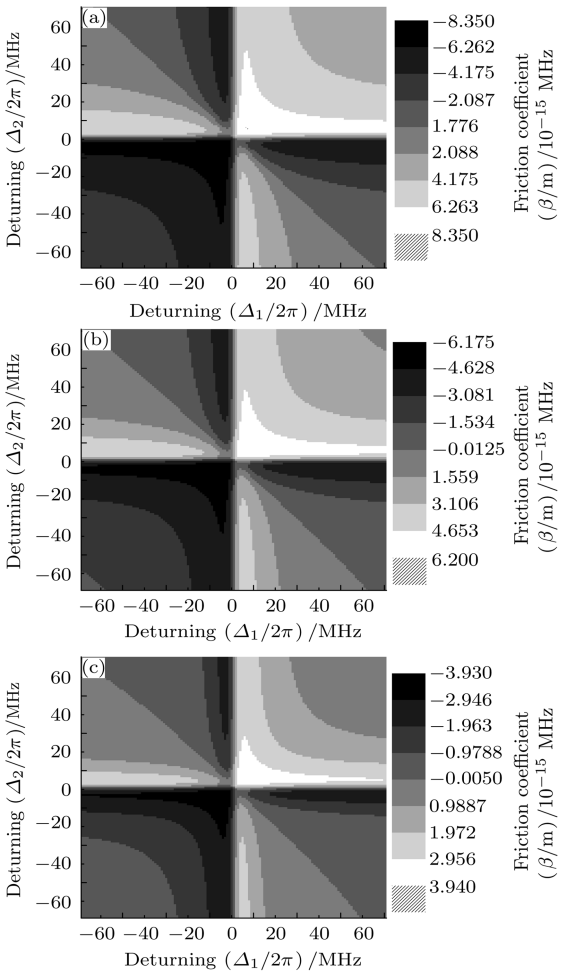


Fig. 3. A average friction coefficients along the cavity axis over a length of $\lambda/2$ as functions of detunings Δ_1 and Δ_2, for different coupling strengths. The parameters are $(\gamma_1 = \gamma_2, \kappa) = 2\pi \times (3, 1.4)$ MHz for (a) $g_1 = g_2 = 2\pi \times 16$ MHz, (b) $g_1 = g_2 = 2\pi \times 12$ MHz, (c) $g_1 = g_2 = 2\pi \times 8$ MHz. The maximal cooling rate is 8.3 MHz, cooling regions are changing significantly for different coupling strengths.

For a better understanding of the strong friction force, the dressed states and Sisyphus cooling mechanism may be used. For a Ξ-type three-level atom coupled with the cavity, the eigenvalues corresponding to the energies are E_1, E_2, and E_3 as follows ($\hbar \equiv 1$):

$$E_1 = \omega_p - \frac{(2\Delta_1 - \Delta_2)}{3} - \sqrt[3]{-q - \sqrt{q^2 + p^3}} - \sqrt[3]{-q + \sqrt{q^2 + p^3}}, \tag{17a}$$

$$E_2 = \omega_p - \frac{(2\Delta_1 - \Delta_2)}{3} + \frac{1}{2}\left(\sqrt[3]{-q - \sqrt{q^2 + p^3}} + \sqrt[3]{-q + \sqrt{q^2 + p^3}}\right)$$

$$+\ \frac{\sqrt{-3}}{2}\left(\sqrt[3]{-q-\sqrt{q^2+p^3}}-\sqrt[3]{-q+\sqrt{q^2+p^3}}\ \right), \tag{17b}$$

$$E_3=\omega_{\rm p}-\frac{(2\Delta_1-\Delta_2)}{3}+\frac{1}{2}\left(\sqrt[3]{-q-\sqrt{q^2+p^3}}+\sqrt[3]{-q+\sqrt{q^2+p^3}}\ \right)$$
$$-\ \frac{\sqrt{-3}}{2}\left(\sqrt[3]{-q-\sqrt{q^2+p^3}}-\sqrt[3]{-q+\sqrt{q^2+p^3}}\ \right), \tag{17c}$$

where

$$p=-\left(\frac{2\Delta_1-\Delta_2}{3}\right)^2-\frac{g_1^2(\boldsymbol{r})+g_2^2(\boldsymbol{r})+4\Delta_2(\Delta_1-\Delta_2)}{12}, \tag{18a}$$

$$q=\left(\frac{2\Delta_1-\Delta_2}{3}\right)^2+\frac{(2\Delta_1-\Delta_2)(g_1^2(\boldsymbol{r})+g_2^2(\boldsymbol{r})+4\Delta_2(\Delta_1-\Delta_2))}{24}-\frac{g_2^2(\boldsymbol{r})(\Delta_1-\Delta_2)}{8}. \tag{18b}$$

