Interaction of two edge dislocations in free-space propagation*

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(Received 21 December 2009; revised manuscript received 9 June 2010)

This paper studies in detail the interaction of two edge dislocations nested in a Gaussian beam propagating in free space. It shows that in free-space propagation the edge dislocations are unstable and vanish, and two noncanonical vortices with opposite topological charge take place when off-axis distances c_1 and c_2 of two edge dislocations are non-zero, and the condition $k^2w_0^8 + 32c_1c_2(w_0^2 - 2c_1c_2)z^2 > 0$ is fulfilled (k-wave number, w_0-waist width). A noncanonical vortex appears when one off-axis distance is zero. However, one edge dislocation is stable when two edge dislocations are perpendicular and one off-axis distance is zero. Two perpendicular edge dislocations both with zero off-axis distance are also stable. The analytical results are illustrated by numerical examples.

Keywords: edge dislocation interaction, optical vortex, free-space propagation
PACS: 42.25.–p, 42.25.Bs **DOI:** 10.1088/1674-1056/20/1/014201

1. Introduction

In recent years much interest has been exhibited in studying optical beams carrying phase singularities (wavefront dislocations) due to their theoretical importance and attractive applications in optical manipulations, atom trapping, high-resolution metrology, optical communications and micromaching etc.[1] There exist two kinds of pure wavefront dislocations, i.e., the longitudinal screw dislocation often called the optical vortex with spiral phase, and the transverse edge dislocation with π-phase shift located along a line in the transverse plane. Experimentally, the optical vortex can be produced by a variety of methods, such as using computer-generated holograms, a mode convertor, a spatial light modulator, and a spiral phase plate etc.,[2−7] and the edge dislocation can be created by using a phase plate to embed a π-phase jump in the host beam.[8,9] The propagation dynamics of different optical vortex beams and vortex-edge dislocation interaction in linear and nonlinear media were studied extensively.[10−12] For example, Indebetouw examined the free-space propagation of an array of optical vortices nested in a Gaussian beam and found that the relative positions of vortices with same topological charge are invariant, whereas two vortices with opposite charges attract and annihilate each other.[13] Kivshar *et al.*[8] predicted theoretically and demonstrated experimentally the interaction of a vortex soliton with a dark-soliton stripe containing an edge dislocation in a nonlinear defocusing medium. The dark stripe soliton bending and breaking up into vortices were observed. Similar effects of the vortex-edge dislocation interaction, i.e., the splitting of an edge dislocation in a linear medium by an optical vortex was found by Petrov.[14,15] More recently, we showed that in the presence of an astigmatic lens some richer vortex evolution behaviour may appear during the vortex-edge dislocation interaction.[16] Up to now, the interaction between edge-dislocations nested in a host beam has not been found in the literature.

This paper aims at studying the interaction of two edge dislocations embedded in a Gaussian beam propagating in free space. Section 2 presents the analytical expressions for the propagation in free space of Gaussian beams carrying two edge dislocations and for the positions of phase singularities. Some special cases of interest are analysed in Section 3. Numerical illustrative examples are given in Section 4. Finally, Section 5 summarises the main results obtained in this paper.

2. Theoretical model

The initial field containing two edge dislocations in a Gaussian beam at the plane $z = 0$ is written as

*Project supported by the National Natural Science Foundation of China (Grant No. 10874125).
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$$E_0(x_0, y_0, 0) = \left(\frac{x_0 - c_1}{w_0}\right)\left(\frac{x_0 - y_0 + c_2}{w_0}\right) \times \exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right), \quad (1)$$

where w_0 denotes the waist width of the Gaussian beam and c_1, c_2 are off-axis distances of two edge dislocations, respectively.

Within the framework of the paraxial approximation, the propagation of the beam in free space obeys the Fresnel diffraction integral of the form

$$E(x, y, z) = \frac{-\mathrm{i}}{\lambda z}\exp(\mathrm{i}kz)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_0(x_0, y_0, 0) \times \exp\left\{\frac{\mathrm{i}k}{2z}\left[(x_0 - x)^2 + (y_0 - y)^2\right]\right\}\mathrm{d}x_0\mathrm{d}y_0, \quad (2)$$

where k denotes the wave number related to the wave length λ by $k = 2\pi/\lambda$.

