Dynamics of Bose–Einstein condensate in a harmonic potential and a Gaussian energy barrier*

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We have studied the dynamics of Bose–Einstein condensate by solving numerically the Gross-Pitaevskii (GP) equation. We examined the periodic behaviour of the condensate in a shifted harmonic potential, and further demonstrated the tunneling effect of a condensate through a Gaussian energy barrier, which is inserted after the condensate has been excited by shifting the harmonic trapping potential to a side. Moreover, it is shown that the initial condensate evolves dynamically into two separate moving condensates after the tunneling time under certain conditions.

Keywords: Gross–Pitaevskii equation, Gaussian energy barrier, tunneling effect
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The creation of Bose–Einstein condensate (BEC) provides a platform for investigating many important phenomena in atomic physics.[1] Recently dynamics of Bose–Einstein condensate has received intense attention and many new and important research results have been reported. These results concern the macroscopic quantum interference,[2−4] coherent quantum atomic tunneling effect,[5,6] the soliton in condensate,[7,8] and the nonlinear effects and chaotic behaviour in BEC.[9,10] However in probing the property of this new state of matter, how to properly solve the Gross–Pitaevskii (GP) equation numerically is essential.[11−13]

In this paper, we solve the one-dimensional (1D) time-dependent GP equation numerically by a symplectic method and carry out some numerical investigation about the dynamics of BEC in a harmonic trapping potential. First we excite the condensate by shifting the harmonic trapping potential to a side, and study the behaviour of the condensate. Secondly we remove the trapping potential suddenly, followed by inserting a Gaussian energy barrier, and then study the tunneling phenomena between the condensate and the barrier. It is shown that two separately moving condensates can be obtained in this tunneling process under certain conditions. This phenomenon is interesting because it provides a method of obtaining two separatedly running condensates which can be employed in studying interference effect. Interference, along with the tunneling effect, is one of the basic tools in studying quantum phenomenon. Usually, interference in BEC is studied through two condensates released from two traps. Therefore our finding provides a way to study interference effect between two separatedly running condensates and thus is of practical relevance.

The 1D time-dependent GP equation for neutral atoms trapped in a harmonic potential near zero temperature can be written as[2,3]

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\frac{1}{2}m\omega^2x^2+U_0N\left|\psi\left(x,t\right)\right|^2\right]\psi\left(x,t\right)=\mathrm{i}\hbar\frac{\partial\psi\left(x,t\right)}{\partial t}. \tag{1}$$

Here $U_0=4\pi\hbar^2a_{\mathrm{s}}/ma^2$ is the strength of interatomic interaction, with a_{s} denoting the s-wave scattering length of the atom; m denoting the mass of a single atom; $a\equiv\sqrt{\hbar/2m\omega}$ is the size of the harmonic trapping potential, ω is the oscillation frequency of the trapping potential, and N is the number of atoms in the condensate. The wavefunction of the condensate is normalised to unity. With $\xi=x/a$, $\tau=\omega t$, and $\Phi\left(\xi,\tau\right)=\psi(x,t)\sqrt{a}$, Eq. (1) can be rescaled into a dimensionless form

$$\left[-\frac{\partial^2}{\partial\xi^2}+\frac{\xi^2}{4}+\alpha\left|\Phi(\xi,\tau)\right|^2\right]\Phi\left(\xi,\tau\right)=\mathrm{i}\,\frac{\partial\Phi\left(\xi,\tau\right)}{\partial\tau}, \tag{2}$$

where $\alpha=8\pi Na_{\mathrm{s}}/a$ is the nonlinear coefficient. Our computation is performed for ^{133}Cs with $m=133$ amu, $a_{\mathrm{s}}=3.45$ nm, $\omega=2\pi\times10$ Hz. For about $N=500$, $\alpha=8\pi$.