with $g_1(\boldsymbol{r})$ and $g_2(\boldsymbol{r})$ being the coupling strengths, and $\Delta_1=\omega_{\rm p}-\omega_1$ and $\Delta_2=\omega_{\rm p}-\omega_2$ the atomic detunings. The ground state energy is E_0. The eigenvalues corresponding to eigenvectors are described by $|\Psi(1)\rangle$, $|\Psi(2)\rangle$ and $|\Psi(3)\rangle$ as follows:

$$|\Psi(i)\rangle=A_i|0,1\rangle+B_i|1,1\rangle+C_i|2,0\rangle,\quad (i=1,2,3) \tag{19}$$

with A_i, B_i and C_i representing the probability amplitudes of the atom in $|0,1\rangle$ (one photon in the cavity mode, atom in the middle state $|0\rangle$), $|1,1\rangle$(one photon in the cavity mode, atom in the ground state $|1\rangle$) and $|2,0\rangle$(no photon in the cavity mode, atom in the excited state), respectively. $|\Psi(1)\rangle$, $|\Psi(2)\rangle$ and $|\Psi(3)\rangle$ denoting liner combinations of $|0,1\rangle$, $|1,1\rangle$ and $|2,0\rangle$. We consider only the four states $|\Psi(0)\rangle$, $|\Psi(1)\rangle$, $|\Psi(2)\rangle$ and $|\Psi(3)\rangle$, where $|\Psi(0)\rangle\equiv|1,0\rangle$ (no photon, atom in the ground state $|1\rangle$). Eigenvalues E_1, E_2, and E_3 each as a function of axial position are shown in Fig. 4. In this atom-cavity system, cavity cooling arises when the system is excited near a minimum of one of the dressed states, and the system will stay in this state while the atom moves further and thus the system energy will vary according to corresponding eigenvalues. For the Ξ-type three-level atom system, after being excited in the potential valley of the dressed state, the stronger the coupling strength, the steeper hill the Ξ-type three-level atom climbs, which can be seen in Figs. 4(a) and 4(b). Hence, the friction coefficient becomes stronger with coupling strength increasing.

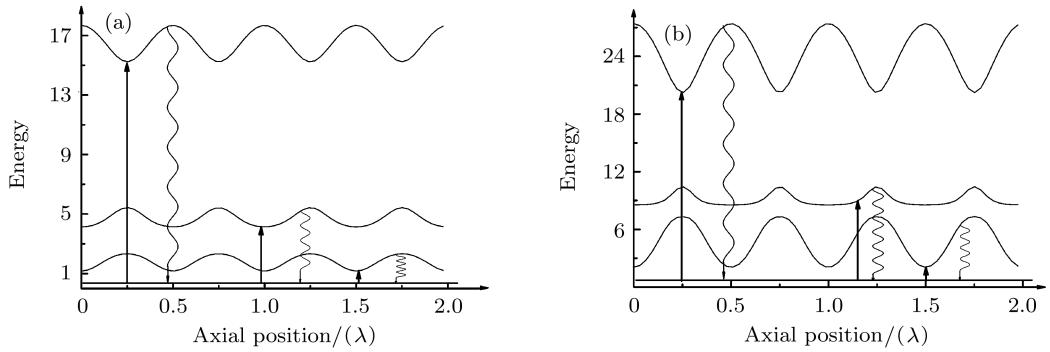


Fig. 4. Energies of the Ξ-type three-level atom within cavity system (dressed states) versus axial position for (a) $g_1=g_2=2\pi\times 8$ MHz and(b) $g_1=g_2=2\pi\times 16$ MHz. The atom is pumped at the maximum antinode, the kinetic energy of the slowly moving atom is decreased.

Moreover, we focus on Fig. 3(a). There are three detuning conditions for cavity cooling, one is

$$\Delta_{\rm c}=0,\quad \Delta_1>0,\quad \Delta_2>0, \tag{20}$$

which corresponds to driving the cavity resonantly, and pumping the atom at the minima of the upper dressed level $|\psi(2)\rangle$, then it has to expend kinetic energy in climbing the potential hills, which is shown in Fig. 4(a). The other cases are as follows:

$$\Delta_{\rm c}=0,\quad \Delta_1>0,\quad \Delta_2<0,\quad \text{and}\quad |\Delta_1|<|\Delta_2|;$$
$$\Delta_{\rm c}=0,\quad \Delta_1<0,\quad \Delta_2>0,\ \text{and}\ \ |\Delta_2|<|\Delta_1|, \tag{21}$$

which mean that the system loses kinetic energy when it is pumped into the minima of the lower dressed levels $|\psi(1)\rangle$ or $|\psi(3)\rangle$ as shown in Fig. 4(b).

When the following conditions are fulfilled: $\Delta_c = 0, \Delta_1 < 0, \Delta_2 < 0$; $\Delta_c = 0, \Delta_1 > 0, \Delta_2 < 0$, and $|\Delta_1| > |\Delta_2|$; $\Delta_c = 0, \Delta_1 < 0, \Delta_2 > 0$, and $|\Delta_2| > |\Delta_1|$, the dressed state is pumped at the maximum antinode, and the atom descends into the potential well, and the heating effect is obtained.