Substituting Eq. (1) into Eq. (2), and straightforwardly calculating diffraction integral,

$$E(x, y, z) = \frac{2\pi}{(kw_0^2 + 2z\mathrm{i})^3\lambda}\exp\left[\mathrm{i}kz - \frac{k(x^2 + y^2)}{kw_0^2 + 2z\mathrm{i}}\right] \times \left\{\left[k^2w_0^4(x - c_1)(c_2 + x - y) - 2z^2\left(w_0^2 - 2c_1c_2\right)\right] + \mathrm{i}kw_0^2z\left[w_0^2 + 2x(c_2 - c_1) + 2c_1y - 4c_1c_2\right]\right\}. \quad (3)$$

The positions of phase singularities are determined by[17]

$$\begin{cases} \mathrm{Re}\left[E(x, y, z)\right] = 0, \\ \mathrm{Im}\left[E(x, y, z)\right] = 0, \end{cases} \quad (4)$$

where Re and Im denote the real and imaginary parts of $E(x, y, z)$, respectively. Furthermore, to determine the kind of phase singularities (screw or edge dislocation) the phase analysis should be performed (see Section 4).

From Eqs. (3) and (4) we obtain

$$\begin{cases} k^2w_0^4(x - c_1)(c_2 + x - y) - 2z^2\left(w_0^2 - 2c_1c_2\right) = 0, \\ z\left[w_0^2 + 2x(c_2 - c_1) + 2c_1y - 4c_1c_2\right] = 0. \end{cases} \quad (5)$$

There exist three cases in search of solutions of Eq. (5), namely,

(i) $c_1 \neq 0$, $c_2 \neq 0$, and $k^2w_0^8 + 32c_1c_2(w_0^2 - 2c_1c_2)z^2 > 0$, the positions of phase singularities at the z plane are

$$\begin{cases} x_1 = \dfrac{\left(4c_1c_2 - w_0^2\right)}{4c_2} + \dfrac{\sqrt{k^2w_0^8 + 32c_1c_2\left(w_0^2 - 2c_1c_2\right)z^2}}{4c_2kw_0^2}, \\ y_1 = \dfrac{(c_1 + c_2)\left(4c_1c_2 - w_0^2\right)}{4c_1c_2} - \dfrac{(c_2 - c_1)\sqrt{k^2w_0^8 + 32c_1c_2\left(w_0^2 - 2c_1c_2\right)z^2}}{4c_1c_2kw_0^2}, \end{cases} \quad (6)$$

and

$$\begin{cases} x_2 = \dfrac{\left(4c_1c_2 - w_0^2\right)}{4c_2} - \dfrac{\sqrt{k^2w_0^8 + 32c_1c_2\left(w_0^2 - 2c_1c_2\right)z^2}}{4c_2kw_0^2}, \\ y_2 = \dfrac{(c_1 + c_2)\left(4c_1c_2 - w_0^2\right)}{4c_1c_2} + \dfrac{(c_2 - c_1)\sqrt{k^2w_0^8 + 32c_1c_2\left(w_0^2 - 2c_1c_2\right)z^2}}{4c_1c_2kw_0^2}. \end{cases} \quad (7)$$

When $0 < c_1c_2 < w_0^2/2$, the solutions (6) and (7) are valid for $z > 0$, when $c_1c_2 < 0$ or $c_1c_2 > w_0^2/2$, the solutions (6) and (7) are valid for $0 < z < kw_0^4/4[2c_1c_2(2c_1c_2 - w_0^2)]^{1/2}$. When $c_1c_2 = w_0^2/2$, the solutions (6) and (7) are simplified to

$$\begin{cases} x_1 = \dfrac{w_0^2}{2c_2}, \\ y_1 = \dfrac{w_0^2}{2c_2}. \end{cases} \quad (8)$$

and

$$\begin{cases} x_2 = 0, \\ y_2 = \dfrac{w_0^2}{2c_1}. \end{cases} \quad (9)$$

The solutions (8) and (9) are valid for $z > 0$.