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The normalisation condition of the wavefunction is $\int_{-\infty}^{\infty} |\psi(x,t)|^2 \,\mathrm{d}x = \int_{-\infty}^{\infty} |\Phi(\xi,\tau)|^2 \,\mathrm{d}\xi = 1$. Here $\Phi(\xi,\tau) = \Phi(\xi)\exp(-\mathrm{i}\beta\tau)$, $\beta = \mu/\hbar\omega$, and μ is the chemical potential. Since the wavefunction vanishes at boundaries, we adopt the zero boundary condition

$$\Phi(-\infty,\tau) = 0, \quad \Phi(\infty,\tau) = 0. \tag{3}$$

We choose the initial condition to be

$$\Phi(\xi,0) = (2\pi)^{-1/4}\exp(-\xi^2/4). \tag{4}$$

Equation (2) can be discretized into a finite-dimension nonlinear Hamiltonian canonical equation, thus we can apply the symplectic method to solve it.[14] We obtain the ground state wavefunctions and the corresponding eigenvalues by symplectic Euler-centre scheme through a procedure suggested in Ref. [15]. The stability of the wavefunctions is tested by dynamic evolution.[16] Figure 1 shows the trajectory in phase space $(R, \mathrm{d}R/\mathrm{d}\tau)$, where

$$R = <\xi^2> = \int_{-\infty}^{\infty} \xi^2 |\Phi(\xi,\tau)|^2 \,\mathrm{d}\xi \tag{5}$$

is the root mean square (rms) radius of the wavefunction, and $\mathrm{d}R/\mathrm{d}\tau$ is the derivative of R with respect to time τ. The breathing mode oscillation is simulated by suddenly multiplying the harmonic oscillator term by a factor of 2 or 0.5.[11] The 10^6 iterations with time step $\Delta\tau = 0.001$ have been computed. It can be seen from Fig. 1 that the trajectories are clean circles suggesting exactly periodic motions of the condensate. It is assured that our method is reliable.

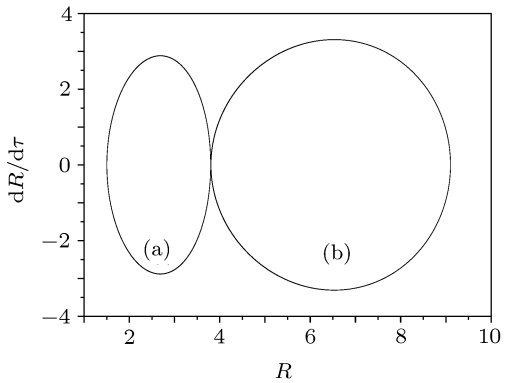


Fig. 1. Phase trajectories in phase space $(R, \mathrm{d}R/\mathrm{d}\tau)$ of the condensate with $\alpha = 8\pi$, initiated at $\tau = 0$ by suddenly (a) increasing or (b) decreasing the harmonic oscillator term by a factor of 2.

First we simulate the motion of BEC in the harmonic trapping potential. We choose the wavefunction of $\alpha = 8\pi$, and the eigenvalue is calculated to be about $4.51(\hbar\omega)$. At time $\tau = 0$ we shift the trap in the ξ direction by a distance of $\Delta\xi$, in coordinate it is equivalent to displace the condensate along ξ by $-\Delta\xi$. The latter is used in our representation for convenience of expression. After the displacement, the condensate is away from the centre of the trap and therefore is expected to perform oscillations about the minimum of the harmonic trap.[17] We present the contour plot of the case of displacing the condensate by $\Delta\xi = -25$ in Fig. 2. It can be seen that the condensate presents periodic motion about $\xi = 0$ with an amplitude $25a = 48$ μm, and period $T = 6.319(\omega^{-1})$. When $\alpha > 0$, the interaction between atoms is repulsive, then this period is larger than 2π. It is a harmonic oscillator. The force on the condensate is $F = -m\omega^2\xi$.

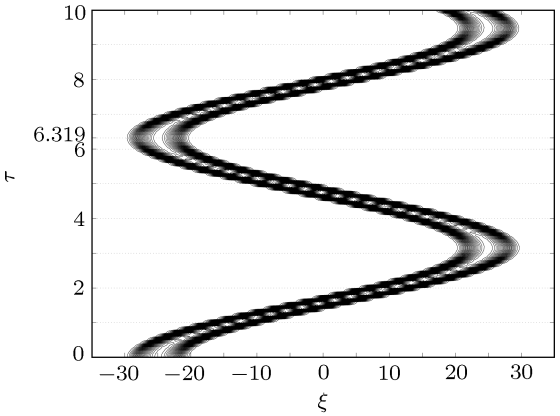


Fig. 2. Contour plot of the probability density of the condensate versus ξ and τ. When the condensate of α=8π is formed at time $\tau = 0$, displace the condensate from the centre of the trap in ξ direction by a distance of $\Delta\xi = -25$. Length in unit of a, time in unit of ω^{-1}, density in unit of a^{-1}.