Furthermore, we illustrate the case of average friction coefficient along the cavity axis over a length of $\lambda/2$ versus detunings Δ_1 and Δ_2 for different pumping strengths (a) $\eta = 2\pi \times 1.0$ MHz, (b) $\eta = 2\pi \times 1.5$ MHz, and (c) $\eta = 2\pi \times 2.0$ MHz, which are shown in Fig. 5. The other parameters are $(g_1 = g_2, \gamma_1 = \gamma_2, \kappa) = 2\pi \times (16, 3, 1.4)$ MHz. It can found that with the increase of the pumping strengths, the average friction coefficient is enhanced. The maximal of the friction coefficient is 17.1 MHz. This is can be explained by using Eq. (8): the amplitude of the field is proportional to pumping strength and the pumping strength can affect the average friction coefficient significantly.

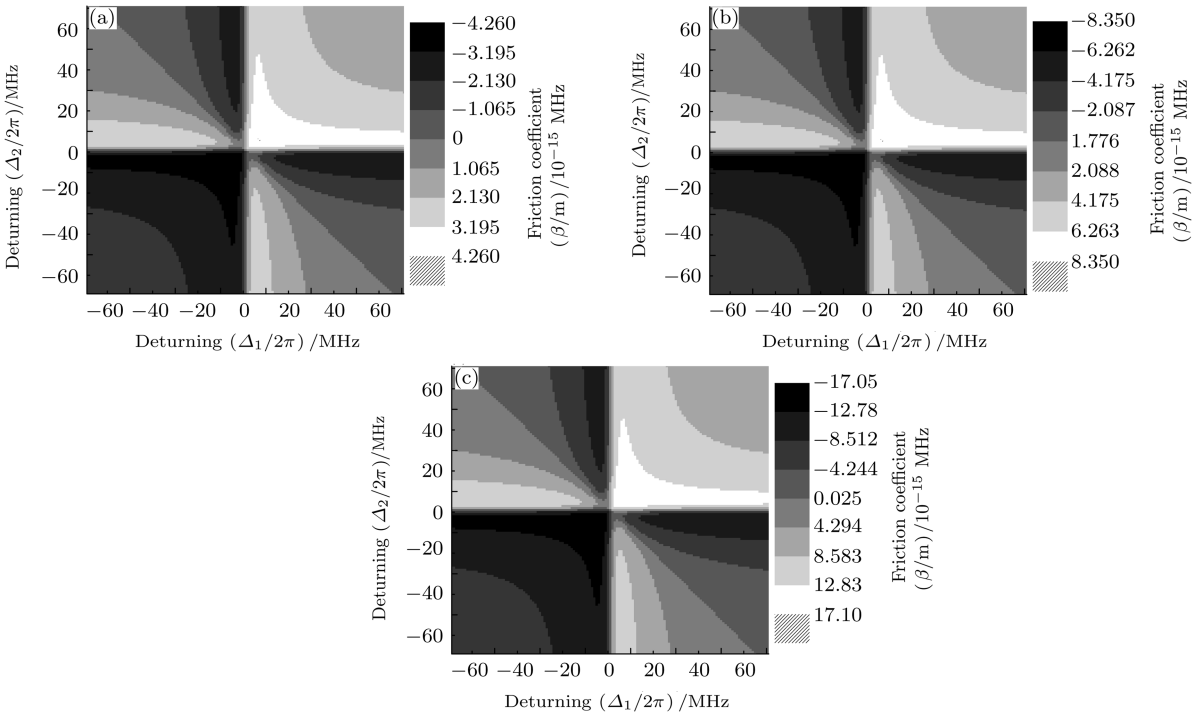


Fig. 5. A average friction coefficients along the cavity axis over a length of $\lambda/2$ versus detunings Δ_1 and Δ_2 for different pumping strengths. The parameters are $(g_1 = g_2, \gamma_1 = \gamma_2, \kappa) = 2\pi \times (16, 3, 1.4)$ MHz for (a) $\eta = 2\pi \times 1.0$ MHz, (b) $\eta = 2\pi \times 1.5$ MHz and (c) $\eta = 2\pi \times 2.0$ MHz.

4. Conclusion

We have analysed the cavity cooling of the Ξ-type three-level atom in the high-finesse cavity. Analytical expressions of the dipole force and the friction force are obtained by using master equation and Heisenberg equations. The cooling process can be significantly affected by the friction coefficient. The friction coefficient is strongly dependent on atomic position, atomic detuning and coupling strength, and the friction coefficient is calculated numerically and shown graphically as a function of controlling parameters. The cooling rate can increase by one order of magnitude than that of a two-level atomic system. The reason can be given by using the dressed states and Sisyphus cooling mechanism, which would stimulate further experimental investigations.

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