(ii) $c_1 = 0$ or $c_2 = 0$

The solution of Eq. (5) is

$$\begin{cases} x = \dfrac{-w_0^2}{2c_2}, \\ y = \dfrac{4c_2 z^2}{k^2 w_0^4} + c_2 - \dfrac{w_0^2}{2c_2}, \end{cases} \quad \text{for} \quad c_1 = 0, \tag{10}$$

or

$$\begin{cases} x = c_1 + \dfrac{4c_1 z^2}{k^2 w_0^4}, \\ y = \dfrac{4c_1 z^2}{k^2 w_0^4} + c_1 - \dfrac{w_0^2}{2c_1}. \end{cases} \quad \text{for} \quad c_2 = 0. \tag{11}$$

(iii) $c_1 = c_2 = 0$

Equation (5) can be rewritten as

$$\begin{cases} k^2 w_0^2 \left(x^2 - xy\right) - 2z^2 = 0, \\ z = 0. \end{cases} \tag{12}$$

Thus, there do not exist phase singularities at the plane $z > 0$.

3. Some special cases

Two special cases are of interest.

(i) Two edge dislocations are parallel to one coordinate axis (say, x axis) which, for simplicity, are called two parallel edge dislocations.

Equation (1) reduces to

$$E_0(x_0, y_0, 0) = \left(\frac{x_0 - c_1}{w_0}\right)\left(\frac{x_0 + c_2}{w_0}\right)\exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right). \tag{13}$$

In free-space propagation the field at the z plane is expressed as

$$\begin{aligned} &E(x, y, z) \\ &= \frac{2\pi}{\left(kw_0^2 + 2z\mathrm{i}\right)^3 \lambda} \exp\left[\mathrm{i}kz - \frac{k\left(x^2 + y^2\right)}{kw_0^2 + 2z\mathrm{i}}\right] \\ &\times \Big\{ \left[k^2 w_0^4 (x - c_1)(x + c_2) - 2z^2\left(w_0^2 - 2c_1 c_2\right)\right] \\ &+ \mathrm{i}kw_0^2 z\left[w_0^2 + 2(c_2 - c_1)x - 4c_1 c_2\right] \Big\}. \end{aligned} \tag{14}$$

The positions of phase singularities are determined by

$$\begin{aligned} &k^2 w_0^4 (x - c_1)(x + c_2) - 2z^2\left(w_0^2 - 2c_1 c_2\right) = 0, \\ &z\left[w_0^2 + 2(c_2 - c_1)x - 4c_1 c_2\right] = 0. \end{aligned} \tag{15}$$

It is readily seen that Eq. (15) has no solution. Therefore, phase singularities vanish if two edge dislocations are parallel at the plane $z = 0$.

(ii) Two edge dislocations are perpendicular to the x axis or y axis, respectively, which are called two perpendicular edge dislocations.

Equation (1) becomes

$$E_0(x_0, y_0, 0) = \left(\frac{x_0 - c_1}{w_0}\right)\left(\frac{c_2 - y_0}{w_0}\right)\exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right). \tag{16}$$

The field at the z plane reads as

$$\begin{aligned} &E(x, y, z) \\ &= \frac{-2\pi}{\left(kw_0^2 + 2z\mathrm{i}\right)^3 \lambda} \exp\left[\mathrm{i}kz - \frac{k\left(x^2 + y^2\right)}{kw_0^2 + 2z\mathrm{i}}\right] \\ &\times \Big\{ \left[k^2 w_0^4 (c_1 - x)(c_2 - y) - 4c_1 c_2 z^2\right] \\ &+ 2\mathrm{i}kw_0^2 z(2c_1 c_2 - c_2 x - c_1 y) \Big\}. \end{aligned} \tag{17}$$