Second, we study the tunneling effect of a condensate on a Gaussian barrier.[18] After the condensate has been excited from the centre of the trap by a displacement of $\Delta\xi$ at $\tau = 0$, we hold the trap for a time τ_{hold} to keep the condensate moving. Then if we remove suddenly the trap at $\tau = \tau_{\mathrm{hold}}$, the condensate will expand ballistically and move with an initial momentum simultaneously. However, if we remove the trap at $\tau = \tau_{\mathrm{hold}}$ followed by inserting a Gaussian energy barrier centred at $\xi = 0$ at the same time, tunneling effect may happen when the condensate reaches the barrier region. The reduced form of the potential barrier we used is[18,19]

$$V(\xi) = b\mathrm{e}^{-c\xi^2}. \tag{6}$$

We choose the height of the barrier $b = 25$ $(\hbar\omega)$, and $c = 1$ (a^{-2}) which controls the width of the barrier.

Different displacement $\Delta\xi$ and holding time $\tau_{\rm hold}$ will result in different phenomenon. The results are interesting and we present the time evolution of the probability density of the condensate in Figs. 3(a)–3(d) for fixed $\Delta\xi = -25$ and different $\tau_{\rm hold}$. At the time of $\tau_{\rm hold} = 16T/32$, as shown in Fig. 3(a), the condensate reaches its maximum displacement. After being released from the trap, the condensate just expands ballistically along ξ axis. Figure 3 (b) is for $\tau_{\rm hold} = 17T/32$, at this release time the condensate gains some kinetic energy, and it moves towards $\xi = 0$ during expanding. But its kinetic energy is small compared with the Gaussian energy barrier, the condensate cannot tunnel through the barrier but reflected back to interfere with the incident wavefunction. Figure 3(c) is for $\tau_{\rm hold} = 18T/32$. At this time the condensate gains more kinetic energy, and it is shown that part of the condensate tunnels through the barrier. Interference can be seen between the reflected wavefunction and the incident wavefunction. We notice that the interference pattern dies down after the tunneling time and a small wave forms, and this phenomenon is more evident in Fig. 3(d) for $\tau_{\rm hold} = 19T/32$.