The positions of phase singularities are determined by

$$\begin{cases} k^2 w_0^4 (c_1 - x)(c_2 - y) - 4c_1 c_2 z^2 = 0, \\ z(2c_1 c_2 - c_2 x - c_1 y) = 0. \end{cases} \tag{18}$$

Equation (18) indicates that there is an edge dislocation $x = 0$ or $y = 0$ at the plane $z > 0$ if $c_1 = 0$, or $c_2 = 0$, respectively. For $c_1 = c_2 = 0$ there exist two edge dislocations $x = 0$ and $y = 0$ at the plane $z > 0$, whereas for $c_1 \neq 0$ and $c_2 \neq 0$ there do not exist phase singularities at the plane $z > 0$.

4. Numerical calculation results and discussions

Numerical calculations were performed to illustrate the above theoretical results. In the following calculations $\lambda = 1.06 \times 10^{-3}$ mm and $w_0 = 1$ mm are kept fixed.

Figure 1(a) gives contour lines of phase at the plane $z = z_{\mathrm{R}}$ ($z_{\mathrm{R}} = kw_0^2/2$-Rayleigh range) of a Gaussian beam carrying two edge dislocations with $c_1 = c_2 = 0.2$ mm. As can be seen, in free-space propagation edge dislocations disappear, and two vortices are presented due to the interaction of two edge dislocations. Their topological charge is equal to $m = +1$ and $m = -1$, respectively in accordance with the topological sign principle proposed by Freund *et al.*[18] The contour lines of phase near the vortices in Fig. 1(a) show that two vortices are noncanonical because the phase increment around the two vortices is not linearly proportional to the azimuthal angle.[19] From the corresponding three dimensional (3D) trajectory of vortices in Fig. 1(b) we see that there are two vortices with $m = +1$ (black dots) and -1 (open dots) at the plane $z > 0$ because the parameters c_1, c_2 and w_0 satisfy the condition $0 < c_1 c_2 < w_0^2/2$. Additionally in free-space propagation ($z > 0$) the total topological charge is equal to 0 and conserved.

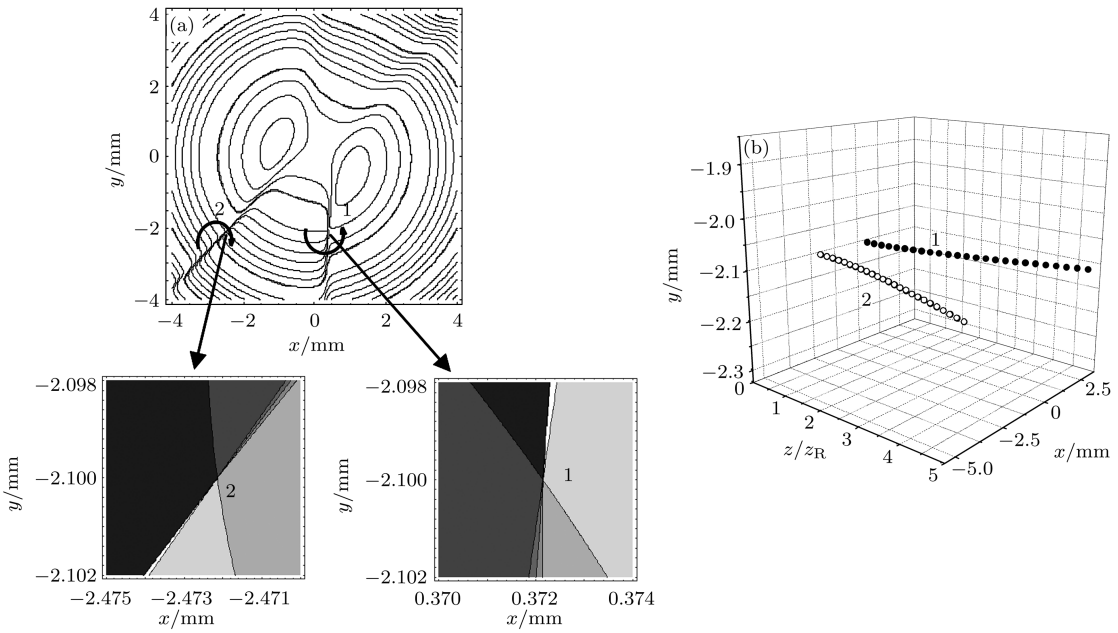


Fig. 1. (a) Contour lines of phase at the plane $z = z_{\rm R}$ for $c_1 = c_2 = 0.2$ mm. (b) the corresponding 3D trajectory of vortices, '•' $m = +1$, '○' $m = -1$. The calculated parameters are seen in the text.

The 3D trajectory of vortices for $c_1 = 0.2$ mm, $c_2 = -0.2$ mm satisfying $c_1 c_2 < 0$ is plotted in Fig. 2, from which we see that in the range $0 < z < 1.702 z_{\rm R}$, there exist two noncanonical vortices with $m = +1$ and -1, respectively (contour lines of phase are omitted for saving the length), which approach each other as the propagation distance increases. At $z = 1.701 z_{\rm R}$, their positions are (1.4500 mm, –0.0001 mm) and (1.4501 mm, 0.0001 mm) respectively, and the distance between them is equal to $\Delta = 0.94\lambda$, resulting in the subwavelength structure.[20] Finally, with a further increase of $z \geq 1.702 z_{\rm R}$, two oppositely charged vortices annihilate each other, and there do not exist phase singularities at the plane $z \geq 1.702 z_{\rm R}$.

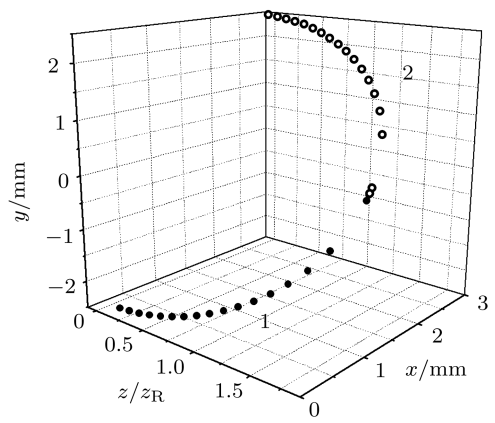


Fig. 2. The 3D trajectory of vortices for $c_1 = 0.2$ mm and $c_2 = -0.2$ mm. '•' $m = +1$, '○' $m = -1$.

The 3D trajectory of vortices for $c_1 = 0.6$ mm and $c_2 = 1$ mm satisfying $c_1 c_2 > w_0^2/2$ is depicted in Fig. 3. It is seen that in the region $0 < z < 1.021 z_{\rm R}$ two noncanonical vortices with $m = +1$ and -1 are presented, respectively, and for $z \geq 1.021 z_{\rm R}$ they disappear, and their dynamical evolution behaviour is similar to Fig. 2.

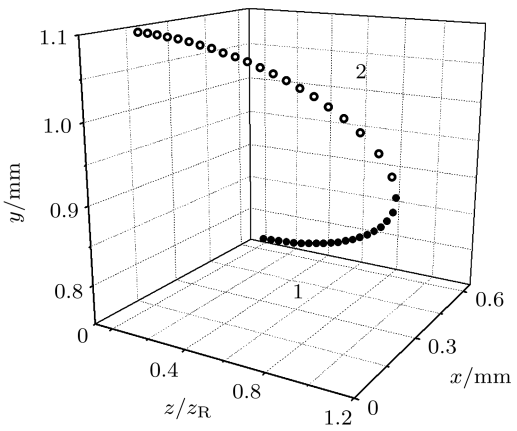


Fig. 3. The 3D trajectory of vortices for $c_1 = 0.6$ mm and $c_2 = 1$ mm. '•' $m = +1$, '○' $m = -1$.

The 3D trajectory for $c_1 = 0.5$ mm and $c_2 = 1$ mm is represented in Fig. 4 which indicates that there exist two noncanonical vortices with $m = +1$, -1, respectively, whose positions (0.5 mm, 0.5 mm) and (0, 1 mm) are invariant upon propagation ($z > 0$) because the condition $c_1 c_2 = w_0^2/2$ is fulfilled.