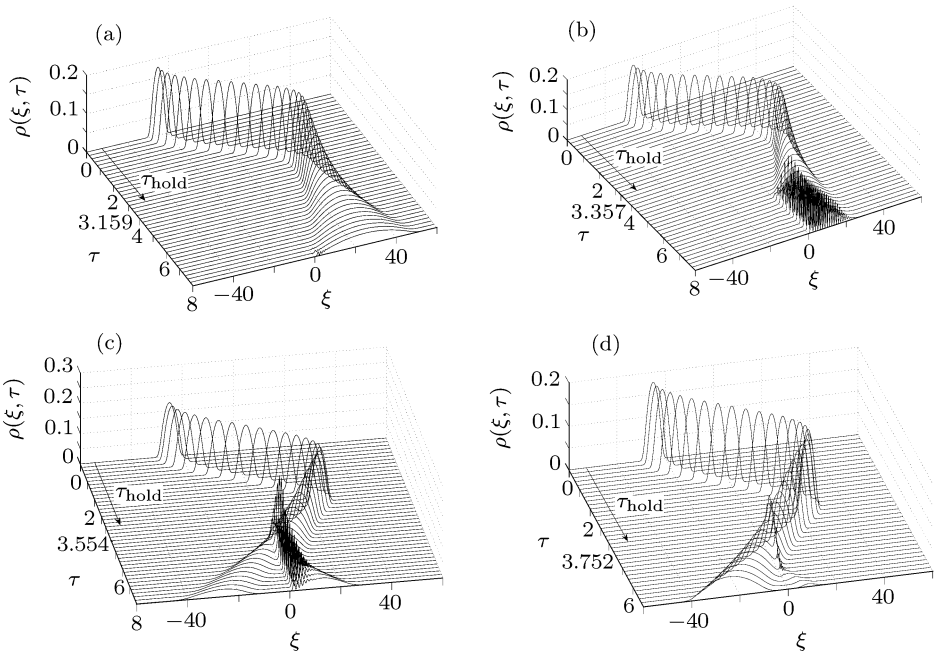


Fig. 3. Evolution of the probability density of the condensate with reduced time. At time $\tau = 0$, the initial wavefunction of $\alpha = 8\pi$ is displaced from the centre of the trap in ξ direction by $\Delta\xi = -25$. The trap potential is held for a time $\tau_{\rm hold}$. At $\tau = \tau_{\rm hold}$ the trap is removed and a Gaussian barrier $V(\xi) = 25{\rm e}^{-\xi^2}$ centred at $\xi = 0$ is inserted. The period $T = 6.319$. (a) $\tau_{\rm hold} = 16T/32 = 3.159$, (b) $\tau_{\rm hold} = 17T/32 = 3.357$, (c) $\tau_{\rm hold} = 18T/32 = 3.554$, (d) $\tau_{\rm hold} = 19T/32 = 3.752$. Length in unit of a, time in unit of ω^{-1}, density in unit of a^{-1}.

Noticing that a small wave forms after the tunneling time in Figs. 3(c) and 3(d), we may obtain two running separated condensates by this method. We choose a time in-between (c) and (d), $\tau_{\rm hold} = 3.653$, and the probability density of the condensate at different time during evolution is shown in Fig. 4. When $\tau = 3.653$ (dot–dashed line), the condensate has just been released from the harmonic trap. It maintains the shape of the initial wavefunction of $\alpha = 8\pi$ but with an momentum towards $\xi = 0$. At $\tau = 6$ (dotted line), part of the condensate tunnels the barrier and interference phenomenon is prominent. At $\tau = 7$ (solid line), the interference is weak and two smaller condensates show their initial appearance. At $\tau = 8$ (dashed line), two completely separated condensates can be identified, and they go apart from each other. However, these two condensates obtained are flat. If two 'peaked' condensates are wanted, we can insert a higher Gaussian barrier at $\tau = \tau_{\rm hold}$ without removing the harmonic trap, then the initial condensate will not expand ballistically during evolution. By modulating the evolution time properly, it is expected that the two condensates coming towards each other can be obtained. Such phenomenon may have important applications in studying the phase coherence or the interaction between the condensates, in addition to providing insights into the nature of quantum tunneling.

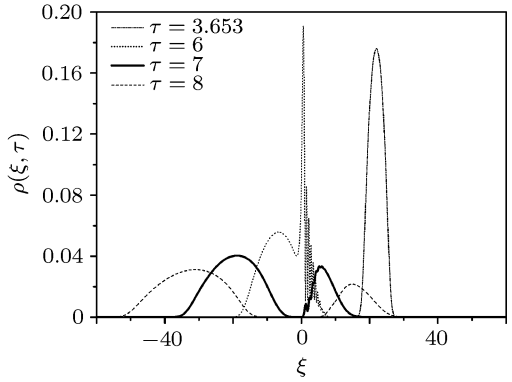


Fig. 4. Probability density of the condensate at different time during evolution, $\tau_{\rm hold} = 3.653$. The other parameters are identical to those for Fig. 3. Dot-dashed line is for $\tau = 3.653$, dotted line for $\tau = 6$, solid line for $\tau = 7$ and dashed line for $\tau = 8$. Length in unit of a, time in unit of ω^{-1}, density in unit of a^{-1}.

In conclusion, we have studied the dynamics of Bose–Einstein condensate by solving the 1D time-dependent GP equation with harmonic trapping potential. The stability of the ground state wavefunction obtained is tested by the breathing mode oscillator simulation. It is shown that the condensate behaves like a harmonic oscillator when the trapping potential is shifted to a side. Tunneling effect is demonstrated when we remove the shifted trapping potential followed by insearting a Gaussian energy barrier. It is shown that two moving separated condensates can be obtained after the tunneling time under certain conditions.

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