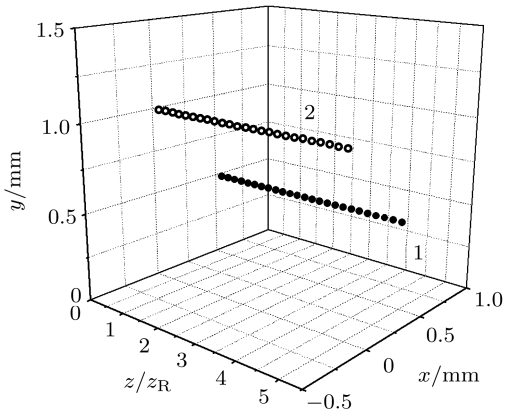


Fig. 4. The 3D trajectory of vortices for $c_1 = 0.5$ mm and $c_2 = 1$ mm. '•' $m = +1$, 'o' $m = -1$.

Figure 5 gives the 3D trajectory of a vortex for $c_1 = 0.2$ mm and $c_2 = 0$, from which we see that a noncanonical vortex with $m = +1$ is presented at the plane $z > 0$. Similarly, it can be shown that there exists a vortex with $m = -1$ at the plane $z > 0$ if $c_1 = 0$ and $c_2 = 0.2$ mm. Therefore, for the two cases the total topological charge is not conserved in comparison with that at the plane $z = 0$, where the total topological charge is equal to zero.

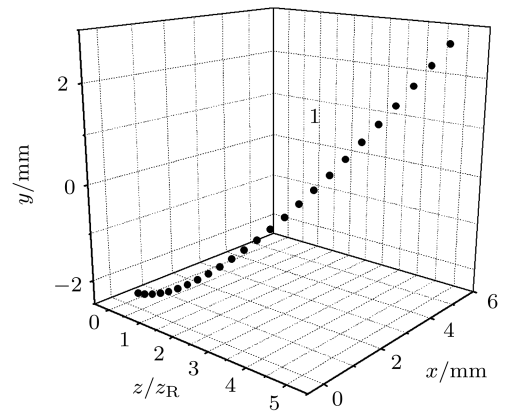


Fig. 5. The 3D trajectory of a vortex for $c_2 = 0$ and $c_1 = 0.2$ mm. '•' $m = +1$.

5. Concluding remarks

In this paper, the interaction of two edge dislocations nested in a Gaussian beam propagating in free space has been studied analytically and numerically. It shows that in free-space propagation the edge dislocations are unstable and vanish, and two noncanonical vortices with opposite topological charges $m = \pm 1$ take place when off-axis distances c_1 and c_2 of two edge dislocations are not equal to zero, and $k^2 w_0^8 + 32 c_1 c_2 (w_0^2 - 2 c_1 c_2) z^2 > 0$. For such a case the total topological charge remains unchanged upon propagation. A noncanonical vortex with topological charge $m = +1$ or -1 appears when one off-axis distance c_1 or c_2 is equal to zero and the total topological charge is not conserved upon propagation in comparison with that at the plane $z = 0$. There do not exist phase singularities for $c_1 = c_2 = 0$, or for two parallel edge dislocations, or two perpendicular edge dislocations with $c_1 \neq 0$ and $c_2 \neq 0$.

However, one edge dislocation is stable when two edge dislocations are perpendicular and $c_1 = 0$ or $c_2 = 0$. Two perpendicular edge dislocations with $c_1 = c_2 = 0$ are also stable. In addition, if there is only one edge dislocation nested in a Gaussian beam whose field at the plane $z = 0$ is

$$E_0(x_0, y_0, 0) = \left(\frac{x_0 - y_0 + c}{w_0}\right) \exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right), \qquad (19)$$

it can be readily seen that in free-space propagation the edge dislocation $x - y = 0$ is stable when $c = 0$, and there are no phase singularities when $c \neq 0$. The results obtained in this paper would be useful for understanding the dynamic evolution and their interaction of edge dislocations in free space and for their potential applications.

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